

Signal Processing for mmWave Communication for 5G and Beyond
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Module - 08
Hybrid beamforming concept and Beamforming in MIMO
Lecture - 44
MIMO Beamforming in Receiver side

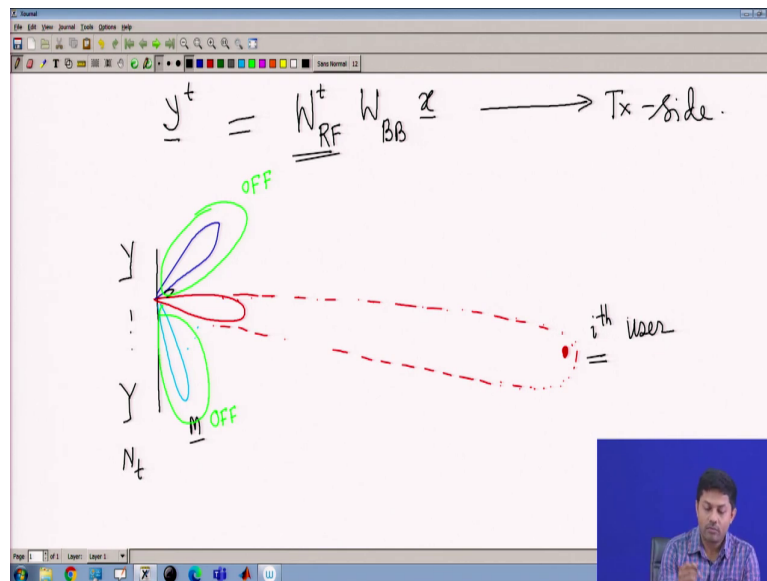
Welcome back, welcome back to the millimeter wave communication. So, far we have talked about the transmit side Beamforming in a MIMO case right.

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Now, today we will be now moving to the, so today we will be covering the like things that we will be covering are the following receiver side.

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So, what happens to the receiver side? So, if I summarize what has been the data model for the transmit side you will, you have already observed that it is nothing but say y bar t is equal to W BB sorry W RF and this probably the x vector. So, this is what my transmitter side. So, this is I would write it like a Tx side.

Now let us understand what happens when it goes through the channel and it comes to the receiver side. Now if it goes to the channel so let us say I am at the transmitter, now I am I have already you know I have already propagated the signal through the air. So, what happens how exactly the propagation happens? So, I do not draw all the antennas but probably a few symbolic antennas and these are all N_t number of beamforming antennas right.

And if I see the beams so this will be multi-level beams will be there. So, M number of if there are M number of data M number of beam splitting you want to do so that many number

of beams will be created ok. So, that we have explained it in the last class. Now it goes through the channel. So now, I am at the receiver side. So, this is the data model whatever has been written in the above equation that is the data model which will create exactly this particular beams ok.

So I will again come back to this Tx side for the exact summer optimization on the W RF part, because this W RF is a very critical component that is the one which is a steering matrix now. Now it is not a steering vector rather I will say it a steering matrix. So, now what will happen to the Rx side?

Now before it goes to the Rx side it has to go through the channel right. So now, as you can notice so I will create the data model for each and every beam, because if you notice that not every beam will finally received finally, it will not be received by all the users right.

Say for example, probably I may have one user here somewhere here somewhere here and most like most likely this is the one which will be covering that that is the one which we will be covering finally right. So, here when I say channel model for multiple beams it is basically with respect to one particular users ok. So, I do not worry about what are the other users. So, it can be just say one of the is a i th users so ok.

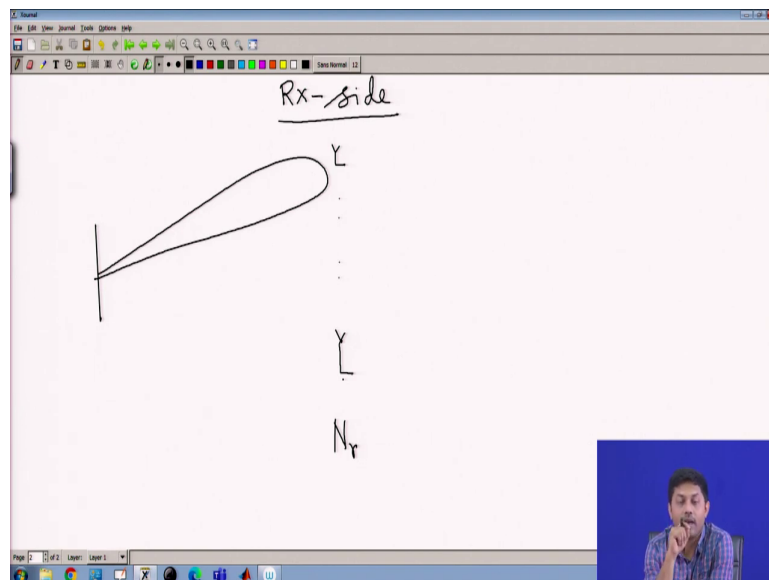
So, I just generalize it so it can be i th user it can be just an i th user. Now with respect to the i th user how things vary; that means, when I say one when I say I am getting a trans receiver model of my data at the receiver it has to be with respect to one particular beam. Now of course, other beams can create some interference because they may have some sort of a side lobe, side lobe can create depends on the distance.

Obviously, the distance is slightly larger this side lobe may not create too much problems. But on an average let us assume that its one beam that is incident upon a particular user. So, now with respect to this user I am modeling it. So, to me it is as if like nobody else is there just one beam is created.

So, what does it mean? It means that to me these things are just off these are all off, off in the sense it is not off with respect to the i th user with respect to the i th user they are off; obviously, because they do not reach so I do not care I do not consider.

So, what do you view it, how do you view this whole system? It is like one beam is coming and the data is just received by one of the user there ok. So, let us now proceed how we can model that ok. Now I am at the receiver side ok.

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Now, suppose it was one beam; obviously, I am getting one beam only. So, it is just like one beam coming and I have multiple antennas sitting there ok. And let us say I have N_r number of beamforming antennas sitting there. Now this N_r antennas are also Rx side beamforming

they need not be the signal of you know of the combining part. So, they are basically meant for Rx side beamforming.

What is the Rx side beamforming? So, what is the purpose of beamforming? If you just look at beamforming is mainly it channelizes the power right to a certain direction that is the beam. So, the purpose is that obviously it boost up the SNR it boost up the received signal again. See that is the purpose when I have multiple antenna at the receiver can it also do the same thing yes.

Of course, provided it depends on how you take the signal and combine them suppose N_r number of antennas are coming there at the receiver and I just transmit from one antenna. So, let us say I take a simple case I come back to this example.

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The slide contains the following content:

- A diagram on the left showing a signal $x[n]$ being transmitted from a single antenna to multiple antennas at the receiver, represented by three curved lines.
- Mathematical equations on the right:
$$\underline{y} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_{N_r} \end{bmatrix} x[n] + \underline{v}$$
$$\underline{y} = \underline{h} x[n] + \underline{v}$$
- The word "Equalizer" written below the equations.
- The number of receiver antennas $N_r = 100$ and the conjugate of the channel vector \underline{w}^* written below.
- A small video inset in the bottom right corner showing a man speaking.

So, there are say I have multiple receive antenna N_r and let us assume I have just having one antenna at the transmitter and it is isotropically radiating ok and so from a DSP point of view I am just sending one data right. So, what is my received data model everything from DSP do not bring the RF and analog part there everything you think from signal processing point of view. If I send x_n through isotropic antenna and I am getting; I am getting the signal received at multiple receive antenna.

So, what is the data model? Data model is very straightforward right, I mean you may have noticed it this is \bar{y} ; \bar{y} is the received data vector at each and every antenna so I just stack it together. So, this would be $h_1 h_2$ these are my channels whatever you think of either it is a millimetre wave channel or it is a normal channel I mean there is no difference just that how you approximate it right.

So I am just generalizing it into x_n plus, I may receive a noise vector figure right. So, now, this is my data model right. So now, ultimately I have multiple choices to decode the data right, I can create some sort of an equalizer ok, I can create an equalizer I can create an equalizer to detect the x_n .

So, what is my final data model? Final data model will be \bar{y} is equal to some $\bar{h} x_n$ plus \bar{g} . So, can I create an equalizer yes there are multiple choices for equalizer like least square like 0 forcing like MRC MMSL MMC anything whatever.

So, I can find out some sort of an equalizer vector w , if I multiply this with \bar{y} then I will get back my x_n . So, that is the way to proceed this is one way of doing, if you want to do an ML decoding you can do an ML decoding.

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ML
 $\arg \min_{x[n]} \|y - h x[n]\|^2$

Design w → MRC
N x 1 → LMSE

So, you have to you know in that case what you have to do if it is an ML decoding. So, why I am talking all this because this will be required at the millimetre wave also how do you finally, receive the signal in what sense you receive it so if you do an ML decoding and the data is a Gaussian. So, what is the data model it will be y bar minus h bar x n.

So, you minimize this distance mod square and you minimize this distance, but its a vector distance. So, its a 2 norm of a vector error vector and over all possible value of your x n. So, if it is some sort of a constellation you take which constellation it has been picked up and then you say ok, this is what the value is if it is ML.

If it is not ML if it is some other way of detecting like equalizer based detection you can create an equalizer w and how do you design the w you can design w based on several

criteria, design w based on one is the MRC maximal ratio combining it can be LM MSE and so on so forth. Based on what cost function you want to optimize for it can have it.

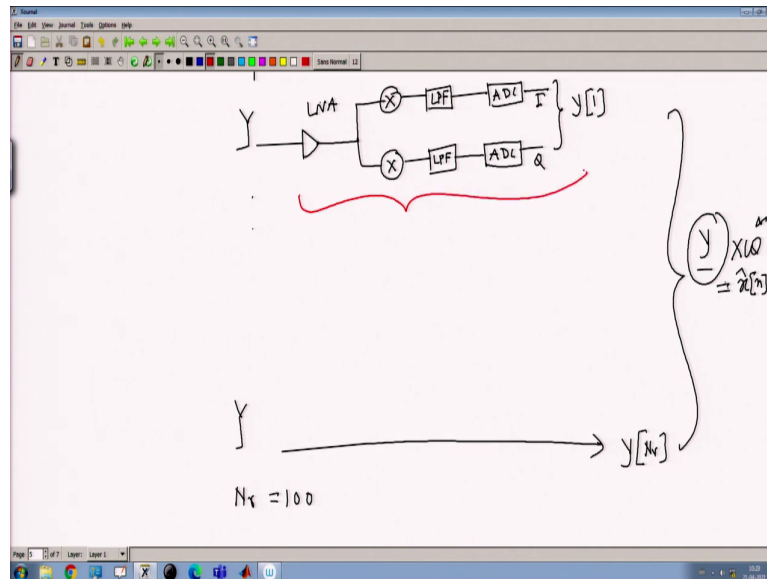
Now, I would prefer any one of them. So, at the end of the day what is this w ? So, if it is say you do not want to do a equalizer based if you do not want to do an ML based decoding sometime it may be slightly tougher for a search point of view. If you want to do a simple linear equalizer based detection what exactly you do you do a some sort of a MRC or some sort of an LMMSE.

So, what is this w ? w some sort of a vector whose length is equal to $N_r \times 1$, because I have at the receiver I also have N_r number of data ok here N_r number of data. So, now why I am talking all this because the same thing I will just carry forward to my millimetre wave as well. So now, the point here is that I am at the receiver. So, when I am at the receiver I have to do some sort of you know jugglery, such that I can get back my data that is my ultimate intention.

So, in this particular example whatever I have given you its kind of a isotropic antenna from the transmitter side. Now if you bring in the beamed case it will not be too different case ok. So, it is your choice how do you design the w . So, you multiply the w and you get back your data from your x_n .

Now, this w the choice of this w either you can do it in digital domain that is preferable right always, because if I receive a y here this is more of a digital data that I received. So, which means that from each and every antenna I have to prepare a digital data right, so if the N_r is say 100 what does it mean? What does it mean? So, what is the actual circuit that will be coming into picture?

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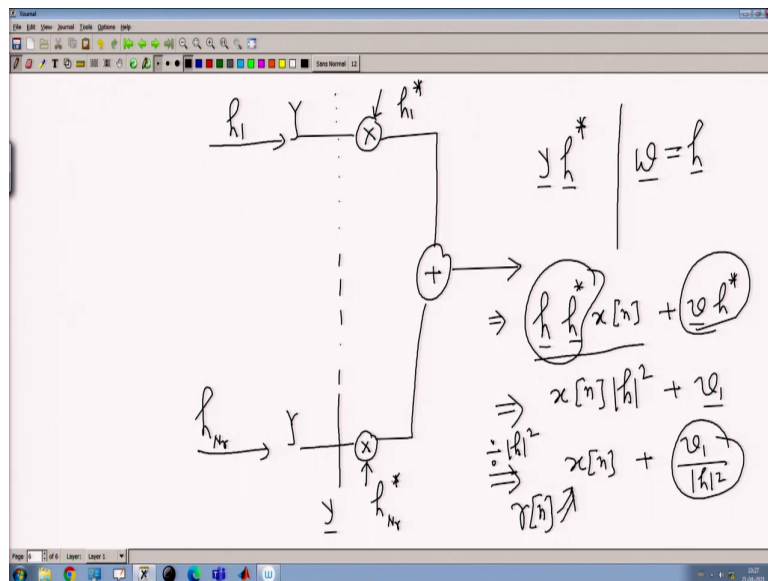
So, at each and every antenna I have to put one LNA gain factor then I have to you know I have to demodulate the signal then I have to pass it through a bunch of LPF filter. The standard demodulation technique then it has to go through an ADC this is your complex data this probably I this probably Q you get one y vectors fast data probably. Now like that you may have a repetition of the same circuit for N r time same thing you repeat it and you get here y Nr.

Now, this whole thing you call it y vector this is one part because I am operating in a DSP domain right, this precisely I was operating on right this is what my point was. Now this is tougher now this you can you have developed the equalizer based on some criteria and you get w as some complex number and then you multiply this w with y and you get back your x n fine.

Or if you even if you want to do you know ML you can do all this, but at the end of the day this is the circuit. So, which means that if N_r is kind of say 100 you need that many number of RF chain and analog chain right. You need that many number of LNA which is an RF circuit that many number of it is more it is actually 2 times of your demodulator.

So, many complex you know analog and RF circuits are present if I want to do the w in the digital domain. So now, once I get this y bar I multiply this with a digital w star and I get back my x_n that is fine. But the point here is this whole thing is happening in the digital domain and precisely I do not want to do that in a millimetre wave part.

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Now, this is ok as long as you are in the normal domain. So, how to design this w there are many ways as you can see you can either say MRC maximal ratio combining you know. What is MRC? If you do not know I can just for your convenience I can explain it what is MRC.

This is basically simple I mean it is it maximizes the SNR in a non Bayesian framework and in fact it is nothing but the least square area.

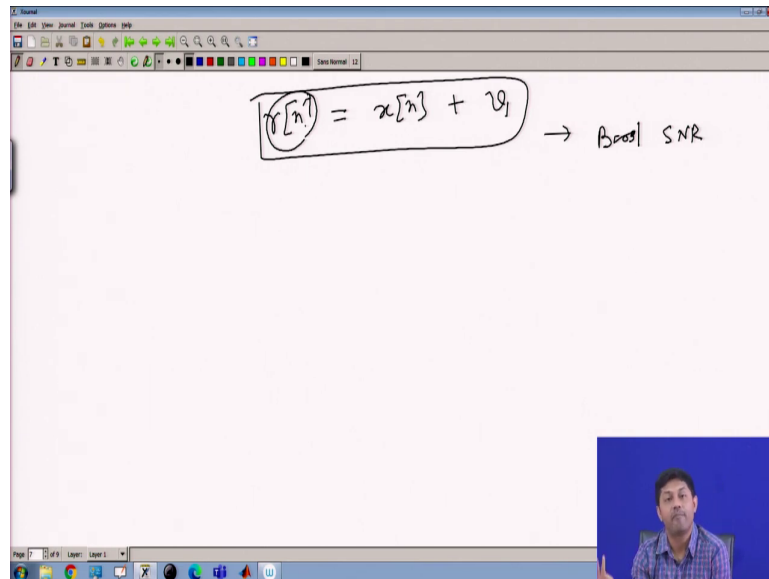
So, what is exactly an MRC? So, if you have a channel here seen at h_1 versus if you have channel here h of N r . What you do? You multiply this data I am assuming you digitize it and multiply it. So, you multiply this data with h_1^* multiplies this data with h N r star ok just multiply it. Then what happens? Then you just add them up and what you get is a very nice form you get.

So, basically here at this stage whatever your data that is your y bar right after your digitizations. Then what I do? I am multiplying with an equalizer, now I am saying that each and every equalizer is nothing but whatever channel you are seeing on that particular antenna just conjugate it and multiply it.

So, what happens? So, you have y bar ok and you multiply this y bar with conjugate of the channel. So, what happens let us see. So, this is your y bar, so let us say and h bar we conjugate it that is what this particular. So, which means that here my w is nothing but your h , because I am conjugate and multiplying so w will be your h ok. So, what happens here? So, what will I get? So what was your y bar? y bar was h bar x n plus there is a v h star, but that is ok this is kind of a equivalent noise.

So, this becomes x n now this will be your total channels norm square right. First let us call it v_1 dash now you divide this now next you divide this whole thing by h mod square. What will you get? You get x n plus v_1 divided by normalized channel square.

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$$r[n] = x[n] + v[n] \rightarrow \text{Boost SNR}$$

So, this statistics would not change much. So now, this will be something like you get $r[n]$ is equal to $x[n]$ plus something like that or in a very simple sense after all this MRC you get a data model which is equal to $x[n]$ plus some normalized $v[n]$ something like that, it is not even a vector it will be some square.

Now, this is the data model that comes into picture, now you can just do some sort of a simple search and find out what exactly the $x[n]$ is. So, this is a so this mechanism will boost your or maximize your SNR. Now, this is one type of equalizer you can have some other equalizer if you have say MMSE equalizer LMMSE equalizer, now instead of $h[n]$ something else will come into picture. But that is ok that is not what I am trying to understand make you understand here.

What I am trying to understand here is that at the end of the day you have N_r number of antennas digitize them and you have to multiply them with an equalizer equal number of equalizer. If there are N_r antennas because you get a y vector whose length is also N_r and you get to multiply it with the N_r number of number of some complex you know coefficient.

If you multiply it you get some data and then you do a simple detection, so you get back your x_n . Now the point here is that all what all you are doing is in the digital domain right, because you are multiplying with w everything you are doing it in the digital domain. But if the N_r is large and that is typically the case when you have millimeter wave how do you solve such problems, because this will be tougher you cannot just go by this mechanism ok.

So, can I do some sort of a simpler mechanism by which I can in the RF domain when I am actually in the millimeter wave, how exactly my Rx side would be. So, if you look at the Tx side some part was done in the RF some part was done in the digital side ok. And you know the obvious reason because when I create a phase different I want to create a phase difference I have to get into the RF domain that will be better, so it was an hybrid.

So, similar approach I also have to take it in the digital domain in this Rx part of the millimeter wave, because your number of antennas are huge. So, it cannot afford to do everything in the digital domain because your components number of components. If you look at if you just leave it like this each and every antenna needs that many number of you know RF and analog chain and you it will be repeated N_r number of times it will be repeated.

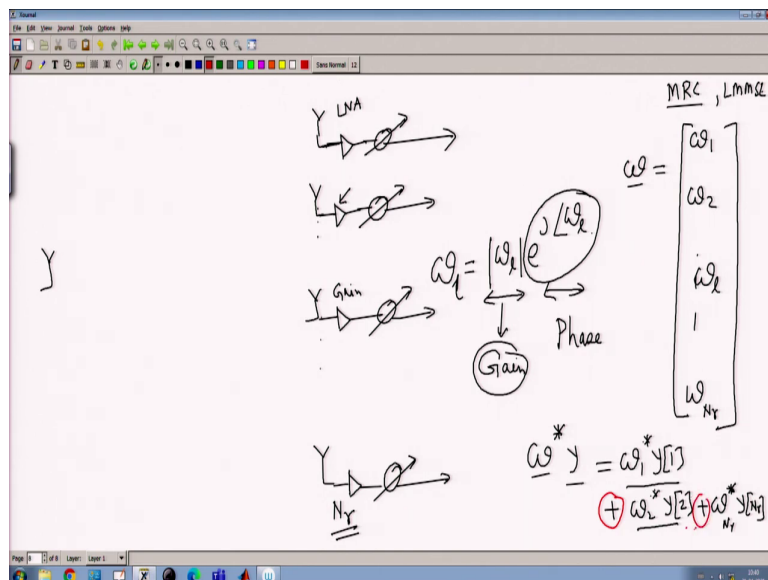
So, the cost of the you know the IC chip the cost of the complete system will be very high and also the power. Look at the power requirement it is very high power requirement right, because each and every point you have a demodulator you have a low pass filter you have an ADC.

So, on a die of you know system on a chip putting. So, many ADC putting so many I PF analog and putting. So, many RF circuits are not advisable you cannot really do that. So,

easily so the better solution is that can I move some of my equalizing concept into the RF domain ok.

So, that is the point that we are trying to make it when I am in the millimeter wave, because I am completely in the Rx now. So, the whatever mechanism I have explained here can I see if some of them can be pushed back into the RF or not ok and let us understand that. Now, let us understand from a single and single transmit antenna point of view because that will be easier for explanation.

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So, if you have let me have one more antenna here ok. So, I have N_r number of antennas ok and let us have the same thing let us have it is a isotropic antenna it really does not matter whether it is isotropic or a beamed antenna. Even if you have a beamed antenna I do not mind it ok. So, let us have one antenna which is a beamed antenna and this whole beam is getting

received by this particular antenna, these are the antenna I am not physically creating the things notion here.

Because in the Rx side I have taken space, but in the Tx side you can see the beams becomes wider it is not the case actually the beam can be even narrower also. Because this distance is really small distance and this distance is really very large distance ok. So, it is just for the understanding purpose.

So, it really does not matter whether its a beamed antenna or its kind of a isotropic antenna because if it is a SISO case it is as if like one antenna sitting which is creating a beam right. I think you have understood by this time ok.

So, physically there may be multiple antennas, but it is like one antenna sitting equivalently one directed antenna sitting creating a beam. So, I am talking of that kind of scenario. So, even if there is a beam I do not mind it ok. So, the point here is that we are in the receiver side and how exactly how much or what we should do to avoid too many RF component too many analog components, specially when my N_r is very large.

So, whatever I have explained so far you can do very well when you have a less number of receive antennas say like a 4, 10 kind of things. Now, you have only 100 you cannot have that you have to minimize the number of components. So, what is the best way to do that ok? So best way to do that is the following. So, I take the approach of n equalizer it does not really matter what equalizer you are puts finally taking up.

Let us take I am taking MRC equalizer or I can take even an LMMSE equalizer it really does not again I am repeating it really does not matter what equalizer you are taking, at the end of the day they are N_r number of complex coefficient. So, I do not want to bring this terms at all I mean you can find out lot of papers and lot of materials where people use this terms MRC or LMMSE or anything like that, I do not want to prefer use this term to prefer to use this term because to me it is nothing but some complex number ok.

So, at the end of the day I have to multiply the data stream with the complex number. So, let us generalize it. So, let us say my complex number as we have described it in the context of earlier case. So, let us have this is the one which I once I get it whatever technique you use either you take an MRC if it is MRC what will be the w_1 nothing but the channel if it is LMMSE channel plus something that is all ok. So, its a complex number.

So, if it is a complex number what should I do? So, what I do here is that I just put now I do multiply it right because this is the one which I would like to have it. Now here is the catch, if I multiply it I mean you can say ok, I am bringing up the RF signal why do not you just multiply this RF signal with the multiplier called w_1 .

So that means, what you are essentially saying that so you have to multiply the w_1^* right, because its a conjugate part this is what I have to do finally ok. So, because that is the notion I am taking it and we multiply the w_1^* there. So, now the question is that can I put a multiplier directly there, do I really need to put a multiplier there ok because I want to avoid the digital part. Now if you say ok do everything in the RF, then does it mean that I just put the multiplier in the RF domain.

Now, my dear friends it will be very difficult to put a multiplier in the RF domain ok, because it is a complex number; having a complex number multiplier in the RF is not a easy task easy task to have it. So, what is the best way to do it? So, each and every w_1 can I think of it is a complex number right.

So, let us say the l th one l th coefficient, can I have it like this something like that. What does it mean? It is a complex number has a magnitude has a phase right; obviously, any complex number has that. So, I split that I said ok I have this which is a gain can I call it a gain this part I call it a phase.

Now mind it that phase ϕ to the power j whatever angle ω_l I have written that is in terms of the digital view. Again your views are still digital it is just the circuit point you are thinking, so it is a like a phase.

So, I have a ωL or $1/\omega L$ as gain and phase right. Now in the RF moment I think this way slight you know rephase can happen here, I have a gain how do you implement a gain in RF? It is kind of it is kind of an amplifier right. Any gain how do you implement a gain in any circuit its an amplifier right its an amplifier.

So, that mean does it mean that I need a RF amplifier well every antenna has an LNA low noise amplifier, so that is usually there anyway. So, I can take that same LNA I can make use of the same LNA and push the gain there.

So, which means that this multiplier whatever I have drawn there instead I just make a small change here I make a small change. What is a small change? Each and every antenna is anyway fed through a LNA ok, now which means that every antenna anyway comes with a gain circuit its an amplifier.

So, that is there any way because every antenna may have a different gain or may have a gain disturbance to improve that gain I have to have an LNA. So, that part I cannot avoid it ok. So, I have an LNA already sitting there. So, this LNA has a gain to improve the you know the sensitivity of my ADC level. So, I push this gain to the LNA itself, if this gain was already if the LNA already having some gain I push the another gain.

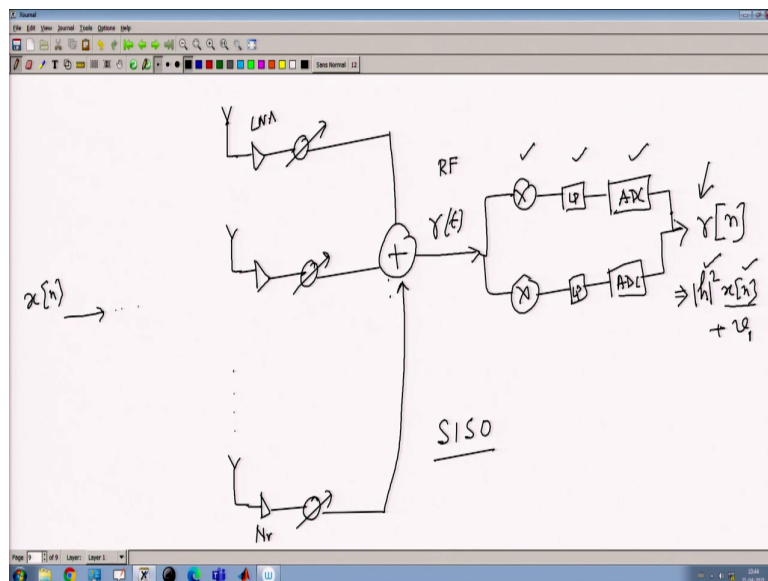
So, which means that now this amplifier whatever LNA is there this gain will be pushed it here. So, if I take this is my you know lth antenna this lth antenna whichever the default gain the LNA was having I put the I put another gain there. So, it is easy to do that because that gain is just an a factor say it is a 1.2 or 0.3. So, I push that gain back to the LNA. So, it is easier to do ok second part was a phase ok what should I do? I can just put a phase shifter here that is all I will do.

So that means, the gain part I push it to the LNA and phase part I implement it as a phase shifter. Now is the phase shifter an easier task to do in RF yes because in a Tx circuit also we have done we have done a phase shifter right for steering, here also I put the phase shifter there ok I put the phase shifter there.

So, I am multiplying definitely I am multiplying with the w , but I am multiplying in a very intelligent way the gain part I push it to the default LNA and only the phase I created as a phase shifter that is all that is the only job I will do next. What should I do? Now, you look at what my intention is I want to ω . Now this y is kind of virtually like a RF y right that is what because this w is now in the RF, but obviously I will view it from the digital domain.

Now, this w star y what does it mean? This is not this is a vector multiplied by another vector in a inner product way. So, which means that this will be ω_1 star into y_1 plus ω_2 star y_2 plus ω_N r star into y_N r right this individual multiplications are done this way. What is left I have to add them up because that is what it is saying right it is addition everywhere there is an addition. So, all these components I have to now add up the same here I have to add up.

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So, what is the last stretch? Last stretch is that if I go up these are my antennas, these are my antennas this is my LNA gain one second let me change the direction of my antenna because I mean a receiver data. So, this is my N_r number of data. What I do? I already have an LNA, I am making use of the same LNA for my gain.

So, this is my LNA part ok then I put a phase shifter I just read out the circuit. Where I do not want to delete the other content there, then what I do? I just add them up right, because this is what it is suggesting me this is what it is suggesting me after multiplication I add them up. So, I put an adder.

Now what is this adder is? It in RF domain or digital domain its in RF domain right, because I have not put ADC now I get only one data stream whatever I was writing right here this r_n instead of r_n I think of its r_n s RF version this is what is coming. So, this is the RF data this is still in RF domain.

Now this I multiply with the I demodulate this one you just demodulate LPF you put it LPF you put it low pass filter then you have an ADC you have an ADC now this one will give me r_n . What is the data model here? This r_n is still not r_n because it is just multiplier, but that is ok it probably h^2 this one you know v_1 bar or some v_1 . Let us not worry about what is the noise here this is what is coming. Now, in the digital domain you just divided by h^2 and do the rest of the processing.

But now you see when I do this in the millimeter wave I have to do it in the millimeter wave because, now you look how many RF components I am having. Just one phase shifter sorry not $1 \times N_r$ number of phase shifter N_r number of LNA which are anyway there.

But the demodulators LPF ADC there are bulky circuit I just have 1, provided my data stream is just one data stream it is like a SISO. So, it is as if like I have sent x_1 it goes through all the transmitting channel blah blah blah, then finally it comes here and my data model becomes just a single one it is the SISO case, that is the reason why I call it SISO. Because I

sent one and but when I receive it through all the processing I get back only one data stream and from there I will extract x that is ok.

So, now this concept is very helpful because this is the one which will be also extended to the MIMO case ok and you can see this particular proposal does not require so many you know ADC just one set of ADC. Of course, couple of ADC because you have an I and Q 1 and 2 pair of one pair of RF circuits and one pair of LP circuits RF demodulator one pair of LPF low pass filter I do not need n_r .

So, no matter whether your N_r is 100 or 1000 I still need just one set. So, this is what is convenient for millimeter wave ok. Now I stop the class today here and in the next class I extend the same concept for the MIMO case whatever we have discussed in the transmitter side. Now, we have already started the receiver side now this is for a single you know single x_n case. Now if I have multiple x_n case if I have $x_1 \dots x_n$ multiple such x_n what will happen to that will be explaining in the next class ok.

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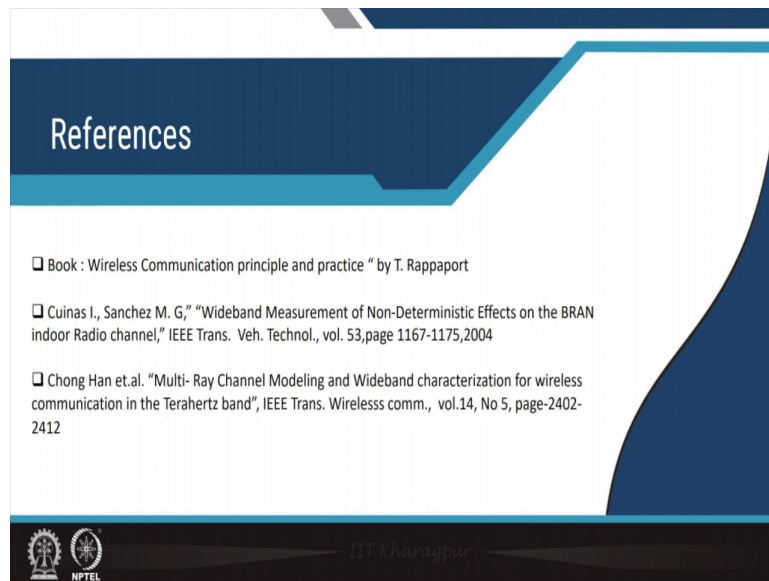
The slide features a dark blue header with the word "Conclusion" in white. Below the header, a white area contains a bulleted list. The bottom of the slide is a dark blue footer containing two logos on the left and the text "IIT Kharagpur" in the center.

Conclusion

- We covered more on MIMO Beamforming
- Description of MIMO in Receiver side

  IIT Kharagpur

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References

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So, in conclusion we have kind of completed the yeah and the reference will be same as what we have.

Thank you.