Signal Processing for mmWave Communication for 5G and Beyond
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Module - 09
Hybrid beamforming concept and Beamforming in MIMO
Lecture - 45
MIMO Beamforming in Receiver side (part-II)

Welcome to millimeter Wave Communication for 5G and beyond. So, last time we have talked about the equalizer part at the receiver for the MIMO case for the single case single meaning it is a SISO case, but multiple antenna. Now, we will be talking more on the. So, today we will be covering the more on the what happens when I go to the MIMO case how the equalizer will be based on ok.

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So, things that we will be covering are the following.

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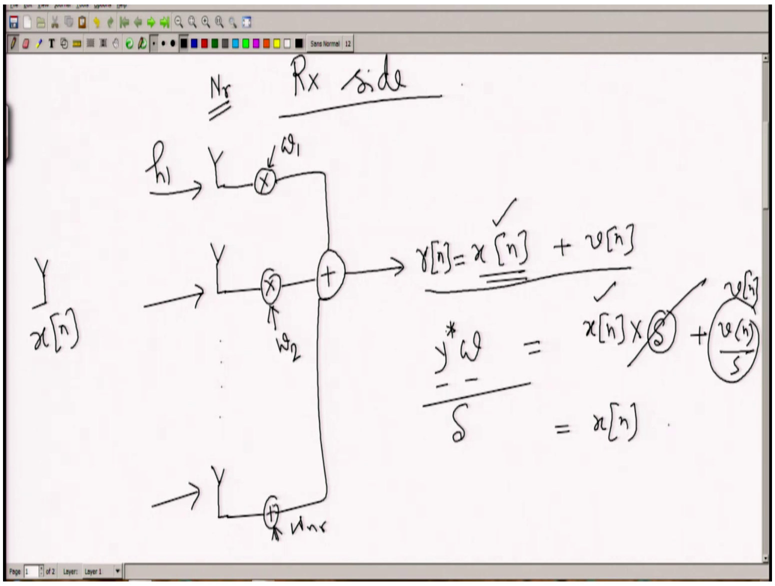

So, I am just redrawing the Rx side ok. So, now, you have some multiple antenna. Probably, I will let me draw few more ok and this is a N r number of antennas are present here ok. And so, what we said last time that whatever channel comes into picture here, if it is assumed I am just sending from one antenna h 1. So, what will happen, then it is nothing but you can create your own equalizers.

So, one of the equalizer technique is like, if I want to use MRC it can also be LMMSE whatever there is no choice it is up to you, but it is some sort of an equalizer. So, let us generalize that concept. So, you can have equalizer here you can have an equalizer here right. So, this is what we have explained it last time. And once the equalizer is built, then what we have to do? We have to combine them together to get the result. Again, it depends on the data model, but usually if it is a single input SISO case, this is what the system. So, this will be your equalized data.

So, this would be more of a I would say r is equal to if I would have sent x n I would say x n plus some noise. So, this is what my data is. This is what my data model is right. This is for the single antenna case single antenna in the sense single data case not the single antenna of course, multi antenna system, but single data case.

Now, think when you have the MIMO case just like your t x you just add them up. Here also, I will do very similar approach. Now, this is one part of the story. Now, when it is a MIMO, which means that it is no longer just one channel right one channel will be coming into picture, because there may be multiple threads or data this kind of data will come into picture it is not just one data.

Now, when you have this kind of scenario, this is now it can be I am just generalizing it here in this case. So, w 1 this would be some w 2 and this would be some W n whatever. So, if there are n number of equalizer or r n, n r number of equalizer. So, this can be the case.

Now, what happens when I have MIMO. So, exactly when I have the MIMO, I cannot just simply take multiple things right, because here, if you look at here only it is possible when the sent data is just one it is easy to do it right, but when I have the whole thing in a in more of a you know in a MIMO system. So, what will happen is that up to first take some of the effect out from the channel in the MIMO context.

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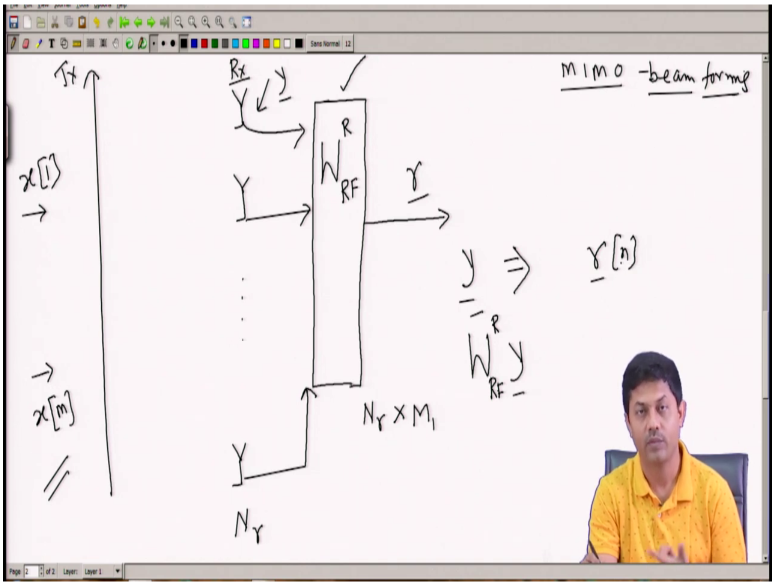

So, in the case of MIMO, the similar thing will be happening here, but there is a cache. So, I still have N r number of antennas, because number of antenna may not change at the receiver just like your transmitter is just the number of feeds or the digital feeds that will be changed right. So, that is the in the case of MIMO. So, this is in the case of MIMO beamforming. Now, I am at the receiver side ok. This is at the at that stage kind of things.

So, here as I said I cannot just use MRC because MRC will not work, because here number of digital data feeds itself are more. So, I can have probably s 1 here or. In fact, instead of s 1, I have written it is x 1 right, because that may be your digital data feed. So, that can be M then it goes from the t x side this is the case right. So, this is from the t x side and then, it goes to the beamforming in between and then goes to the R x side. So, this is my R x side ok.

Now, here what I do? I do a slightly different way. So, here I may have the LNA and all. So, let me not draw the LNA part, because this is more of an implicit part. So, what I am trying to say is that this N r I will send it through some sort of a RF matrix kind of multiplier ok. So, this I will send it here this I will send it here this I will send it here ok. And you can have your own dimension of the data.

So, let us say I am making a dimension N r cross. You can have say M 1 we can have M 1 also now M 1 can be N r equivalent and it is up to you how do you want to get. Now, this I call it W RF which I did it in the t x side also, there was a W RF which was more of a beamforming part. Here also, I do a similar set of combining.

So, what happens? So, basically first you do combine the data such that you optimize your s N r or optimize some of the cost function, then you go for a detection that is the. I mean in the sense in case of single thing it was not so prominent though it was implicit. For example, the output of what is the output of equalizer? This is not the exact output of my equalizer, because there is more to it if you look at the way it is it was y vector and it was multiplied by some sort of the equalizer vector ok.

Then it is not exactly coming out to be an x n rather it will be some sort of a x n plus some value will be there may be some constant or something plus v dash n, because this equalizer will have the additive and then some adjustable things are there. Now, as it is single this was not an important thing, because I can always divide this whole thing. So, you can say whatever. So, let us call it some s constant. So, you can always divide it and make this fellow this I call it a new v n.

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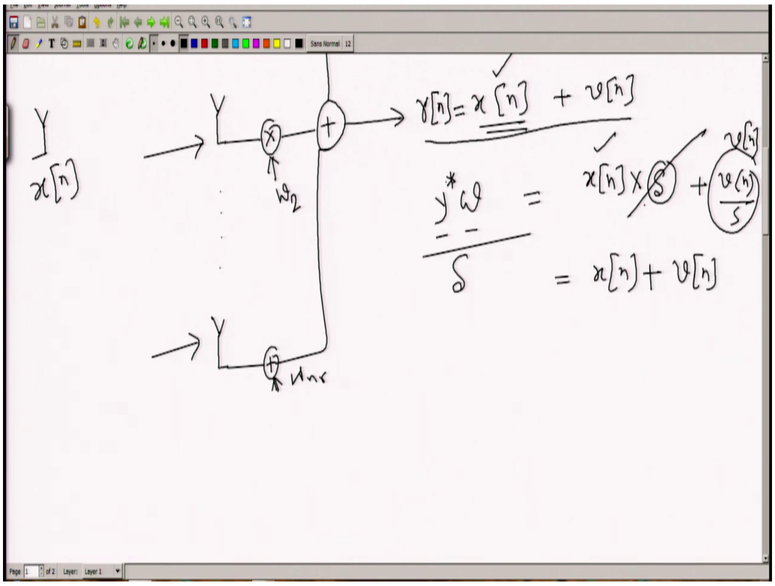

So, it is as if like x n plus v n sorry it is as if like x n plus v n. So, this factor was there. Now, this for single case it was very easy to handle, but when you go for a multiple data stream, then you cannot simply take out the that particular thing. So, here W RF I call it R part just to indicate that this is at the receiver side. So, this was more of a combining part ok. So, you have a multiple data stream and then you combine them with some matrix. So, I would say this is some sort of a matrix ok.

Now, moment that goes into a matrix. Now, this was my r vector ok. Now, what happen? So, at the end I was receiving y vector and I was getting r vector from there how I get the r vector. I multiply the y vector with this matrix. Now, this is purely in the RF domain.

Why this is in the RF domain? Because in the single case also, I have shown a part of this can be RF, because the equalizer I can just put it as a phase shifter you have seen it right in some of the earlier classes, this equalizer it has a amplitude part, it has a phase part the amplitude part I push it to the LNA and phase, but I just implement it.

Similar concept I will put it here as well ok that it is a combined thing. So, the it will include that itself will include l n a you know phase shifter and all these things. So, this is some sort of a R F matrix the completely it is in the R F domain.

Now, once I get this r you can notice that I have not written r n ok, because I have not digitized it. So, this was more. Now, we will explain it more how this can be you know estimated or this can be found out.

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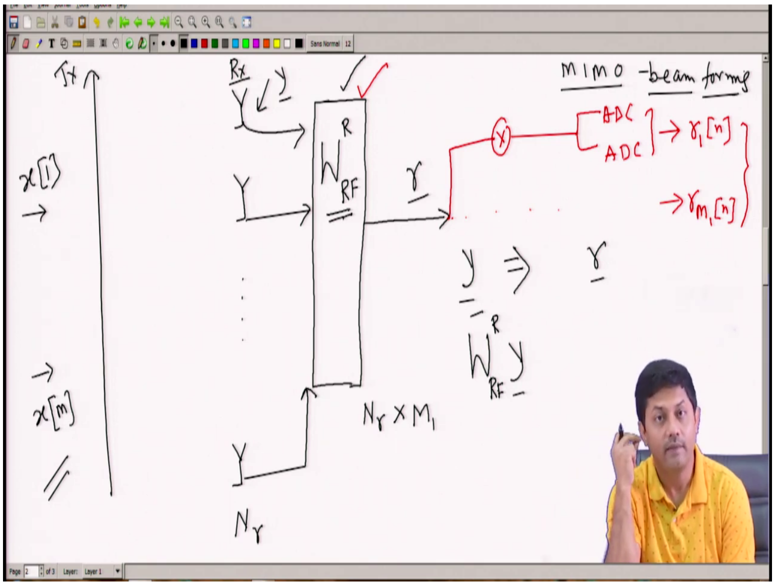

Now, this r you can this is a vector now from here you can have you know you can have your normal A D you can have your you know demodulator and ADC you can have your ADC here and then, you can create actual r 1 n and similarly, I will get it r M 2 n. If there are M 1 number sorry this would be M 1; this sort of things I can get it.

So, earlier what is the difference between the earlier single case and multiple case multiple antenna case. Earlier, this W RF was nothing but one vector ok, but now it is no longer just one vector rather it will be multiple level of such vector ok. So, in the earlier case what happens, let us do some more surgery into the exact structure of your W RF. If it is a single data stream what would have been the case?

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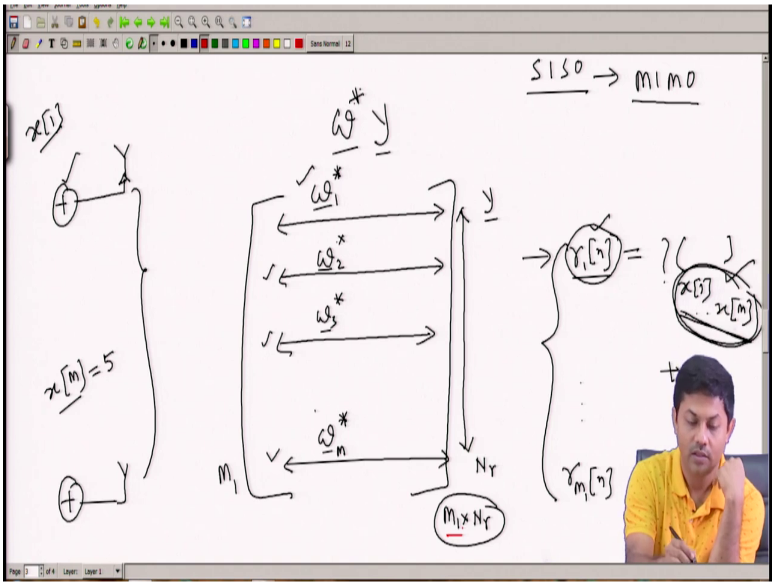

In the single data stream I was multiplying this y vector with the equalizer vector right. So, either you can say y bar or. So, w star vector. This was the case right earlier that is how the case. So, which means that I have one for example, I have one w star vector here and then I multiply with my y vector it is not a single case. From here, I was getting that r and then I digitize it I get r n. This is what the case right.

Now, because now it is a MIMO case, what will happen. It is no longer just one w will be acting why? In the t x if you remember, how the feeds are there. If you look at the in the t x side, it there was an adder right there was an adder here. So, which means that each and every data is each and every data from one antenna is already having multiple you know multiple digital data stream thread.

So, if I just use one w I cannot write r n is equal to what which data will come? Which is first data, second data, third data you do not know right, because there are x 1 to x M number of data's are present right. So, which data will be coming here, because every data is there right every antenna is transmitting weighted sum of everything right.

So, that mean it will be some summation of x 1 to x M will be there; that mean from one thread if I just use one equalizer and this r n will be some weighted sum of x 1 to x M plus a noise, I cannot reconstruct x 1 to x M, because it is one observation and I have multiple input data. So, I cannot handle that.

So, what I do. I create multiple such vectors equalizer and that is the fundamental principle of equalizer; that mean instead of w I will say w 1 dash this will be w 2 dash then it will be w 3 dash and so and so forth.

I can have if this is M I can also have M number of such things no hardy or no harm in that or you can have your own. It should be at least more than this M more or equal to M, because if it is less then again, it will be more data compared to my observation.

So, the idea here is that how do I create more and more different sets of equalizer. So, that instead of just r I will get r 1. Instead of r I will get r M n. So, now, with this whole thing becomes a vector when I go for a matrix thing. So, that is the basic you know fundamental attire.

So, I hope you get my point here. So, if it is SISO to MIMO the transition is SISO to MIMO the transition is that I cannot use just one equalizer vector because one equalizer vector will always give you an output which is a linear combination of all your input data x 1 to x? Because if you look at the antennas all antenna will have x 1 to x M right.

If had it been SISO what is the difference this x 1, it will this will not be x 1 right because if the same x is fed to all the antennas. So, in that case this is one equalizer is fine right ok. But when it is multiple things what will happen? One equalizer and the multiplied by y will always give you a single r which is you know is a linear combinations of all my x n; that is precisely going to happen right, because this has everything.

So; obviously, whatever I decode what whatever I do that also contain everything. So, which means, that I need more observations for my data decoding suppose, I have five input vector say.

Let us say I am sending five data stream. So, can I just have one r 1 not possible right, because if from one output I will have that contains all x 1 to x 5. So, it is not possible to do. So, I need at least five number of minimum observation to get my x 1 and x. So, which means, that I need some sort of a different linear combination of x 1. So, that is the reason I have multiple such equalizer coming.

So, as a whole if I put it as a whole, I can call it some sort of a matrix. Suppose, if this length is N r and I have say, M 1 number of such things. So, this is more of a you know M 1 cross N r matrix I can think of. So, that is precisely this W RF is ok.

So, if it is a SIS case, this W RF is just a vector; a vector of equalizer, but moment it is a MIMO case, this W RF is no longer a vector. It is a set of vectors, because one vector multiplied by y it will always give you one observation; obviously, vector multiplied by another vector if it is inner product give me one observation.

Now, in that observation itself I have M number of you know independent unknown variable. So, one observation mine versus multiple you know inputs. So, I cannot decode such kind of things right so; obviously, I have to increase my observation.

So, what is the best way to increase my observation? The best way to increase my observation is that instead of this W RF being one vector, consider it as a matrix. That is all just like this fundamental principle that mean I will create multiple such W such that I get variation in my observation ok.

Moment I get a variation in observation what does it mean? It means that I will now have M 1 number of observation. From there, I have to get back my M number of input variables. So, this is the fundamental principle. So, in summary if I go back to my diagram in summary. So, what is my data model now coming up? Now, the question is that, how this W RF will be implemented?

Now, if you look at. It is like as if like I have parallel number of such w's. So, how did you implement one w it is LNA again multiplied by your phase shifters right. So, N r number of LNA adjustment multiplied by you know phase shifter. So, you have M such parallel things coming into picture. So, if I just summarize or if not summarize if I just kind of brief about what exactly this implementation would be.

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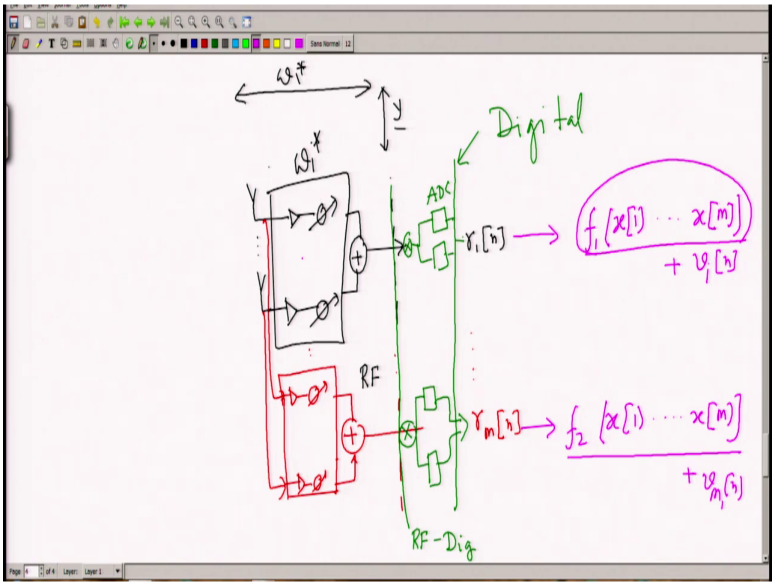

So, one w 1 star multiplied by you know y observation. What was the result? This was antenna this was the antenna and this was the antennas right. So, this was implemented as an LNA multiplied by a phase shifter this whole thing is your w 1 star right.

Now, after that what we do? I add them up. So, this is still inside my RF domain and then I go to the digital part. So, then you can put an ad demodulator and then ADC and you get digital data stream this is the part.

Now, what happens when I have multiple such thing? This such boxes will be repeated M times if the dimension of this matrix is M 1. So, I will have M 1 number of such boxes ok similar thing. So, how the data line so, this line will be fed here and this line will be fed here ok.

So, now again there will be LNA again there is a phase shifter again there is a LNA again there is a phase shifter and you have again adder. Now, this is again in the RF domain this. After that what will happen? You have your demodulator ADC converter and all these things and finally, I get my digital data here I do not get the digital data be very careful here.

So, in between there is a in between each and every line will go through demodulator then ADC, because I and q you will have. So, this becomes a complex number.

So, this is the case here also the same thing we have multiplier, we have an ADC; we have an ADC and that gives you RF. So, that is all it is. So, this is the RF to digital converter part. The output side is digital this precisely what happens ok and that is the point now. I am still not done ok.

Now, what I said? That each an individual r 1 each an individual r 1 will be what? Will be a function of it is a linear function of your actual data plus a noise. So, this part is there ok. Now, when it is a linear function, why it is a linear function? Because everything is a linear system why it becomes suddenly a non-linear equation it will not be it is a some you know some combination of x 1 to x m.

Here also, it will be some combinations. I will say some function here, but it is a linear function. Some combination of this is what is this is what is going to happen right. So, now, this whole thing would be my you know would be my data. So, now, plus some noise. So, if it is v 1 I will have v M 1 n this my noise part.

So, ultimately this part is there always, because that is the resultant part which is coming out even after multiply an equalizer, because equalizer will have you know residual components and all these things like some multiplication factor and those kind of things right. So, those will be anyway there you cannot take it off there. So, all these components will be presents here. So, basically this individual r n will be a linear function of my x 1 to x M system. So, if it is a linear function. So, so how do you visualize this r ok?

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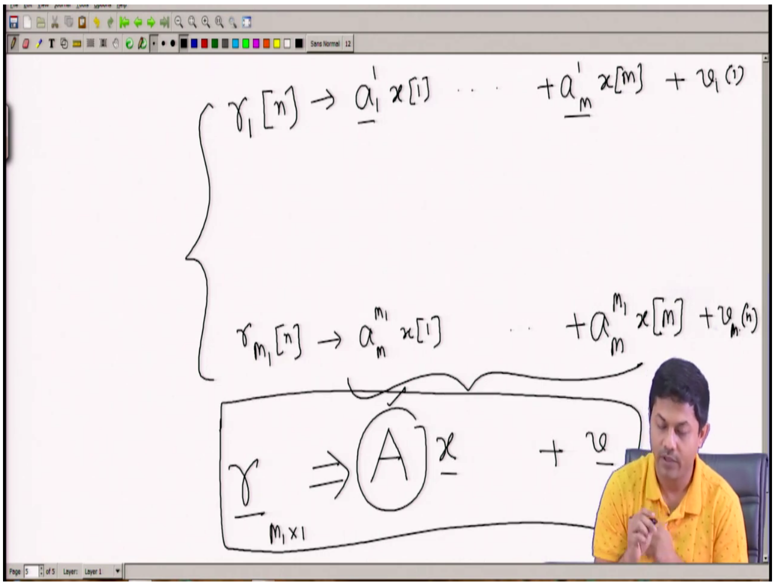

So, this is very important how do you visualize this r. Now, I can visualize this r as like this if r 1. What is that? I said it is a linear function of x 1 to x m; obviously, because that is the resultant of my equalizer multiplication ok. So, that mean I can say some you know some a 1 of x 1 plus you know a M of x M that is the linear function plus v 1.

Now, I really need not to know what exactly my a 1 and a M will be there, because it will be automatically coming up if I know the channel; obviously, if I know each and everything a 1 and a M everything will be known I will explain that one.

Now, what is the r M 1. Let us generalize this will be some you know a M 1 ok. I let me put a different notation here a dash sorry a 1 here. This should be a M 1 x 1. This will be automatically popping up. You really do not have to not this one. You really do not have to worry how exactly this a 1 and I will explain it what is this linear funda here. So, this will be v M 1 n ok.

Now, as a whole I can think of it. Now, this is a vector right. So, I can call it r vector what is its dimension M 1 cross 1. So, that is the dimension that r comes. What is this one if it is a linear combination? So; obviously, this whole thing I can think of it as a matrix A ok and a vector x plus of vector v right.

So, what is this a matrix A matrix individual row of a A matrix is like this whatever a 1 a M last row is a M M to a M M 1. So, those kind of things right. So, this is that A matrix.

Now, how do you get the A matrix? It is easy to get. I mean it is not something that you have to estimate, because that is the effect of my channel. All this W RF estimation those kind of thing, but finally, this is what will be coming up in your data at the end of this data part at the end of my RF extraction part ok. Now, this RF extraction part when I do that this is what my digital data come, but I am still not done from my detection prospective. Now, let us come to the point how what is this A funda what is this. Now, let us go back to my data model.

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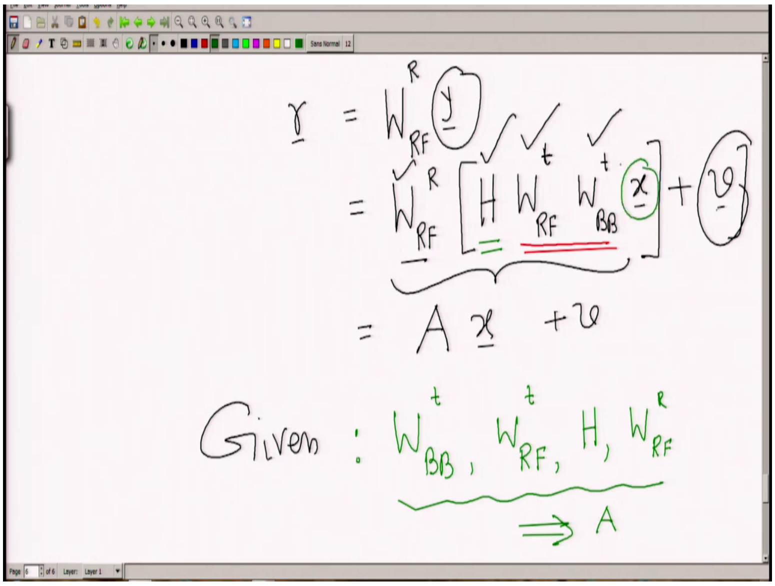

So, what is my data model? Data model is that r vector is nothing but let me not draw 2 r vector is nothing but W RF at the receiver side and then, you have this y vector plus w not plus w this is the this is what is coming what is coming.

Now, what is this y; this y is nothing but W RF into r multiplied by that whole channel matrix that we have talked about right that was your N r cross n t that channel model then you have this W RF t; that is the transmit side beamforming vector beamforming matrix we have talked about.

And then W BB base band side. And finally, your x vector x or s vector whatever we have decided plus a noise let me not consider the noise, but currently, because noise is anyway implicit in this case right plus anywhere whatever your noise part that is not in it is not my important thing, because I can always take this fellow off and I call it this whole thing W RF multiplied by this v some resultant matrix. So, I just always you know keep that fellow separate.

So, which means that a matrix I was talking right it is a impact of everything that gives you the A matrix plus the noise. Now, if I know this if I know this if I know this and if I know this, I can get A no big deal right so; that means, given all these four matrixes.

So far, the these are the transmit matrixes these are the transmitter side this is the channel part this is the not I should not say the whole three things are transmitted say these two are transmitted side and this is my channel part.

Now, if I know my channel if I know my W RF t W BB t and if of course, W RF R is also known to me, because I am the one who is designing its if all these four matrix are known; obviously, A is known to me. So, it is not so difficult to find out A ok.

Now, we will see whether this A will be really required or not. We will see that, because there is no point in finding out a because you know all the value of A right. So, that is not a big deal. So, you know all the A ok. So, that mean given I just make a small comment that mean given W BB t W RF at the t side and H W RF R side all three things are known you know A say A is not a big issue for us ok.

So, now we will see in the next class that is that what my story ends. I mean is it where my story end, do I really need to A do I really need to know A? Ok. Actually, it is not required to know A, but my what is my goal? My goal is to get back my x that is my data, but that is what I transmitted it right. So, that is my goal I think if I am not wrong I could have used a different notation in my earlier one.

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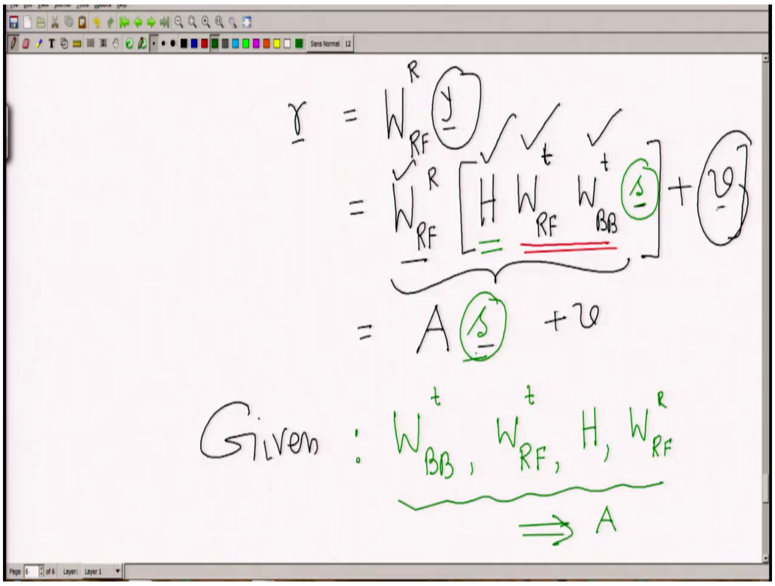

So, probably this was s bar, but that is ok as long as you understand that was the transmitted vector and then W BB n this was the digital pre coder side. This was the beamforming part and that is the channel. So, three things here right.

So, what is my ultimate intention? Ultimate intention is to let me put a different notation. So, that it would not ultimate intention to get back my s, because that is my information data ok, but I am still not done. I am just proceeding step by step.

So, in the next class what I do. I will now go for detecting my s part ok. So, with this I end the session here, and in next class we will be detecting the last part of my beamforming once.

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So, in conclusion we have kind of completed the yeah.

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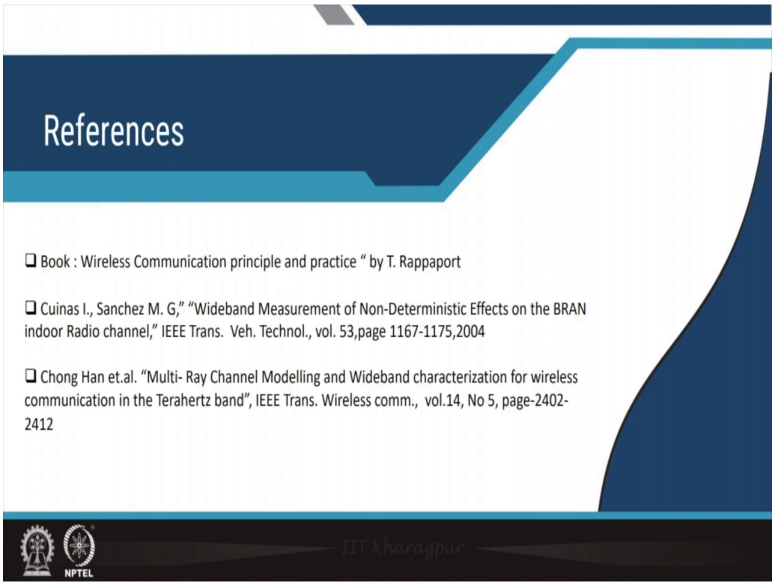

And the reference will be same as what we have.