

**Signal Processing for mmWave Communication for 5G and Beyond**  
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**Module - 08**  
**Hybrid beamforming concept and Beamforming in MIMO**  
**Lecture - 42**  
**Different Level of Beamforming**

Welcome back to the 5G Signal Processing for millimeter Wave.

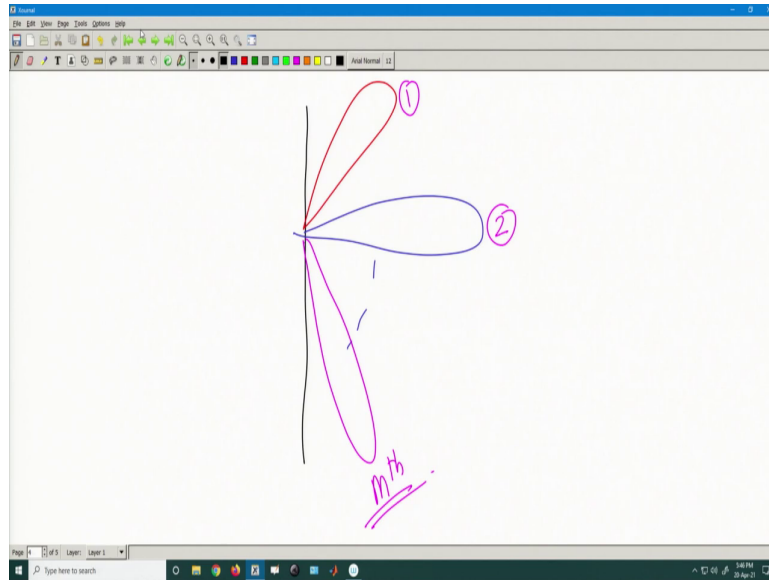
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So, today we will be covering the like things that will be covering are the following. So, now, we were talking about the MIMO beamforming part and in the last class we have seen about the structure exact structure that is going to be implemented or that is being implemented as

part of the MIMO beamforming structure. Now, today we will be talking more on the exact mathematical part of W RF which is the precoder part RF precoder part ok.

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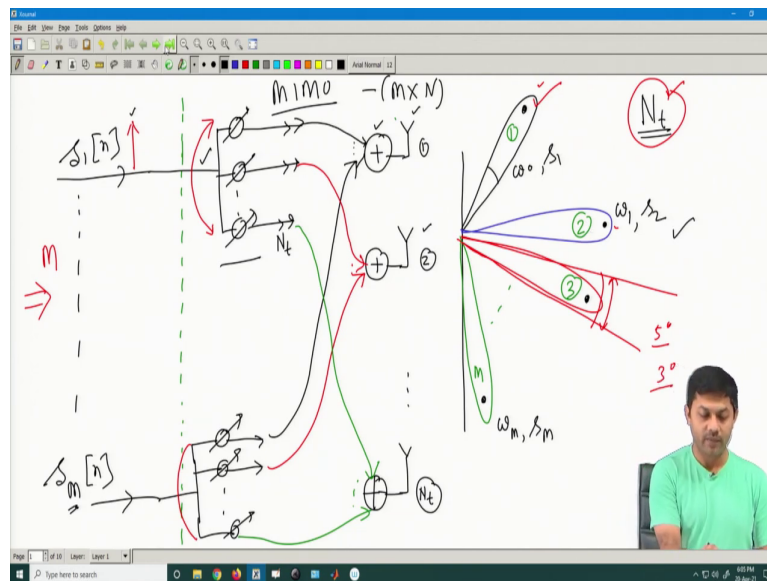
So, let us go back to the diagram that we have earlier discussed here.

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The image shows a whiteboard with handwritten mathematical diagrams and equations. On the left, a diagram shows a vector  $\underline{s}$  with dimension  $M$  entering a box labeled  $W_{RF}^t$ . The box has a checkmark below it. Two output vectors are shown:  $\underline{y}^t$  and  $N_t$ . To the right, the condition  $N_t \gg M$  is written. Below this, a smaller diagram shows the same vector  $\underline{s}$  entering a smaller box labeled  $W_{RF}^t$  with a checkmark, producing output  $\underline{y}^t$ . Below this is a boxed equation: 
$$\underline{y}^t = W_{RF}^t \times \underline{s}$$
 with dimensions  $N_t \times 1$ ,  $N_t \times M$ , and  $M \times 1$  respectively. Below the equation, it says  $W_{RF}^t \rightarrow$  tall matrix. In the bottom right corner, a small video inset shows a man in a green shirt.

So, this is what the actual mathematical model.

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And this W RF if you can see it is a  $N_t$  cross  $M$  antenna right. So, that is the point here. So, this is  $N_t$  cross  $M$  antenna the  $N_t$  cross  $M$  antenna. So, which means it has  $N_t$  number of row and  $M$  number of column it has.

So, who are the rows and who are the columns we need to understand that. Now if you can construct in fact the individual you know the columns we are done with it. So, now, if we can guess can we guess what should be my you know the columns? So, if I know I know there are  $M$  number of columns. So, who are those columns ok see if you can go to the diagram the first diagram here. So, let us put some thought here.

So, how many such blocks are here or chunk are here which I was referring as a chunk? So, there are  $M$  number of. So, there are  $N_t$  number of you know phase shifter and such phase shifter are  $M$  right. So,  $M$  number of chunks and within the chunk there are  $N_t$  number of

phase shifter points that is there. So, now, we can guess what should be the W RF because I have M I has I have N t also. So, if you can think of slight deeply here. So, ultimately this W RF is nothing but so if I see the structure of W RF.

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So, it is a matrix it is a tall matrix right because N t is much larger than M. So, it is a tall matrix. So, let me put it that way, so that it represent somehow a tall matrix N t cross M here. So, which the first column which the it is a anti length elements will be there. So, which is the first column?

So, the first column if you think properly, the first column should be responsible for creating the first beam steering I should not say first beam rather I should say first beam steering the second column is responsible for creating the steering for the second beam and so and so.

So, the  $M$ th one is responsible for the  $M$ th beam steering. So, which is the first column element this first column element is nothing but the element present inside the phase shifter in the first one ok. So, which means if we go back here so which means the first element will be nothing but it  $1 e$  to the power  $j \omega_0$ , if  $\omega_0$  is the angle I am trying to you know I am trying to shift by the firsts the first beams. So, that is the point.

So, it will be then  $j^2 \omega_0$  the  $e$  to the power  $j N t \text{ minus } 1 \omega_0$  that is my first column that is my first column because the first column is nothing but the phase shift component of my first block, what about the second one?

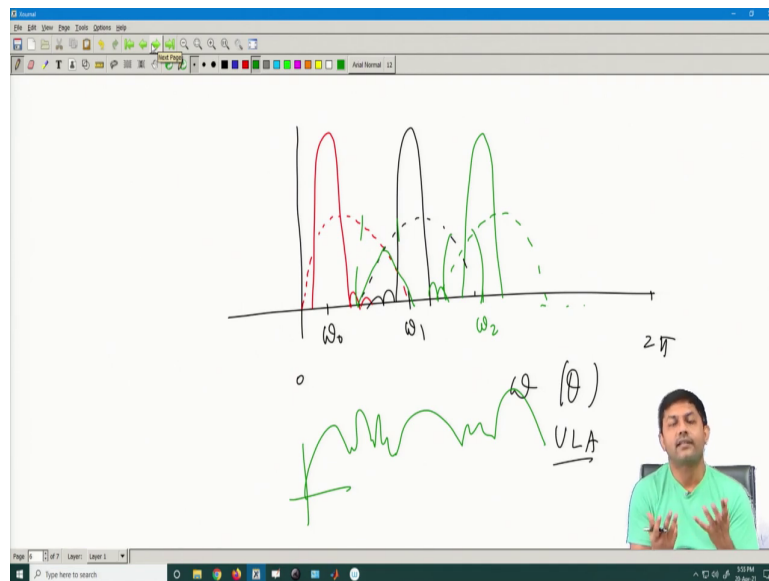
It will be  $1 e$  to the power say  $j \omega_1$   $e$  to the power  $j^2 \omega_1$  then  $e$  to the power  $j$  that is the point like that how many such there?  $M$  number of such column would be there. So, you can just generalize it probably I will ok. So, that is the point here ok this is my first one, this is the second one, this is the  $M$ th one.

So, that is my WRF that is a structure right. So, this should be multiplied by  $s$  vector. So, this can be multiplied by  $s$  vector and it will be generating the  $yt$  vector. So, this will be the as a it will have a dimension  $M \text{ cross } 1$  this will have a dimension  $N t \text{ cross } 1$  all matches everything match.

So, this is basically my WRF now this is fine. So, what about the  $\omega_0$ ,  $\omega_1$ ,  $\omega_M$  ok, how are they placed? I think this part also we have discussed in the one of the previous class that they should not be overlapping right because if they overlap the spectrum will be overlapping on that ok, the electric field will be overlapping and that creates a distortion.

So, you have to ensure that they are kind of you know discrete an orthogonal in the sense that it is not just orthogonal because beam also has to come into picture. So, the with the beam the beams every individual beams can be distinctly put there. So, otherwise there is no point of choosing those kind of  $\omega_0$ . So, for example, let us put some more light here.

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Say for example, if I create if I take say 0 to 2 pi that is my omega I am varying I am seeing that over that omega it can be theta or whatever. So, you can say it is a ULA in that case omega represents the elevation angle in radii ok, so that part. So, it can be theta if it is ULA ok. So, that can be done.

So, now let say I have beam something like here there are small beams here small small side lobes are there this is my main lobe of the beam. So, my omega 1, so if this is my omega 0 suppose this is my omega 0. So, omega 1 should not be such that it overlaps on that the main lobe at least should not overlap.

So, I should obviously, want something here. So, this is my omega 1 ok and now there will be bumps here omega 2 and so on so forth. I do not want a scenario where the omega 0 remain. So, let go back to the same diagram I do not want a scenario where because the beams are.

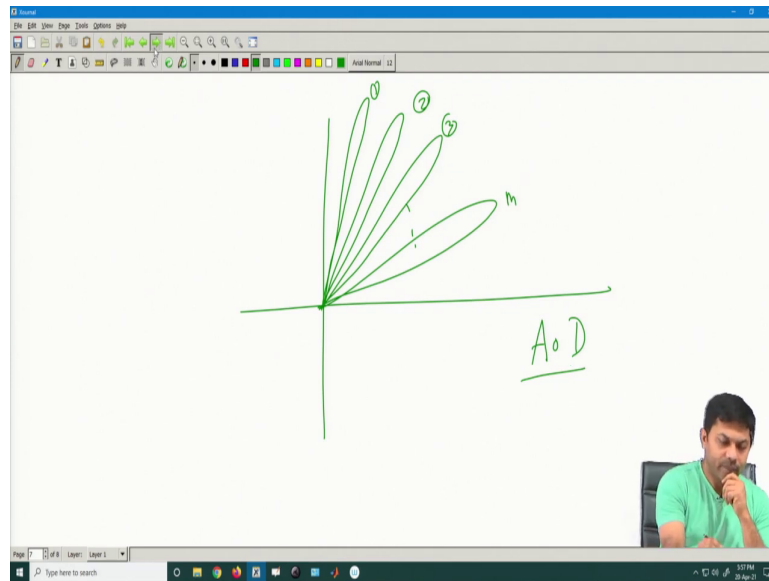
So, wider beams it comes somewhere here and the black comes somewhere here and greens come somewhere here I do not want this scenario because here you can see the main lobes overlap with each other it is very difficult to distinguish I mean when they overlap at this part.

So, the ultimate the resultant beams will be it depends it is either it can be destructive or constructive. So, there will be distortion here; because the beams will be something around something like this and then you do not know what happens then again something then again do not know what happen something like that the beams will be created ok because this distortion; I mean this not distortion this interference will create that.

So, the beam should be tentatively separation, so that I can create a very clear beams on that ok so, that is the only requirement that I have. So, accordingly you have to choose your  $\omega_0$   $\omega_1$  and so and so forth. Now it is not the choice of angle at the same time you have to understand what is the what is the beam width because it is the same angle if your beam width is very smaller even if the angles are you know squeezed they can support the particular thing.

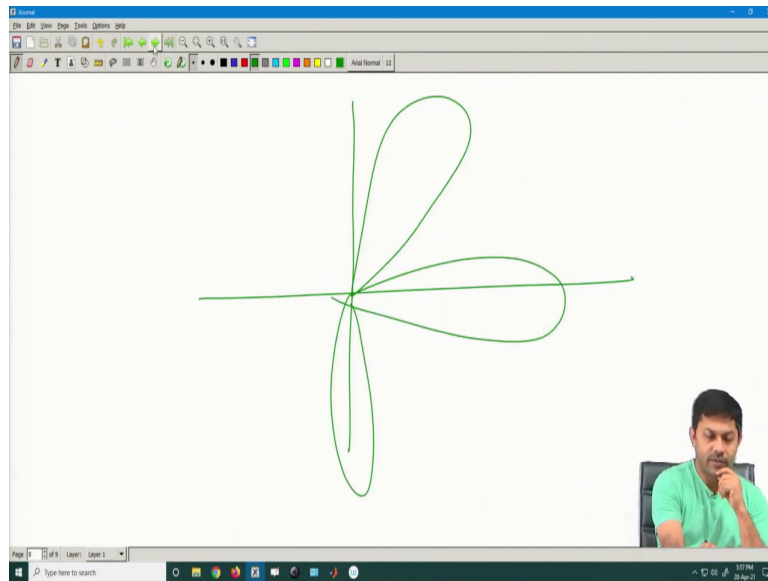


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What I am trying to say is that you may have a scenario where the beams are very narrow say for example, and then I can placed multiple such beams and probably my M th beam itself within here. Because beams are very narrow in this is my first one, this is my second one, third one and M th one because beams are narrow.

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Whereas, I can have a scenarios if the beams are wider then probably the second one has to be here the third one some has to be here ok. So, then I need a wider you know wider choice of my  $\omega_0$ . So, gap between  $\omega_0$  and  $\omega_1$  will also. So, that depends. So, this is not something you can just do it like that.

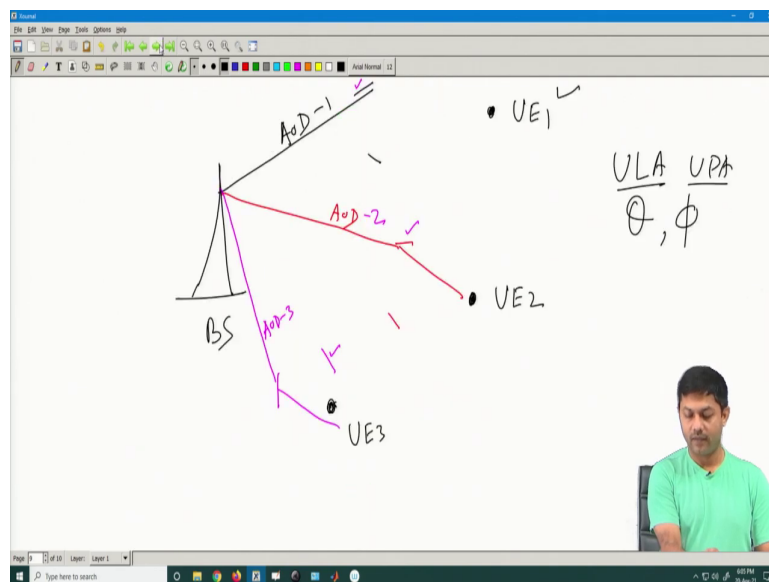
So, you can have the same choice of  $\omega$  set of  $\omega_0$  to  $\omega_M$ , but you have to be very careful what is the beam width, because unless the beam width is such that it actually separates it then all the beams will interfere with each other ok. So, this is the basic idea of how to create the choice of my angles and all these things. So, ultimately WRF depends on that. So, if you choose the  $\omega_0$   $\omega_1$  and  $\omega_n$  based on that it will be.

Now, the question is that on what basis you choose  $\omega_0$   $\omega_1$  who are these points representing we understand that these are nothing but the beam direction right or in a simple

sense these are the direction like a which direction my beam should be sent here, how do you get that? How do you know that these are the direction at which I have to set my beam? Who determines that? Because I am in a transmitter, I am at a transmitter.

So, being a transmitter how do I know these are the direction at which I have to send my beams first point. So, who which parameter actually influences that ok? Now if it is a ULA or UPA does not matter it is nothing but angle of departure it is your angle of departure that determines how my beam should be ultimately put here. So, which means that lets say I have a transmitter.

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I have a base station here. So, this is my base station ok I have one user here. So, this is my user 1. So, I have another user 2, let us say I have one more user here user 3 ok. So, let say I

have 3 users ok. I want to serve this 3 users says simultaneously without interfering. So, I can create a beam obviously, beam can create a special you know difference.

So, I cannot suppose I want to send UE 1 some separate data, UE 2 some separate data, UE 3 some separate data in that case I can create the beam. So, what does it mean? This base station has to understand or has to know the angle of departure with respect to the first UE 1 ok.

Suppose the UE 1 first UE 1 may see some reflector here he may see some other reflector he may see some other reflector ok. So, let me put a different color code. So, that it will be easy for understanding. So, he may see a different reflector and he may see a another set of reflectors ok. So, each and every angle of you know departure has to be tackled. So, if UE 1 sees the this particular reflector, he sees this particular reflector, he may see this particular reflector ok.

So, for him this is the angle of departure this is my A o D for the first case ok, say probably for the second case this would be the angle of departure with respect to this reflector and for the third case probably it may come like that. So, this is my angle of departure for the third case this is my angle of the departure for the second case. So, this three angle of departure has to be known to the base station who does that? Either base station does angle of departure estimation.

So, I will be talking about the A o D estimation A o estimation later in this course itself, but just for your own understanding. So, it is the base station who has to know that either the UE itself can estimate its angle of parabola and angle of departure angle of parabola it can estimate and from the base station point of view he can estimate the angle of departure.

And then he then he can choose accordingly the  $\omega_0$   $\omega_1$  and  $\omega_2$  see if it is a ULA if it is for ULA probably he has to only know the  $\theta$ , if it is for UPA he has to know  $\theta$  and  $\phi$  both if it is a UPA configuration that is the only difference, but he has to know

this angle in which angular position my reflectors are present such that if I send in that particular direction.

Finally, after getting reflection it will reach that particular UE that is the only part he has to know it ok. So, he has to, so these information's are given to the base station or the transmitter, I should not say base station because base station is just specific because it is a transmitter.

So, somehow the transmitter has to know either receiver measures and give feedback information or transmitter itself can measure the information by different methods ok. So, given that it is known I can reconstruct my W RF ok. So, now, the point here is that falling point here is that is this whole system complete or does it mean that? If I go back to my first diagram let us say this diagram itself ok lets I am here ok. So, what I have done? I have M number of data stream, I can create M number of beams alright simple.

So, the first beam if any user sitting in the first beam he does not even know what is going on in the second beam a user sitting in the second beam he does not have any idea about what is going on in the first beam and other beams right. If there is a if there is a user here if there is a user say every user knows the signal characteristic within its own beam he it has no idea about what is going on in the other beams ok.

So, the question that arise that is arising here is that. So, it means that the s one n is meant for the first user that is what it coming out to be correct right. See if the user is sitting in the say suppose I have one user sitting in the first beam, I have another user sitting in the second beam, I have another user sitting in the third beam and I have M th user sitting in the M th beam let us say take a very simple example.

So, each and every user there is one one user each and every beam there is one one user sitting there ok and because that is how I have known my A o D angle of departure and I have created the beam in that direction.

So, what does it mean? For the first beam or the first user rather. So, who is the data that is reaching the first user which data of  $s_1$  to  $s_n$  reaching the first beam if you notice the first beam is created completely out of  $s_1$  because it is the  $s_1$  that is going to the phase shifter and that goes to multiple  $N_t$  antenna and that creates a beam forming with an angle of  $\omega_0$ . So, it is nothing but the  $s_1$  data.

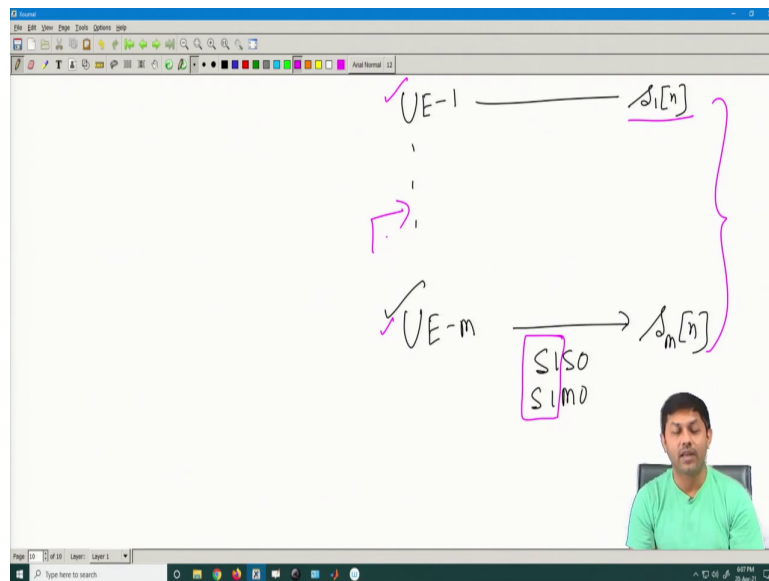
Similarly, for the second beam it is the A o D second A o D and second data for the last one it is this and  $s_m$  clear that is how it is coming. So, that mean the  $M$  th position the the user sitting on the  $m$ th beam he receives only  $s_m$  data. So, what can we think of this kind of scenarios it is nothing but  $M$  number of parallel you know user sitting and they are not interacting with each other.

So, it is like every individual user it is as if like a single input single output system he is not utilizing the MIMO the power of MIMO it is not utilizing it right he is not utilizing it because the first user sitting in the first beam will not receive anything from  $s_2$  to  $s_m$ .

The second user sitting on the second beam will not receive anything from  $s_1$  to  $s_m$  except  $s_2$ . So, this particular scheme what I have drawn here still has some inaccuracy in the data model it is not inaccuracy it has the problem because we are graduating right. So, you have to first solve one problem then find out what is the problem and then go for the second problem solution and so on and so forth.

So, this structure solves the beam creation problem beam you know splitting and all these things it creates it and then it also creates the notion of I mean multiple too many antennas to a smaller number of antenna set that also it does it, but still it has a big problem in terms of the data. That means, the first beam is meant for the first data  $M$  th beam is meant for the  $M$  th data. See if there is a user in individual beam they are finally receiving only one data stream. So, it is not a MIMO scheme for one user.

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So, that means, what I am trying to say it means if I have user number 1 it is taking  $s_1$  data correct it goes to UE number M it takes this data. So, for this user it is as if like I am creating a SISO transmitter at least single input transmission this particular you may have multiple antenna its either SISO or SIMO, but it is the SISO S I part coming Single Input it is not MIMO.

So, I am not really utilizing the strength of MIMO in this particular figure configurations the strength of MIMO meaning every user should receive data from all the antenna, all the data stream not antenna all the data stream then only I can utilize the actual strength of MIMO other it is not utilizing the strength of MIMO.

The notion of MIMO is kind of lost for individual user though it is a MIMO system, but I would say it is a multi user MIMO, but individual point individual user is is receiving data

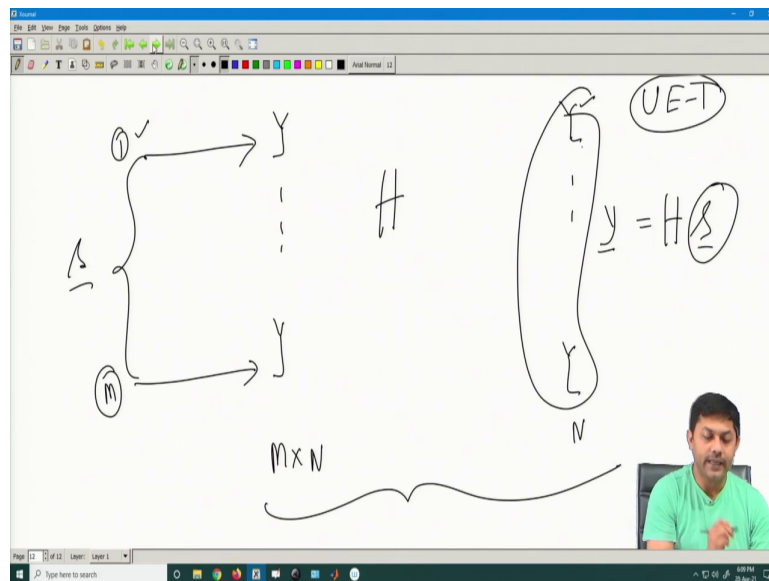
only from a single antenna. So, it is not really utilizing the power of MIMO. So, that is the one of the issue of this particular configuration ok. So, how do I solve that? So, that means, how do I ensure that the data that I want to send it to UE 1 is not just from s1 data stream, but from all these data stream get my point suppose UE is having a video transmission it is a video transmission.

So, it is not enough if I just send it from one link I have to utilize multiple links otherwise how can it be how can it serve a 20 or 30 mbps data link it is not possible right because it is just 1 s1 that is going to serve UE. So, unless I utilize each and every sorry each and every data stream the UE 1 will not be getting the complete volume of the data.

Similarly, for other users as well, so that means, how do I ensure that every user is receiving data from all the antennas all the data stream because that is precisely what the MIMO does if you remember what exactly you do in a MIMO.



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So, MIMO suppose I have  $M$  cross  $N$  MIMO's I have  $M$  number of transmitter side  $N$  number of receiver side  $N$  number of antenna here ok. So, what I do here suppose it is a single tap channel, I am not getting into the multiple multi tap channel here. So, this is my first one this is my  $M$  th one ok.

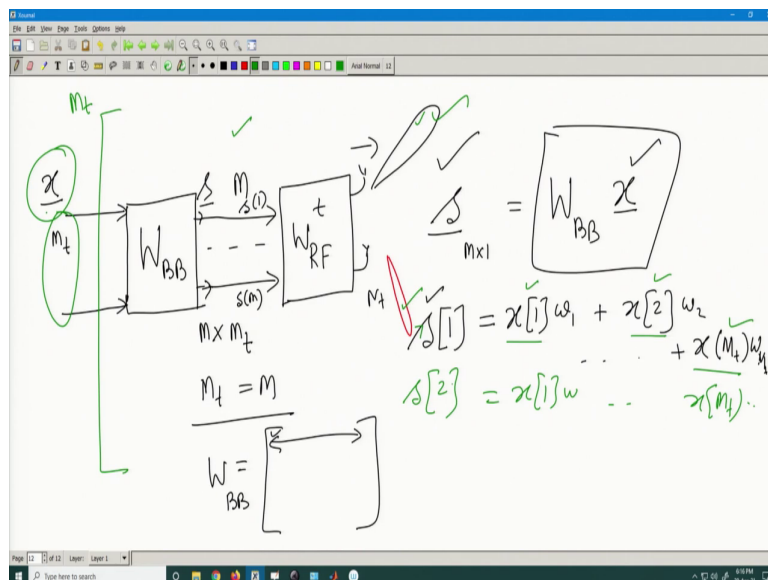
So, this is a user 1 or one of the user, you know what I do here this particular user you see it gets data from everybody because the data vector I will get it  $y$  bar is equal to see if the channel is  $H$  s bar say I say s bar I am sending it.

So, which means that this user which has  $n$  number of antenna it takes into account the data from all the data stream ok, but what we have just seen in the beam in the MIMO beam

forming is that s1 the s1 is receiving only the first user though it is a MIMO's case. So, it is not possible to have a scenario in that case.

Now, can I create a similar notion where a particular user if it has does not matter whether it is a MIMO multiple antenna and the receiver or a single antenna and receiver it receives all the data from all the data stream ok. So, that is why I need another structure in the earlier diagram ok. So, what is that structure? Ok. So, that is one of the motivation for that particular block of course, it has something else or as well.

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So, here what I was drawing it let me draw it mathematically let me draw the diagrammatically let me draw it. So, that it will be slightly clear for you see this is M number of data then it goes to W RF, let us not worry about the lets not worry about the exact because

this part we know how what exactly the structure. So, from here  $N \times t$  number of antenna this is going here right and here you have the  $s$  vector.

Now, what I do here is that following I put one more matrix here ok let us call it some  $W_{BB}$  ok let us call that matrix. So, which dimension is let us say  $M \times 1$  or  $M \times M \times 1$  or  $M \times t$  rather I should put it ok. So, what does it mean? It means that it needs  $M \times t$  number of data stream and it does something inside it and it creates  $M$  data stream ok.

So, that is the only thing it does and this  $W_{BB}$  is also a some sort of a matrix some digital matrix you can think of no complex structure nothing it is a simple of vanilla matrix, I think from this point of view.

So, what I get now we will describe we will describe what is this  $M \times t$  what is this  $M \times t$  data stream. So, this  $s$  this  $M$  data instead of having  $M$  data directly I am putting it pass it through one more matrix whose input numbers are  $M \times t$ . That means, instead of  $s$  number of  $M$  number of original digital data that I want to send it you send it  $M \times t$  digital data that you want to send it and then pass it through this  $W_{BB}$  and make it  $\bar{s}$  another vector ok. So, what is the advantage of it?

Now, you can have for the time being you can make it equal also, but there is no restriction you can have any number of  $M \times t$  and  $M$  ok, but if you make it equal then what does it mean? It means that lets call this vector is your  $x$  vector ok. So, what does it mean?

I get  $s$  vector whose dimension is  $M \times 1$  which is equal to  $W_{BB}$  a matrix multiplied by  $x$  vector ok. So, this is my original vector which is  $M \times 1$  that remain same, but before that I multiply I do this particular signal processing nothing just a one matrix whose dimension is  $M \times M \times t$  and this is  $x$  vector.

Now, this  $x$  I think of it is my original digital data that I would like to send it, it is just gets you know processed internally and becomes  $s$ , now this  $s$  is the one which is ultimately

feeding the beam forming part. Now what is  $s_1$  the first data element, what is  $s_1$ ? This  $s_1$  is the first row of  $W$  multiplied by  $x$  right.

So, if your  $W$  is some matrix. So, what is  $s_1$ ? It is the first row multiplied by this  $x$  vector. So, that means, it will be something like  $s_1$  some first element of this  $1$  plus  $x_2 w_2$  whatever blah blah plus  $x_N$  or  $M$  multiplied by some  $M$  here something like that.

Now, what I am trying to say here is that, every individual beam depends on individual  $s$  that mean here  $s_1$  is creating the is feeding the first beam  $s_M$  feeds the  $M$ th beam, they are orthogonal in space, but now notice this particular beam or any beam you take it that depends on individual  $s_1$ , but within  $s$  that is a linear combination of all the data. So, now I shift my focus what exactly my digital data that I would like to send it its basically  $s$  data that I would like to send it.

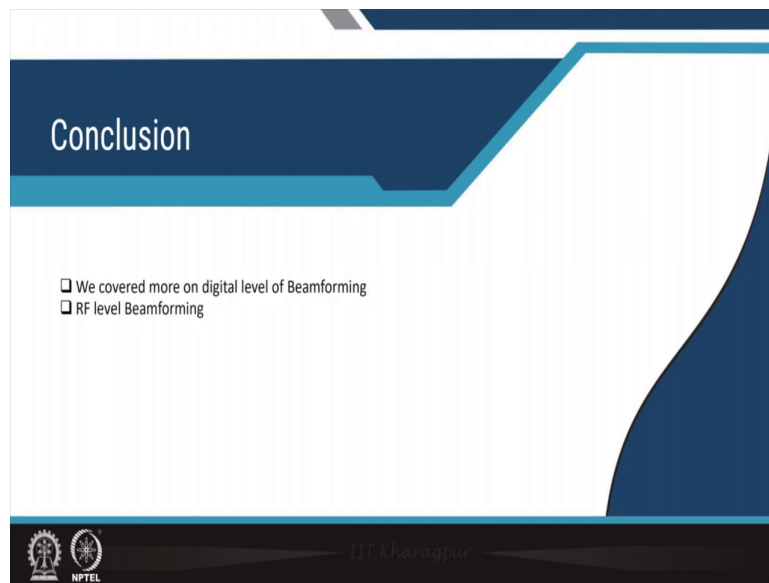
Now, this  $M$  number of  $x$  data I would like to send it to all my users, now actually we see no matter which beam you take it everybody will have some contribution of  $x_1$  to  $x_M$  correct. Suppose  $x$  is my actual constellation data that is my you know that is the data I would like to send it ok that is my information data. So, what I do I just mix it is like a masala you just mix it with  $W$  and create an  $s$  vector which will contain which will contain all the  $x$ . So, now, if you go to you know  $s_2$  what it will contain? It will be  $x_1$  plus some coefficient here it will be  $x_M$  multiplied by some coefficient.

So, that means, the weighted part of my  $x$ . So, that means,  $s_1$  will be here only for the first beam  $s_2$  will be in the second beam, but  $s_1$  will contain each and every  $x$  there ok. So, what I do I just mix up I just mix up all my data and I send it to one bundle. So, weighted bundle and that weighted bundle will goes to 1.

So, that means, now each and every beam will contain all the data. Now think this is my MIMO system I was thinking from here I should consider as  $M$  cross  $N$  MIMO right, I think this is from here I can think of my MIMO. So, it is like a  $M$  cross some MIMO system where I have  $M$  number of digital data and I am creating a beam forming on that.

So, each and every beam will be getting data from all the data digital data, all digital datas weighted part will be coming to each an individual beams ok. So, with this notion I end the this particular class in the next class we will be talking more about this W BB characteristic and so on ok.

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So, in conclusion we have kind of completed the yeah.

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## References

- ❑ Book : Wireless Communication principle and practice “ by T. Rappaport
- ❑ Cuinas I., Sanchez M. G,” “Wideband Measurement of Non-Deterministic Effects on the BRAN indoor Radio channel,” IEEE Trans. Veh. Technol., vol. 53,page 1167-1175,2004
- ❑ Chong Han et.al. “Multi- Ray Channel Modeling and Wideband characterization for wireless communication in the Terahertz band”, IEEE Trans. Wireless comm., vol.14, No 5, page-2402-2412

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And the reference will be same as what we have.

Thank you.