

Signal Processing for mmWave Communication for 5G and Beyond
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Module - 08
Details of Beamforming in mmWave and MIMO
Lecture - 40
MIMO Beamforming

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Concepts Covered

- MIMO level description
- Single antenna to multi antenna system.
- steering is done in Beamforming

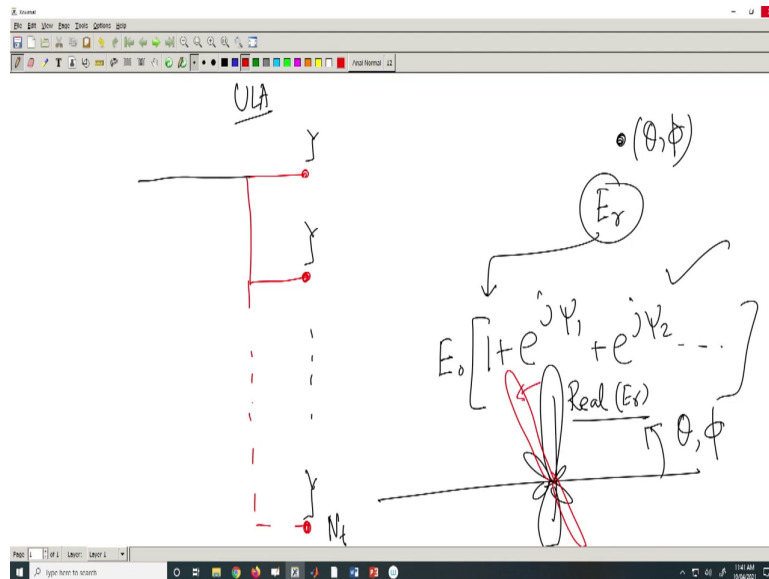
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Welcome back. Welcome back to; so, today, we will be covering the, like things that will be covering are the following, Signal Processing for Millimetre Wave for 5G and Beyond. So, last week or last classes basically, we have talked about the beamforming, and we just gave a glance of what exactly meant by the steering, ok. Steering is basically movement, right.

I mean just like you can think of what happens to a steering to a car or a vehicles. So, based on your you know steering control, I mean car can moved to a certain direction, the same thing, the name comes from the same thing. Now, we will see from the beam point of view how exactly a steering is done. And that has to be done from the electrical sense. So, let me just redraw the diagram for your convenience.

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So, this was the electrical signal, I mean you can think of whatever I am drawing it physically on the layout this is how it looks like, ok. So, probably I will put it slightly up, so that will be easy for you to understand. So, this is where a physical wire is coming. And say; this is where my antennas are present, ok. And what are the different types of antennas configuration that we have explained it in the last class. Like 1D, 2D, 3D, and based on that different set of beams will be created, right.

Now, let us say this is how a physical connection is. I am making it red, so that it will be easy for you to understand. So, this is the first antenna, and this is how a physically this is connected. This is the last antenna. Let us say I have N number of antenna, ok. And we have seen in the last class, how exactly a beam patterns will be formed.

So, that means, if I stand somewhere here at a position θ , standing at a position θ and ϕ , the cumulative electric field or I would say the field, do not do not do not worry about electric or magnetic, it is a field. Predominantly we dealing with electric field only. So, that would be nothing, but summation of this part $e^{j\phi_1}$ plus $e^{j\phi_2}$ dot dot dot.

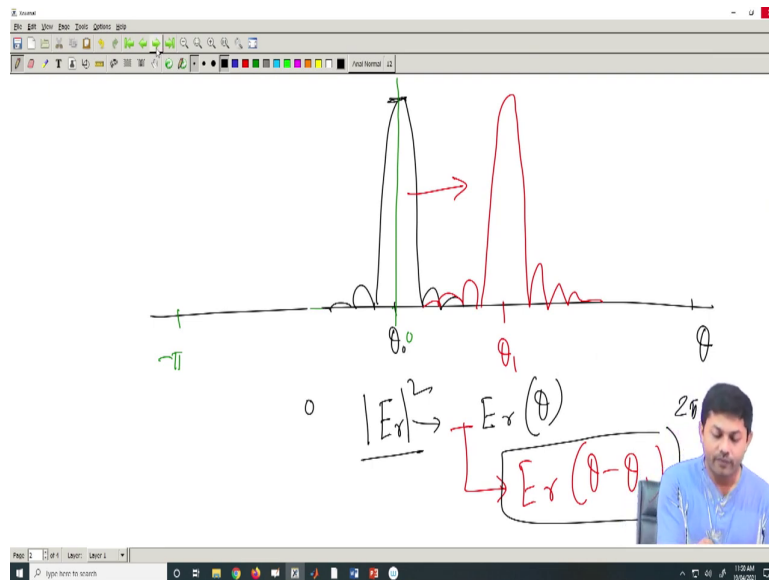
Then, if you plot this E_r part, it is a complex number. So, either you can plot it mod of E_r or just the real part of E_r or imaginary part of a , its your choice. So, if you just say mod that mean you only get the positive value of it if you get the real part, suppose, I just want to plot real of E_r , it is ok, you can plot anything, whatever. See and you create a polar diagram, so you may see something like that, some beams here, and there is a beam in between.

So, you can try it at your own end using MATLAB, and there will be a lot of assignments on that, how exactly your you know this beams part can be created. So, you can plot it whatever we want. So, and this would be a polar diagram with respect to θ . And you know if it is not ULA, had it been UPA, θ and ϕ , both will come into picture. In that case it cannot be just a two-dimension case, it will be a three-dimensional beams will be created here.

Beams will be three-dimensional, but it will be planned on a three-dimensional disc kind of things, ok. So, this is what we have learnt it. So, what we have not, yeah, in fact, concentrated on is how do you move the beam. Now, to move the beam, obviously, it is something with the angle, right. So, let us say I am in a ULA configuration, its easy to understand. Suppose, I am in a ULA configuration, I want to move my beams to this level, whatever. So, this is some θ angle I want to move it, ok.

This one way of looking at it or you can also look at it from a normal theta versus you know X-Y coordinate point of view.

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Instead of a polar diagram you can also you know draw it like that. You can also draw it, this is my angle theta, and you can plot it between either between minus pi to pi or 0 to pi or 0 to 2 pi, whatever, your choice. Let us say I plot it between 0 to 2 pi. So, you will see some places with respect to theta, if it is just ULA, it is easy to understand.

So, you will see at some point of you know theta 0, it will has a, it has a peak, and there will be some small bumps here and there, and so and so forth. So, this is how things can be, all right. And there may be a some other beams here some lower beams here. I want to shift the same thing to a angle say like that. It is more of a shift, ok. So, some angle say theta 1, ok.

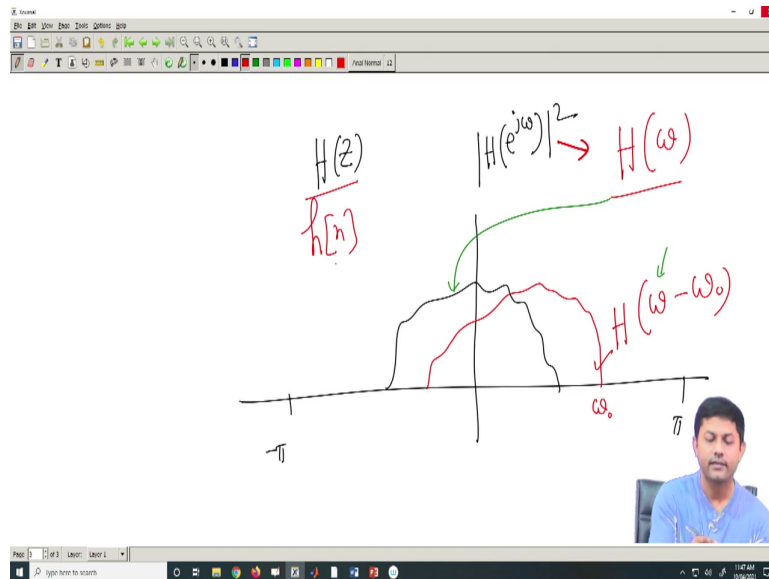
Shift, I want to shift it to an angle θ_1 , how exactly the beam pattern should be. So, what does it mean? It means that wherever it is from there I want to make a shift, right.

Now, let us assume for the time being that this θ_0 is nothing, but your 0. So, it will be easy to explain it. So, let us not worry about where the 0 here. So, let us say my 0 is starting exactly here. It is easy to understand. So, this is where your θ_0 and I am plotting it between say minus π to π . So, it is easy easier easy to visualize the things. But you can plot it between 0 to 2π , also the way you want to vary the θ , that is ok.

Now, from here I want to shift the beams from θ_0 to θ angle. So, what exactly I it, how do you map such concept to your spectral or spectrum kind of thing? So, if you notice very carefully what I have plotted is a electric field. So, instead of electric field, I could have also plotted some sort of a e , mod of e square which is kind of a power. So, it is mod of a power spectre, very related kind of thing. It is kind of a power spectral density kind of that, but it may not be the same thing.

So, it is like if it is some sort of a power representation, so what I am doing is that I am shifting the power. So, what does it mean? It is I mean kind of shifting the spectrum, right.

(Refer Slide Time: 07:13)



So, now, in a normal electrical case, suppose I have a filter whose transfer function is say $H(z)$, ok. How do you plot its spectrum? So, you plot the spectrum on a scale of minus π to π or 0 to 2π . What you plot exactly? You plot $\text{mod of } H e$ to the power you know $j\omega$ square you plot it, right. That is precisely what you plot it, right.

So, if you plot that what you exactly get? You get something, right. You get something. Suppose, somebody says, ok, you shift the spectrum. So, what you do? So, ok, suppose, you I want to shift the spectrum to some amount say ω_0 . What I need to do? So, here what I need to do here is that instead of you know, so it is I can say this is kind of a, this is kind of I can just make it H of ω , is easy for me to understand. Suppose, this one what do I what do I represent it? See, this will be H of ω minus ω_0 , right, right. That mean wherever ω is there, I have to make a ω_0 shift.

So, this is your this $H(\omega)$ is your black part and this red part your $H(\omega - \omega_0)$; so, that mean, if I want to shift my spectrum I have to just replace the actual ω by $\omega - \omega_0$. Similar, thing I can think of it here as well. It is kind of a spectrum shift, all right.

So, if it is a spectrum shift and say let us say I have plotted, I have plotted $|E_r|^2$ kind of a power, either $|E_r|$ or $|E_r|^2$, whatever you want to plot it. So, this is also some sort of a spectrum, it is showing with respect to the angle, right. I mean it is also with respect to angle. If you notice it, here also I am doing with respect to the angle, right, it is ω .

So, that mean in an angular domain, if I want to shift a spectrum, the only thing I have to do is that, so this was $E_r(\theta)$, I can think of. The only thing this red part, then it will be; it will be something like that, right. If I want to shift it m by θ_1 amount, somewhere wherever there is a θ , I have to make it a $\theta - \theta_1$.

So, now, let us go back to the normal system of linear equations. If this is your LTI system, and I want to shift the spectrum from ω to you know ω_0 , what is what I need to do in the time domain, because what you receive is a you know it is a time domain signal. So, instead of $H(Z)$, what I what you receive is basically $h[n]$. So, that is your time domain signal. So, shifting a frequency from ω to ω_0 , what is the impact on the time domain sequences, that you need to understand. I mean that is pretty simple to understand from a DSP point of view.

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$$H(z) \rightarrow h[n] \rightarrow e^{+j\omega_0 n} h[n]$$

$$\begin{array}{ccc} h[0] & h[1] & \dots & h[n-1] \\ \downarrow & \downarrow & & \downarrow \\ x[1] & x e^{j\omega_0} & & x e^{j\omega_0 (n-1)} \end{array}$$

So, which means that if $h[n]$ is a sequence, if $h[n]$ is some sort of a time domain sequence and I want to shift the spectrum of $h[n]$, the only thing I need to do is, I need to multiply it with e to the power $j\omega_0 n$, $h[n]$. That is the only thing I have to do, right. Sorry, this should be plus, that is the only thing I need to do. Plus or minus depending on whether it is a positive side or negative side, I can put plus or minus. That is the only thing I have to do.

So, which means if I multiply it e to the power $j\omega_0 n$, n , n is very important. This you remember. It is not e to the power $j\omega_0$, n has to be there, that is the time domain. So, which means that if I think of a sequence, so let us say $h[0]$ have a sequence, and I am just putting the sequences like that, $h[n-1]$ and so and so forth.

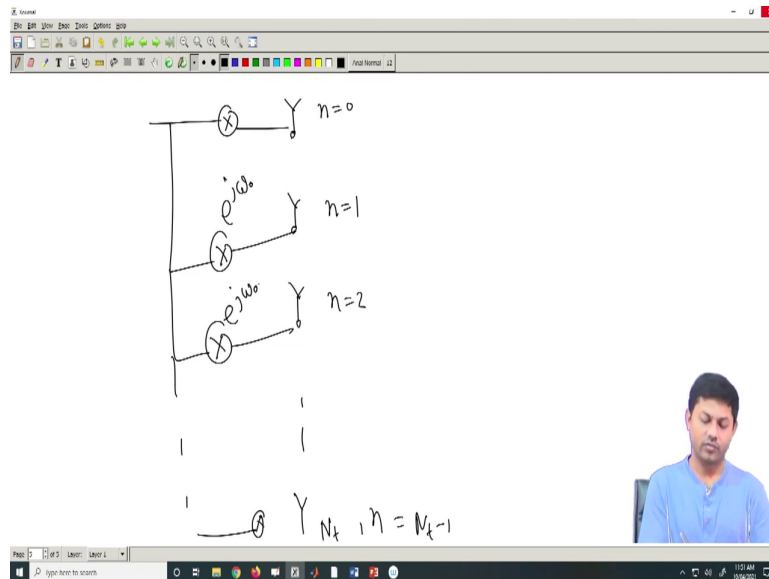
So, what is the impact on this time domain sequence such that there is a shift in frequency domain? So, I have to multiply this by 1, I have to multiply this by e to the power $j\omega_0$,

and so and so forth. This sequence I have to multiplied by e to the power $j \omega_0, n$ minus 1, that is the only thing I have to do. That means, at every time domain sequence I have to just multiply it with the phase, but there has to be an n there, so that it will be $\omega_0^2, \omega_0^3, \omega_0$ and so on and so forth. Then only there will be a shift in the frequency domain.

Exact similar, now why I am talking all this? Because I want to create an analogy, analogy between this antenna spectrum with respect to the normal DSP spectrum that we see. So, you just think from that analogy, this is the similar thing that I also need to do here. So, to create a shift of my spectrum from whatever angle it was, θ_0 , say, to another angle θ_1 , the only thing I have to do is that I have to just multiply in some time domain, somewhere there is a time domain that we have to see it what exactly the time domain concept comes into picture.

That is nothing called a time domain here, ok. So, what is the analogy from there to here, something we have to do it and so that way I can create a shift. Now, now here it is easier, right. So, I can always have a $H Z$, and then I can always create some sort of a time domain. So, it has a time domain sequence. On that time domain sequence, I can multiply. Now, what is the; what is the similarity of this part with my antenna sequences?

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Now, from the antenna point of view, the only thing I can think of is, this is the first antenna, this is my second antenna, this is my third antenna, and so on and so forth, this is my say N_t antenna. Now, this I call it say n equal to 0, this I call it n equal to 1, this I call it n equal to 2 and so on, so forth and here it will be n equal to N_t minus 1.

Now, you can get, you can imagine what I am going to say next. So, it is like the value at each and every point, the relative electric field at each and every point, I can think of it as a kind of a equivalent time domain sequences. I mean if I create an analogy between this part and my you know DSP part. Now, the point whatever I am you know I am doing it the same thing I can it is as if like it is a N_t number of you know time domain sequences and they are just getting said.

So, which means that this if you look at the what I am doing it here, multiplying with this way I am multiplying it similar thing I can do it here as well. So, which means there is a sequence here, multiplied by 1 of course. This sequence I need to multiply with, you can imagine what I am multiplying with, right.

(Refer Slide Time: 14:38)

Phase Shifted Antennas

Phased Array antennas

$n=0$

$n=1$

$n=2$

\dots

$n=N_t-1$

$|e^{j\theta_1} E_r| = |E_r|$

\Rightarrow

θ_1

See, if I multiply this with $j\omega_0$ this will be $j\omega_0$, instead of ω_0 , I should write it θ_0 or θ_1 because that is the amount of shift, I want to do it. That I need to multiply it, ok. So, if I just create that multiplication the spectrum will be automatically shifted.

If I just make it equal will the spectrum be shifted no because the only thing that will happen is that E_r , e to the power $j\theta_1$, the common part will be coming out. If the common part

comes out when I take the mod it will be equivalent to E_r . So, no shift in the spectrum happens.

To shift in the spectrum individually I have to do the same similar things. It is as if like I have a you know time domain n number N t number of time domain sequences, I want to shift the spectrum by θ and θ angle, individual lines I have to multiply it accordingly as of it. Now, this one whatever configuration you are seeing it, it is a ULA configuration, this one will definitely create a shift of my spectrum, ok. So, if you redraw the spectrum, so you will see that instead of default value, so at θ_1 the spectrum may have been shifted.

Now, because of this there may be slide increment, slight rapturing happens in the minor lobes that you have to see it how the that gets impacted, slightly sometime happens. but more or less the spectrum gets shifted to an angle θ_1 . Now, this is for a ULA case. A similar things you can draw it when it is UPA or 3D, any other configuration, whatever configuration you would like to have it, you can just do that.

So, now, in summary if I want to do a steering what I need to do is I need to appropriately multiply the sequences with these values. Now, what are these? What I have multiplied with? If you notice it, I mean RF domain, right. I am not in digital, neither I am in analogue, I am completely in RF domain, right. So, what does it mean? This signal, this is high frequency signal, it can be say 60 gigahertz also.

So, what is this consequence of multiplying with e to the power $j\theta$? The consequence is nothing but phase shifter. So, these are all phase shifter, so whatever this multiplier I have drawn. So, individually they are all called phase shifter. They are called phase shifter, ok. So, in the RF domain what I am doing? I am basically changing the phase, ok.

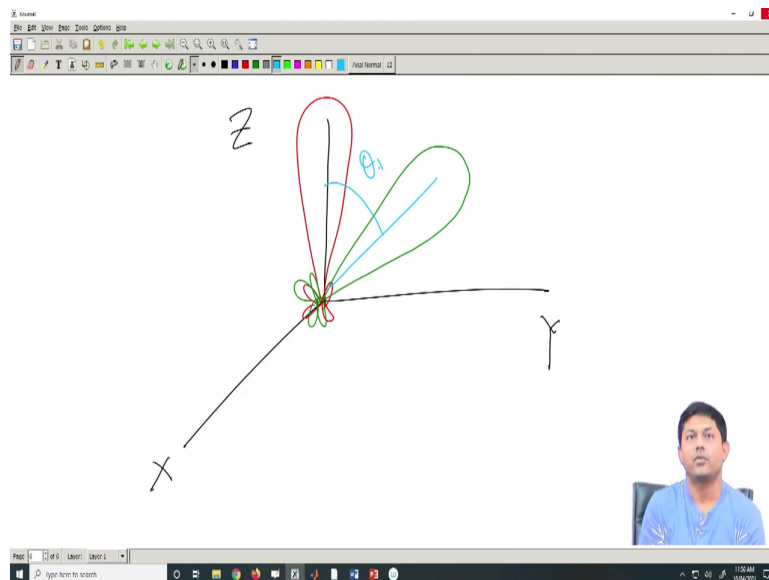
Now, this is also called a phased array antenna. It is a phased array antennas. This is array antenna, but it is a phased array antenna. So, you can create a phase of your choice and you can steer it, ok. So, here I really, now, if you think the difference between this one and the

rudder, right in a rudder also you actually you know shift the beam, right but there it will be a mechanical system.

So, what people will do is they rotate the rudder, ok. If you rotate the rudder, it will obviously, create a beam, but here we are not rotating any antennas because our antennas are very small scale antenna, they are very you know they are like fixed point antenna. So, what I am doing is I am creating a phase. So, I am creating a different phases, I am multiplying with the different phases and that creates a steering, ok.

So, which means that if I just little bit of more drawing if I create it, I may have a beam like that. So, this is say X, this is say Y, this is say Z, ok. I may have a beam like this by default something like that, ok.

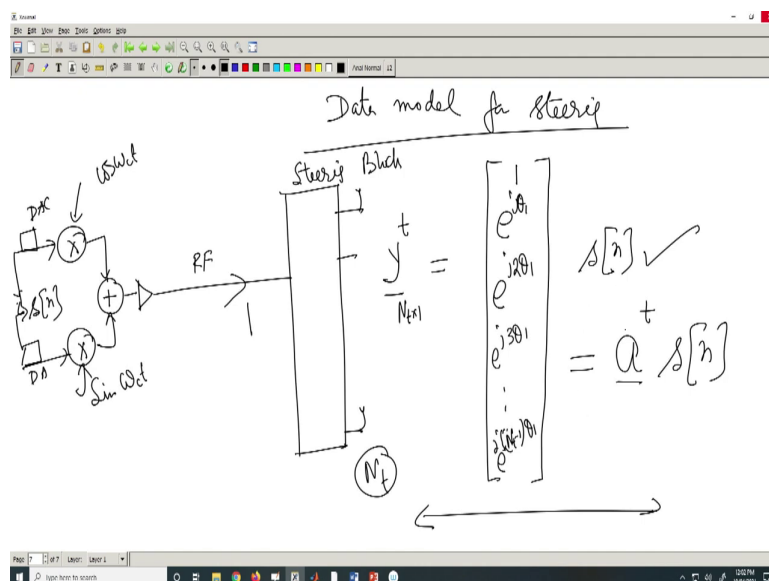
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Now, because of the steering the beams will be rotated. Now, there may be slight disruption in the minor lobes, but more or less the shape remains the same, more or less, ok, there is no guarantee as well. Slight rupture happens, but you can think that this is what I can. So, this is the amount of you know rotate happens. So, this is the impact of you know rotation, ok.

Now, this is at the transmitter side, ok. This is exactly at the transmitter side, ok. Now, we are signal processing people, can I think from a mathematical modelling point of view? Ok. So, everything is kind of a mathematical modelling. So, this is what it is done physically, ok. There is a phase shift there is something, but can I think from linear algebra, those kind of data structure point of view. Because it will be easier for us to proceed for further processing. So, now the whole thing I can think of it as a linear algebra point, and this is what the data model part.

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So, now, I will be talking more on the data model part. How do I do the data model for steering? So, the data model for the steering. So, here what I can view it? I can view that, I have a signal, and that goes through a certain phase shifter and that individually gives me the data. That is all I can think of, right.

I do not get into how the structure of the RF is, what is the circuit is, nothing, for me is like a one line data and multiple line of phase shift. Obviously, in internally, there will be all like, this is my steering part, this is my steering block. This part is nothing but you are here, this block diagram, this block diagram part, ok. This part, ok.

Now, how do I mathematically model it? So, I get one input, ok, and I get multiple output. So, I can think of this is one input and there are N output. So, how do I define it? Suppose, my signal; now, whole thing you know you view it from; you know view it from a baseband point of view, just think from a baseband point of view.

So, which means that you are standing, so, here of course, this is RF. So, there will be there may be a power amplifier here, if you wish. Let us not worry about the power amplifier multiplication part. So, this is my addition, this is my i and q part, this should be my RF multiplication my modulation, this should be say $\sin \omega_c t$, this will be multiplied with $\cos \omega_c t$, and this is my input from DAC part. This is what we have drawn it, right.

So, this is coming from a DAC. This is coming from a DAC. This is DAC. This whole thing you can think of a digital s_n . So, this s_n is going to the DAC, to DAC, i and q , then it will be you know going to two different multiplier, that RF multiplier, then added a power amplifier and from there it got split, the steering part. And then, it goes to different different antenna. That is precisely the actual circuit is.

Now, how do I model it? Now, I am standing from here. If I stand from there the only thing, I can think of is that it is as if like I am sending one digital signal and it gets the split. So, can I think of it like that? So, if I say this is my output vector, ok at t side, y side, right. That is exactly my you know transmitted output signal.

So, can I call it like this? I am completely modelling from a DSP point of view. I am standing in a digital domain, and I am trying to view it from there. I am not really interested in how exactly RF time domain sequencer. For me is everything is like a digital. So, I send s_n , how do I view it from that?

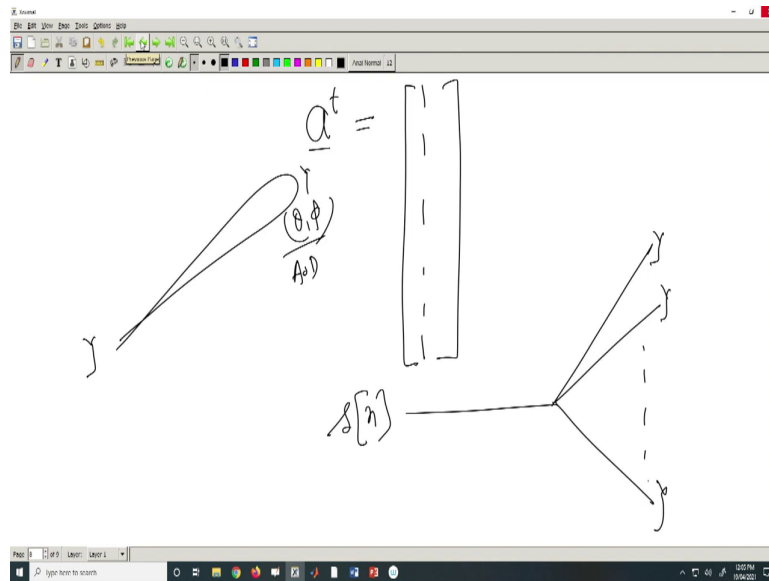
So, if you notice the way we have you know model the retracing channel, right. At different point I am standing and I am viewing the thing. So, it here I am standing at s_n and this is how I am viewing. So, how do I? So, it is something like a this is a vector y_t vector, $N \times 1$, right, and this is my s_n vector. So, how do I treat that? It is 1, then it will be $e^{j\theta}$ to the power you know $j\theta$, it can be $j\theta$, whatever, how much angle you want to shift it say, let us say θ angle I want to shift it. This will be $e^{j2\theta}$, it will be $e^{j3\theta}$, it will be $e^{jN\theta}$ minus θ , right.

So, this I can think of it is like a some complex number, my s_n is multiplied and this is what I am getting it is an y_n . That is all my view of my data. That is the view of my data which is coming out of my antenna, that is my digital view, ok. So, this is what I am sending it, got it. Now, this part is mainly for you. So, that is your data. Now, it will go through channels, so and so forth, all these things.

Now, can I put this whole thing in a much more compact form? This is precisely your a_t . If you notice, if you look at your earlier notes, this is precisely your steering vector, a t vector. So, that is my view from digital pointer. So, a t vector s_n and that is basically my. Now, this will be of course, used, this whole part, every part will also be coming into my channel of course, because that is my steering vector and that steering vector will definitely come to my channel part.

Now, if I do not use any steering vector, what was the configuration if I do not use any steering vector? So, it was like a as if like it is like a broadcast kind of thing, right.

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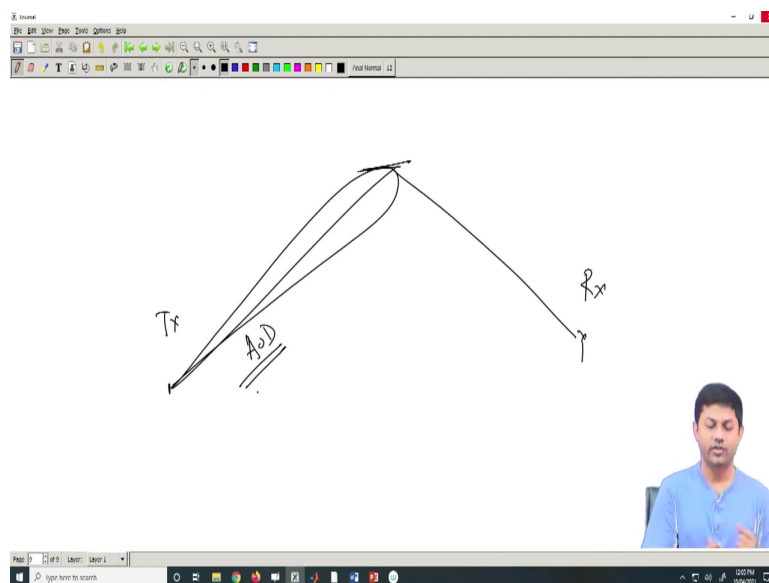
So, there your a^t vector will be all 1. So, what is the physical configuration? Physical configuration is like from this antenna I am getting. So, let us not worry about this RF and DAC, and all these things, is as if like I am sending $s[n]$ and it is going everywhere. This is where my first antenna, second antenna, and N antennas, ok. So, this is how my view. Say here, my a^t is all 1, right, that is it. So, how exactly you know your view of the system happening, based on that, you can have your configuration.

So, in summary, I can say that steering depends on how much you exactly want, how much you want to steer. And also, who gives you that feedback, that I want to shift it on this much angle, on θ and this much angle of ϕ , it depends on your angle of departure, right. So, that was, that will be discussed more. We have given a basic idea of what is angle of

departure and what is angle of arrival, of course, in subsequent classes how exactly you estimate it, that will be clearer.

So, basically this is the angle on which I need to you know transmit my beams, so that there is a receiver at that angle and he is receiving it. So, as simple as, is as if like I have a beam of antennas and some somebody is sitting here in an angle of theta and phi.

(Refer Slide Time: 27:29)



So, this is your angle of departure because that is the angle if I send it, it will reach it or if there is a you know if there is a reflectors, angle of arrival, angle of departure will be more dependent on how the reflector is present. Say for example, this is where I am sending here, this is my receiver here, and this is where my transmitter here, ok.

And so, here angle of departure is mainly on this side because if I send it to this angle then there is be a reflector who will send it further in a different direction. So, here, this will be the angle of departure in that case. So, depends like how you have your angle of departure and that same angle of departure will be fed to the transmitter and that fellow will create a right steering, ok.

And how to estimate that angle of departure, the it will be talked about in subsequent classes, how angle of departure, angle of arrival will be estimated, and that feedback should be given to the transmitter, and based on that transmitter will create the steering, ok. So, I hope the steering part is clear. So, it is very straight forward for a single antenna input, single antenna output, it is only a phase at your you know input side antenna data, ok. So, with this I conclude the session of Steering Part.

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Conclusion

- We covered MIMO level description
- Single antenna to multi antenna system.
- steering is done in Beamforming

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So, in conclusion, we have kind of completed the, yeah.

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And the reference will be same as what we have.

Thank you.