Signal Processing for mmWave Communication for 5G and Beyond
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Module - 07
Details of Beamforming in mmWave and MIMO
Lecture - 38
Basics of Beamforming patterns (part-II)

Welcome back. So, in the earlier classes, we have shown you this diagram, this configuration of the diagram; that where we have a two antennas and we would like to have the received electric field.

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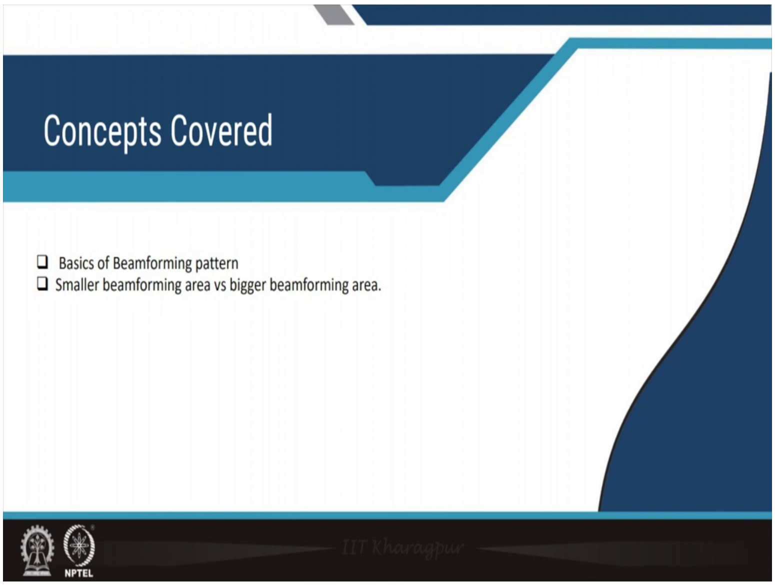

So, today we will be covering the like, things that will be covering are the following and how exactly the received electric field would be with respect to the angles, ok.

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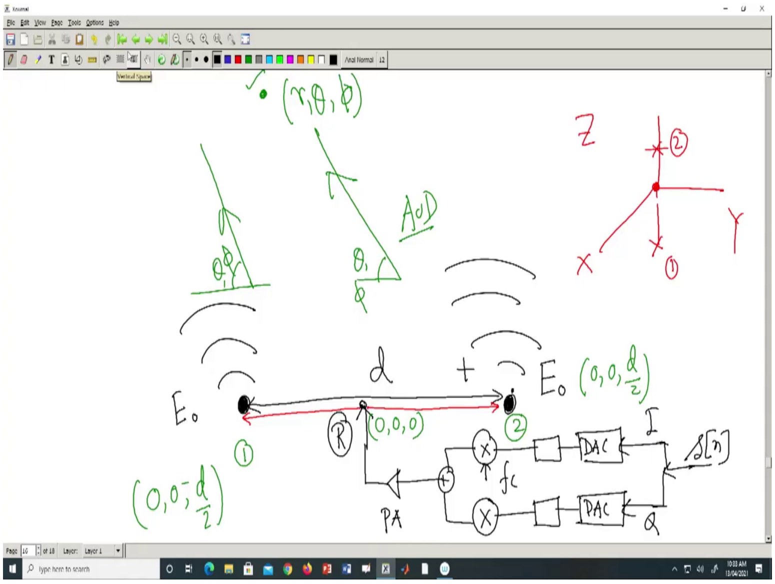

Now, had it been an isotropic case, what would have been the case, what would been the received E r; it is just like a one E coming into picture, right. So, it will be like a ball, football kind of thing.

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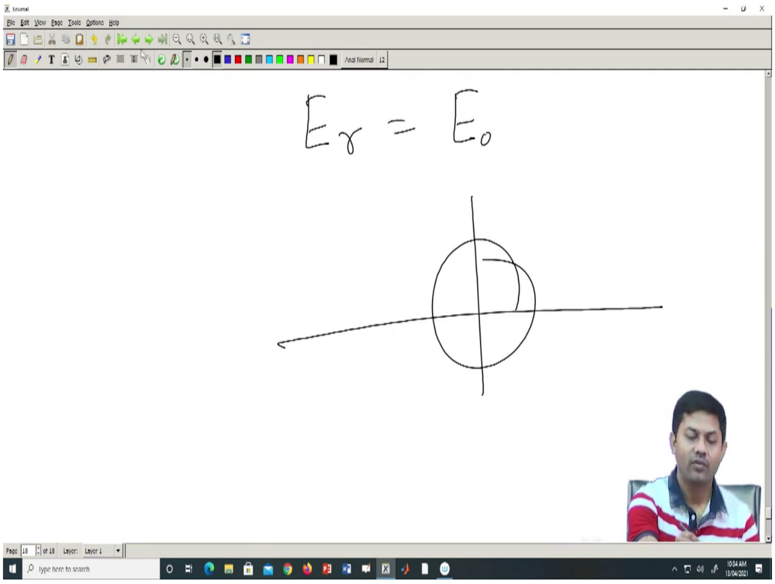

So, if it is just one antenna, your E r is nothing, but E. So, what does it mean? If I plot it with respect to any of the elevation and azimuth angle, that will be like a ball; there is no difference, that is what the isotropic case. But now because I have another antenna, which are physically apart; so what is the resultant E r that we have shown in the last class, this is the equations, right.

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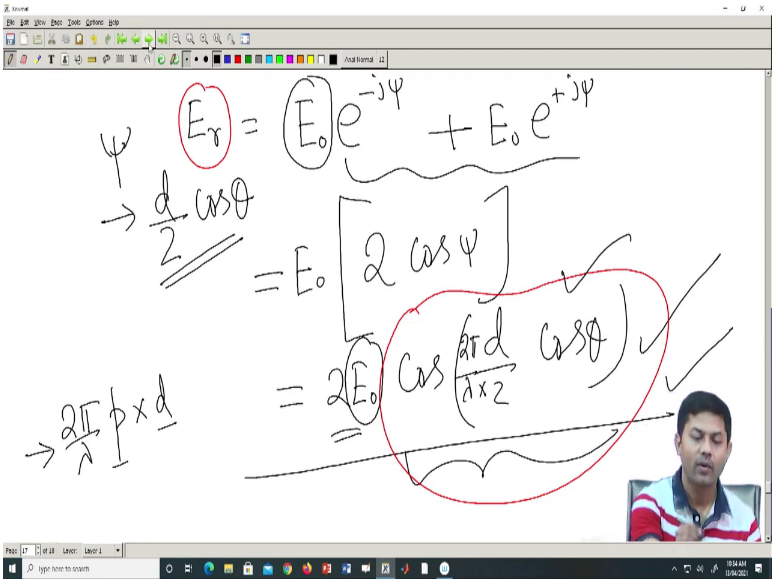

Now, I will be plotting this particular E r with respect to which variable; you can guess with respect to only theta here, because this is the only angle which is available to me. Had it been some other configuration of antenna, maybe phi will also come into picture; but as it is just a u l a, antennas are sitting on the z axis, this is what I am getting, I would like to just plot it and let us see what happens, ok.

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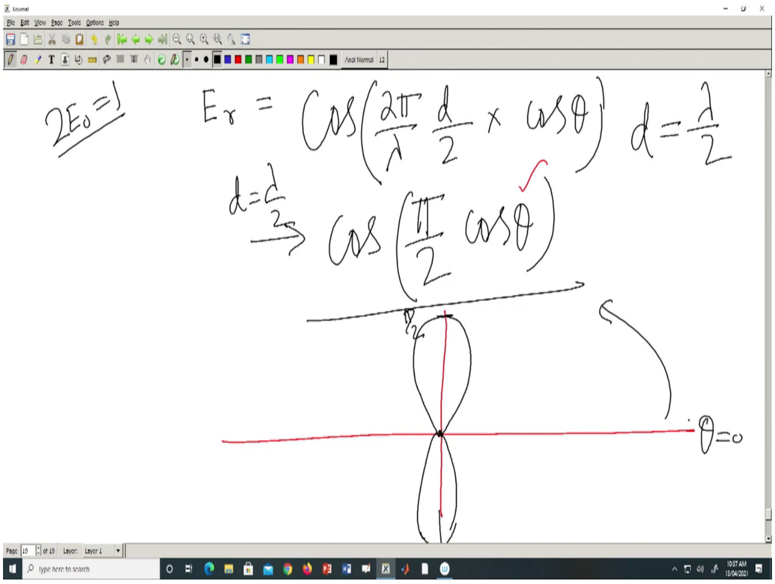

Now, E r let me redraw it, E r it is 2 E 0; let us assume 2 E 0 is 1 for us, because as we said the amplitude of the spectrum is not so important. What is what matters to us is the phase part, ok. So, let us not worry about the 2 E 0 part. So, finally, I will be getting cos 2 pi by lambda into d by 2 multiplied by cos theta, ok.

Now, you can see the d matters, because d is the separation of two antennas. Let us assume that for this particular case d is lambda by 2, ok. So, that is the bare minimum requirement for any of the antenna processing. So, what it would be then? With d is equal to lambda by 2, this will be cos, you just put lambda by 2 here. So, this would be 4, which will be pi by 2 right; that is what it would be ok and multiplied by cos of theta. So, that is the final E r comes into picture, multiplied by 2 E r that is ok, I am not so bothered about that.

Now, you plot it, what it would be? Let us say I plot with respect to only theta, because that is the only angle I am getting it, ok. What it would be? You see when theta is 0, what will happen? When theta is 0, this will be pi by 2 cos pi by 2 is 0. So, when if I just plot, so this is my theta. So, this is theta equal to 0, I go like that, ok. So, this is theta is equal to pi by 2 and so and so forth; this is how normal coordinate system is, right.

So, when theta is 0, what happens here? This is cos pi by 2 which is 0, so the value is 0. But when it is maximum, when theta is cos theta is pi by 2 theta; when theta is pi by 2 what happened, this cos pi by 2 is 0, then cos 0 1. So, it is 1. So, it will be shape will be something like that; this is how it turned, this how the shape is right, that is interesting point.

So, now you see, it has formed the beam ok at an angle theta equal to 90 degree. So, what does it mean?

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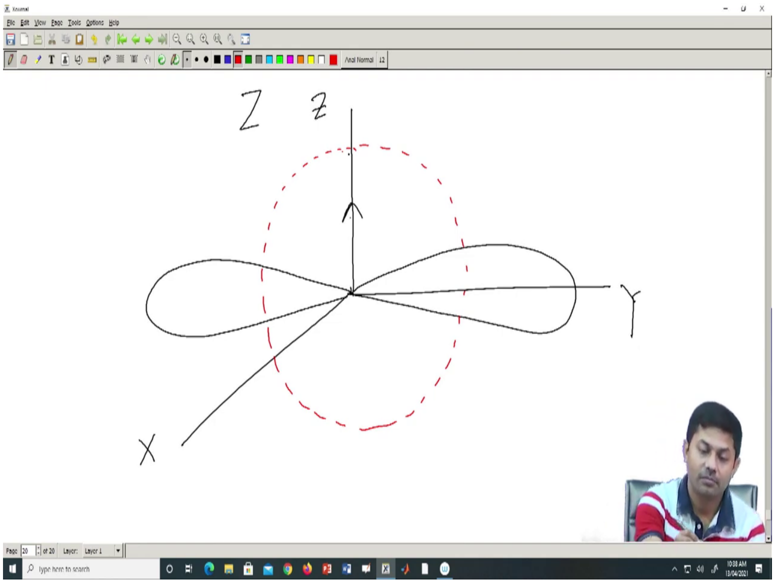

It means that, if I physically now try to understand what it means. So, let us say this is my X, this is my Y and this is my Z, alright. So, what it means? It means that, so this is my z angle, ok. So, act so, that mean my antennas are placed like that; but at theta equal to 90 degree, which is like this I have the maximum power like this. So, this direction my antennas are placed and in the Y direction mainly I can say my beams are radiated.

So, now it is no longer, it is no longer an isotropic radiation. So, if I draw the red part, that is more of a single antenna isotropic radiation; just because I have added one more antenna with such configuration, the whole thing changes here, ok.

Now, from the same angle, from the same configuration what if I take d is equal to lambda? You can play with it, you can put it in any software like octave or MATLAB or in even c also; you can just try to program it and see what is your E r coming for different value of d.

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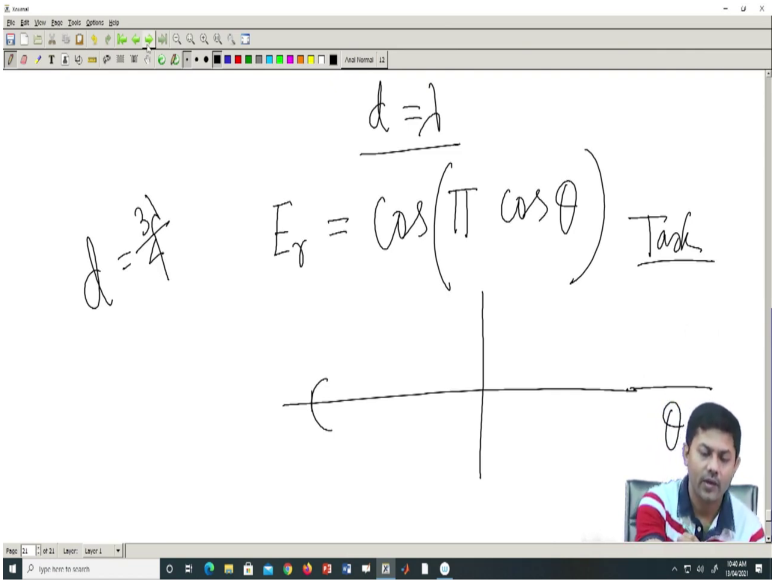

So, for example, what happens, probably it is a task you can try it. What happens when d is equal to lambda, the same configuration. So, what will happen? So, that will be how it is be, how it will be varying? It was d is equal to lambda, so it is a pi now; this pi cos of theta, right. Now, what does it mean? So, when theta is half, then only cos will be 0 right and otherwise it is not.

So, you can have, I mean you can plot it, try to plot it and see what happens. So, see what happens when cos theta is equal to 0; what does it mean? So, when cos theta is 0 here, this will be 1; so, cos pi is minus 1, so it will be somewhere here, ok. So, all things can you know start there, ok. You can see how the prediations would be, you can plot it; I mean that may be a task from my side to you, you can try it, I do not want to draw it, you can try it. See you plot it theta and see the polar diagram there, ok.

So, I can just give an hint when theta equal to 0 what will happen; this minus 1 and plot it, what happens when theta equal to pi by 2. When theta equal to pi by 2, this will be 0, this cos 0 will be 1. So, you can try what happens, I do not know just see what happens there.

So, these are the different ways that you can plot it, ok. So, another task is what if d is equal to say 3 lambda by 4? How the spectrum would be, ok?

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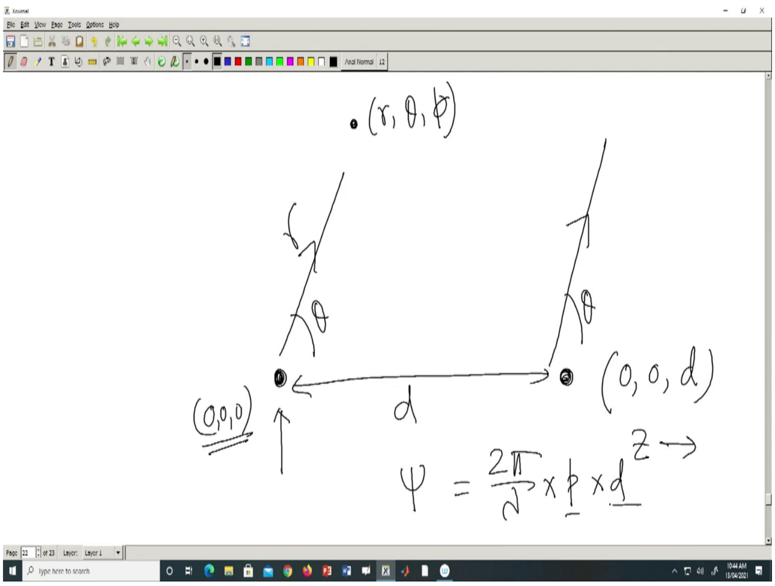

Now, let us take, let us take some more example. Let us say instead of these two antennas configuration, what if I change my antenna configuration, so that my feed is from here and this is my reference point, ok. And so, the distance is d; so let us assume 0, 0, d, it is in Z direction.

If it is in Z direction, so what is my; so I am standing here at a position, I am standing here at a position r, theta, and phi here. So, how would I see my radiation? So, it is like as if like one ray is coming and another ray is coming like that; never draw this, that is confusing. Why this diagram is wrong? This diagram is wrong, because we are thinking of a far field and this distance d is very small compared to this r.

So, this will never appear, this will never appear, always it will be parallel, ok. So, I should not even draw this; I drew it only for your clarification, this is how always in the parallel direction, ok. So, next you know what exactly we need to draw here. So, this is angle theta, this is angle theta. So, what is the phase difference? So, how do you, how do you get the E here?

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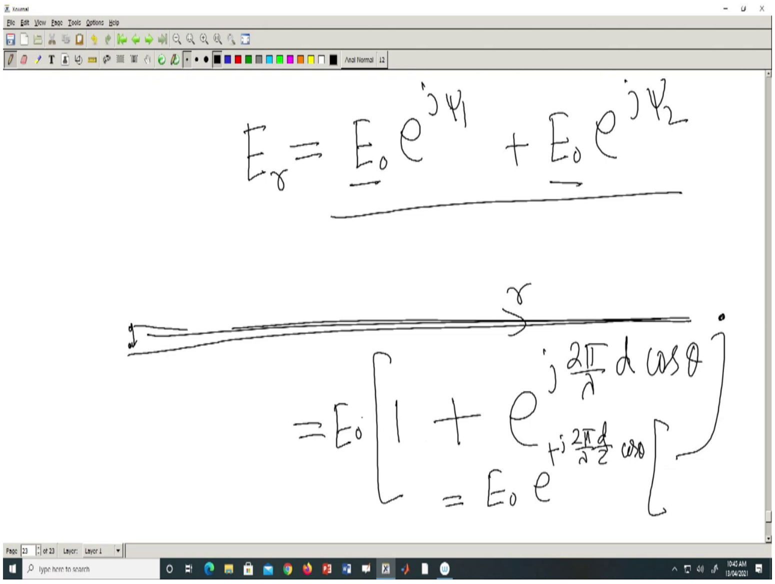

So, this E will be received E r is equal to E 0 into e to the power j phi 1 plus E 0 e to the power j phi 2. By the way, will the E 0 be different? Because it is coming from two different sources right; so the magnitude and all the phase will also be different, right. Logically yes, but practically no; because again I am taking a far field approximation, so which means that my actual physical antenna might be so small here, what I am observing is much far. So, this r is much larger.

So, the actual path and this actual path may not have two different gain, ok. Only thing that they have is the small phase difference and that too only because of, only because of the position between the antenna, not because of the absolute path of the antenna, ok. I think we have explained it, when we talked about the channel part; similar assumption, a similar concept I can also hold it here.

So, now this is what my E 0 to E 1. So, you will look at this configuration, how much E 0 E 1 would be. Now, this part will be vanishing 1; because why, because position of the antenna is 0, 0, 0. So, what is the phase here? So, every time you always write phase is equal to 2 pi by lambda multiplied by position of the antenna vector multiplied by position of your own antenna, receive antenna vector. So, that is the basic difference of the phase, ok.

Here also I will do. So, this will be 0, because reference point itself has the antenna plus 1 j 2 pi by lambda into, if it is a d cos theta. So, this is what it is, ok. Now, you can see that there is a; now this is a complex number, so how do you plot it, ok? Because electric field now it becomes a complex here. So, in this particular case, you take the absolute part out. So, it means that E 0 e to the power you know j, half part you just take it out and again. So, there will be one part which will be 2 pi by lambda d by 2 cos theta by 2 for plus.

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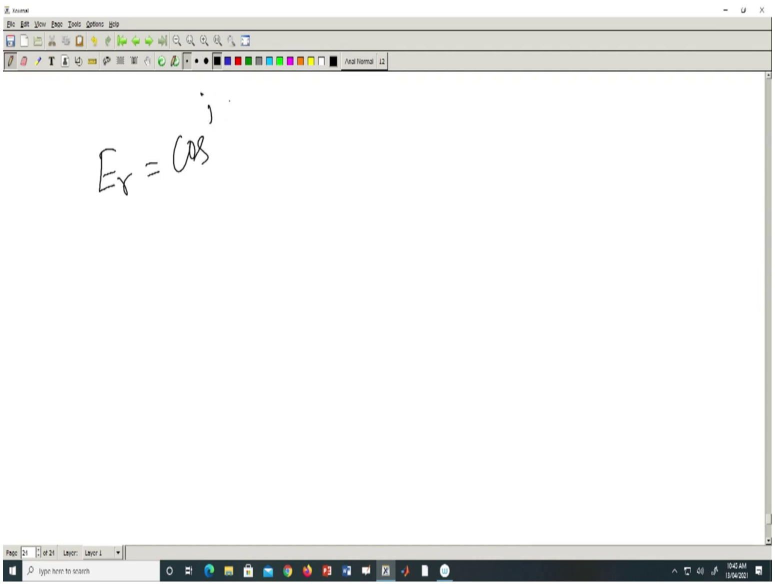

So, let me just put it in the second part. So, you E 0 when you have E r; when you have this complex part, take the common part out just for your own benefit. So, E 0 you do not worry again as said, e to the power j 2 pi by lambda.

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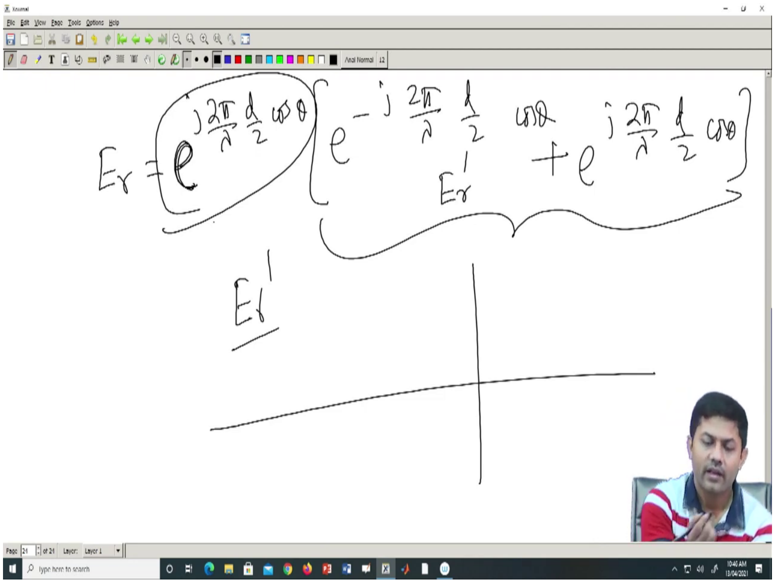

Let us say it is I take it half part of it cos theta, e to the power minus j 2 pi by lambda d by 2 cos theta plus e to the power j 2 pi by lambda d by 2 cos theta.

Now, this whole part you do not worry; this is a one common part right, that it does not make much difference ok, because this is only the magnitude part, you see this is only the magnitude part, I mean this part magnitude is always one. So, I am interested to know how this part varies ok. Now, this part you can plot it and see how the phase will be right; how exactly the pattern should be, E r would be. Now, I call this part as a say E r dash, plot E r dash what is the big deal here, just take the.

Because the relationship between E r and E r dash is only the phase part, one common phase; because I am not interested in a common phase part, I am interested always in the how the shape would be as I repeat. The phase repeat part, I mean the common part I am not interested ok, this is how it is. Let us say I make the point even complicated; let us say instead of one antenna, what if I take multiple antenna?

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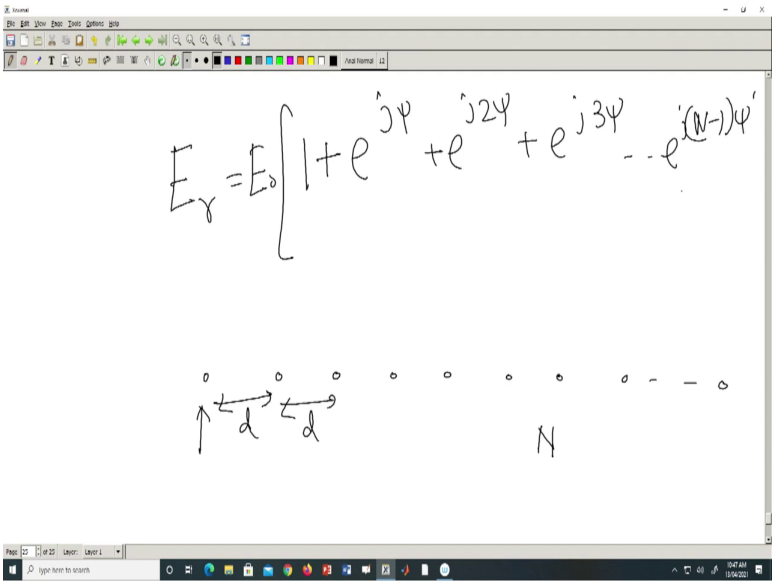

All in the z direction, I am taking N number of antennas, how the and this is my feed point, ok. So, what is my effective E would be? Effective E r is nothing, but E 0 again something that I am not so interested; 1 plus e to the power j phi plus e. If everything is a d distance, what it would be? J 2 xi plus e to the power j 3 xi e to the power j N minus 1 xi, this is how it is, right.

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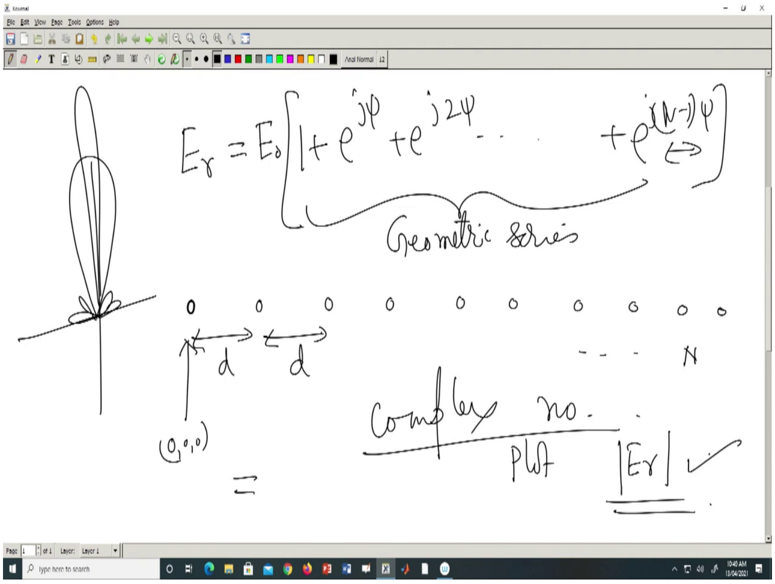

So, let us say these are my antennas, ok. Let us say I have N number of antennas here. So, this is my feed point, where my coordinate is 0, 0, 0 and the distances is d like that, ok. So, what is my E r? E r would be E 0 1 plus e to the power j phi e to the power j 2 phi 2 xi plus e to the power j N minus 1 xi, correct. Now, what it would be? It is nothing, but some geometric series; take the geometric series and see what happens, ok.

If it is if you just take a geometric series, it will have a complex number. So, we just some complex number, ok. Now, for your simplicity, you can plot mod of E r; just like the one which we did earlier, but it was a real case, so there was no need. But if you do not want to create other things, you can just plot the E r part itself; mod of E r and see how it looks like, we will see that the plotting with respect to theta will be showing something like that, something like that will be seen.

So, as the N grows, this will be sharper and sharper, ok. So, this is exactly what happens when you do for multiple antenna. Now, what happens when I take a different antenna configuration.

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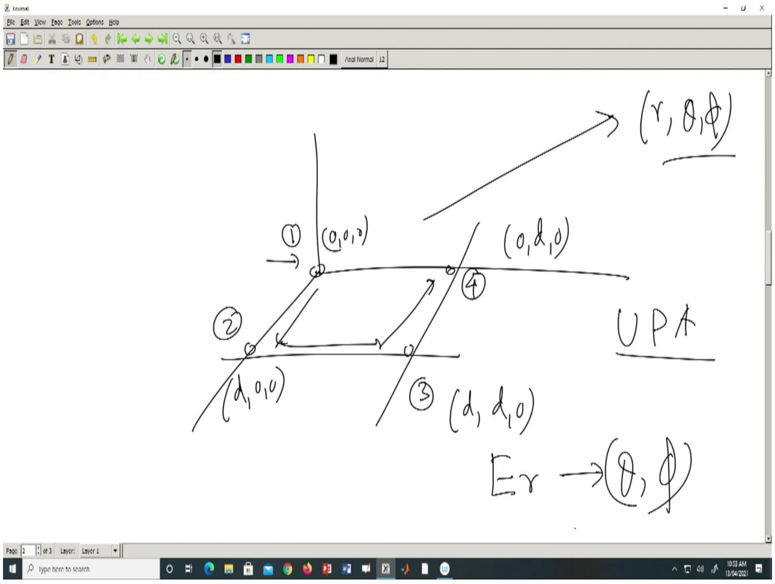

Let us say my antennas are like that, I mean the similar thing that I have done when you did the channel kind of. So, this is my feed 1, this is my feed 2, this is my 4; how this is how the radiation is going, I mean the electrical connection is going the order and this is my reference point.

So, reference point is say 0, 0, 0. So, this would be d 0, 0; this would be d, d, 0; this would be 0, d, 0 these are my four antennas, right. I want to know how the radiation pattern would be at a point theta phi. So, how many antennas are there? Four antennas. So, what should I do? So, E, again E r, ok.

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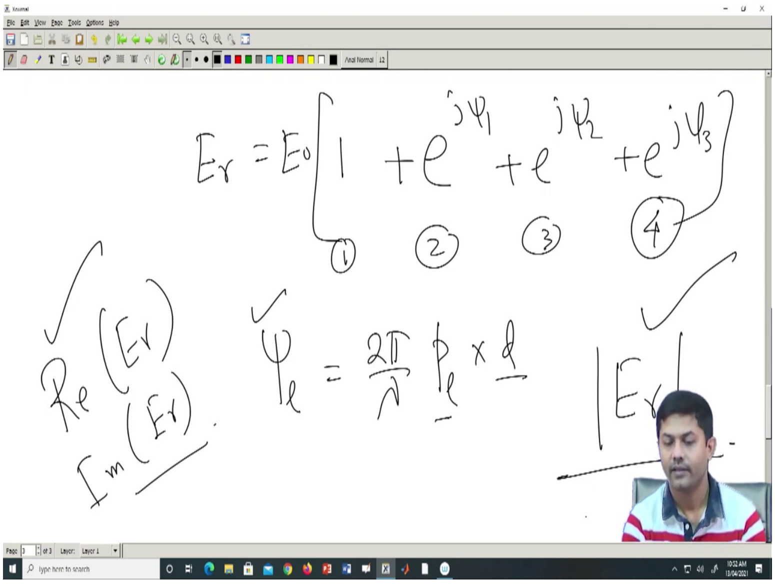

So, what should I do? For the first case it will be 1, because reference point there is a one antenna. Similar thing what we have tried it in the edge; but there we are creating a vector, instead of vector now everything will be added up, because it is the combined four antennas will be come we will be the case, right.

So, this will be j of phi 1, this will be j of phi 2 e to the power j of phi 3; this is antenna number 1, antenna number 2, antenna number 3 and number 4, ok. How do you get a particular phi l? For that you have to just say 2 pi by lambda, position of that particular antenna that vector multiplied by a general d vector, which is your position of your receive antenna, multiply it you get a xi everywhere, ok.

You get a complex number right, because this finally if you add them up, you got a plot mod of E r; because you cannot plot complex number, you have to take a real and imaginary part, take mod of E r. Sometime people can also plot real part of E r or imaginary part of E r, that also you can do, it depends how do you want to plot it. But if you plot this one, that will be giving you kind of a picture of how exactly it may look like, ok.

You can plot individually real E r or imaginer E r; you can plot mod of E r also, it is up to you if it is a complex number. But ultimately we will get some shape there, you will not get a you know, you will not get isotropic case, ok. So, that is the point here ok now you can choose your configuration of your antenna and try to see what are the different plotting by, but be careful here. So, there is an interesting observation here.

So, here when you have UPA kind of configuration, ok. So, if it is a UPA, now as you see the E r will be a function of theta and phi both, it is not one. Then, how do I plot it? Because it is both a function of theta and phi you will not be able to plot it using just theta, right. If you plot it with respect to theta, then you have to say for what phi you are plotting it. So, which means that for such kind of scenarios, the plotting will not be a simple polars 2 dimensional polar, you have to go for a 3 dimensional polar diagram.

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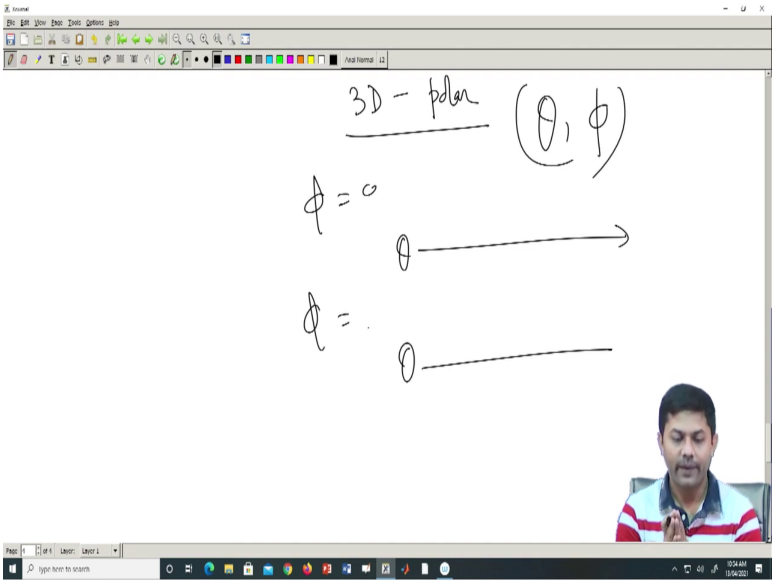

So, what does it mean? It means the plotting has to be there, but there has to be a phi also. So, you have to have a 3 dimensional kind of plotting, where the plotting has to have a theta and phi both way, ok. So, I mean lot of tools are available, where you can do some sort of a 3 D mesh structure, where you have to vary theta, you have to verify phi and see the 3 D mash structure kind of things, ok.

So, this is a 3 D polar diagram you have to plot with respect to theta and phi. If you go to MATLAB, you can you can create it; it is like take phi equal to 0, then you vary the theta, then take phi is equal to another resolution vary the theta, create 2 dimensions mesh and then plot the whole mesh structure in the 3 D world. So, that is kind of the point that we are talking of, ok. So, now you can make the point, you can make the whole thing even complicated.

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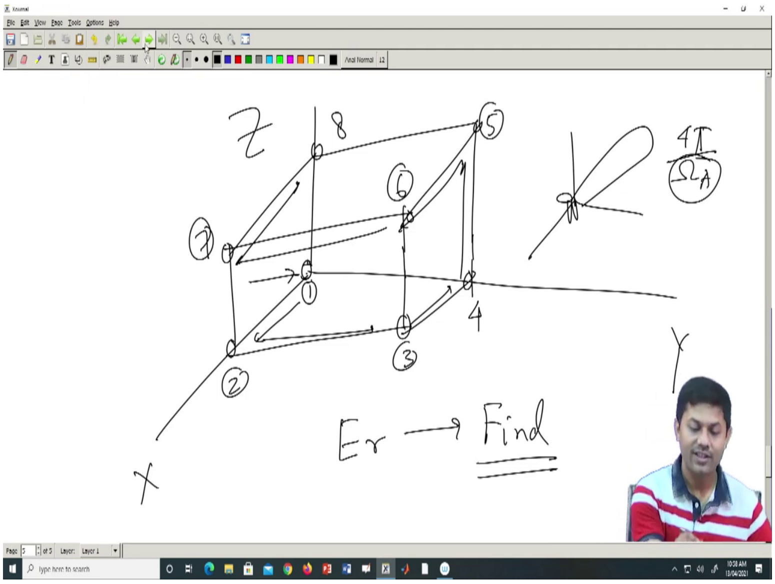

In the sense that, what if my antennas are not 2 D, if it is 3 D; how the beams would be, ok? So, for example, this is my antenna alright, this is one more antenna probably. So, let us say this antenna; this X, this is Y, and this is Z. Can you do that? Yes, so this is say 1 and you can number it. Number depends on how? How the physical feed connection. So, from here the first feed is going, let us say this is how the feed is going.

So, you can number it 1, 2, 3, 4, 5, 6, 7, 8 the order has to be there. So, this may be 2, this may be 3, this may be 4, this may be 5, this may be 6, 7, this is 8, this is the antenna order and you can find that coordinates, find out E r, find E r. So, now, you see how easy it is you know to define the complete part here, ok.

Now, with this I kind of conclude the sessions of how a beam can be created in the electric field patterns context. Now, this beam is important for us in the context of finding the angle part; because that is where our interest is, how exactly the beams angle would be, ok. It is not the value of the electric field, rather how exactly the beam area and those kind of things.

Now, next classes I will be talking from this context, how do you find out the directivity of the antenna; because this is what we have done fine, this is how the beam patterns, but what about the directivity, we need to find out. We also need to find out how exactly the hpv w's and those other parameters are present there; how exactly the power part will be, because this is only the electric part, electric field part, what about the power part ok?

So, those parts I mean, if you know the electric part, the power part is just a mod square, ok. So, if you have a E r, mod square E r is what your power part would be, it will be easier part. But what about the directivity, because that is important; we need to find out with this configuration, how my antennas directivity.

We only know the beams, but that is more of a plotting; from plotting I know what the beams would be, right. So, here there are various structures, various configuration and the beam can be; I do not know how the beam should be, depends on how the structure is, so for example, here, so here.

So, I do not know how the beam would be right; you could just theta and phi, probably the beam would be something like this, I do not know, I am just guessing, right. Now, what about the directivity of this particular. So, that mean I need to find out 4 pi by sigma; how do I find sigma A for this given electric field. So, those are the things which will be taught about little bit; because we do not have to know very details, but at least in the context of you know beam forming from an isotropic antenna processing point.

We need to know little bit about how exactly those parameters vary for us; because those are the ones which will be finally, interest to us ok. So, with this what we learnt today? So, far what we have learnt is the channel modeling; how exactly in the context of 3 D, how the channels should be modeled and then how the beams will be created there.

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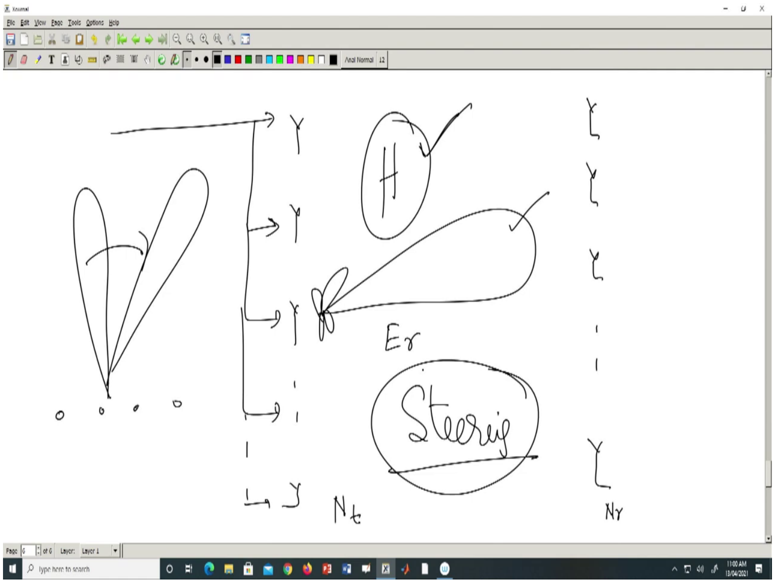

So, which means that if I just draw a overall diagram; so it means I have a set of antennas in the t x side, I have set of antennas in the r x side.

So, this would be my N t, this is my N r and this N t can be a physically spaced, N r can be physically placed ok, now whatever physical placing we have talked about. Then what we talked about? How H can be constructed. Then what we talked about? Ok, if I feed it like that, feed it like that, how exactly the beams can be created. So, this is our, so we know how to given. So, that mean if somebody gives you this N t configuration, N r configuration of antenna, you can easily find out H it is easy.

If you know how to find out H, you can also find out the E r; how exactly the beams would be from this N t antennas, ok. So, this is our physical configurations of the complete diagram.

Now, in the next classes some of the antenna, some of the beam related parameters will be found out with some more example and most important part is the steering part.

Because this may be the beam, but I may not be interested in the direction of the beam; I may be interested to create a beam in some other direction, this is the default. So, far what I have put; you just give the antenna, it will create a beam in a certain direction, you have no control. But now we will show that yes I have a control to change the direction of the beam; that mean if I place my antennas like that and if it creates the beam like this, I would like to shift the beam like that.

So, this is your steering part. So, in the next class, I will be next class onwards the steering part and some of the beam patterns related parameters will be discussed in more; because that is the beam forming aspect of this millimetre wave context, ok. So, with this, I conclude this class here.

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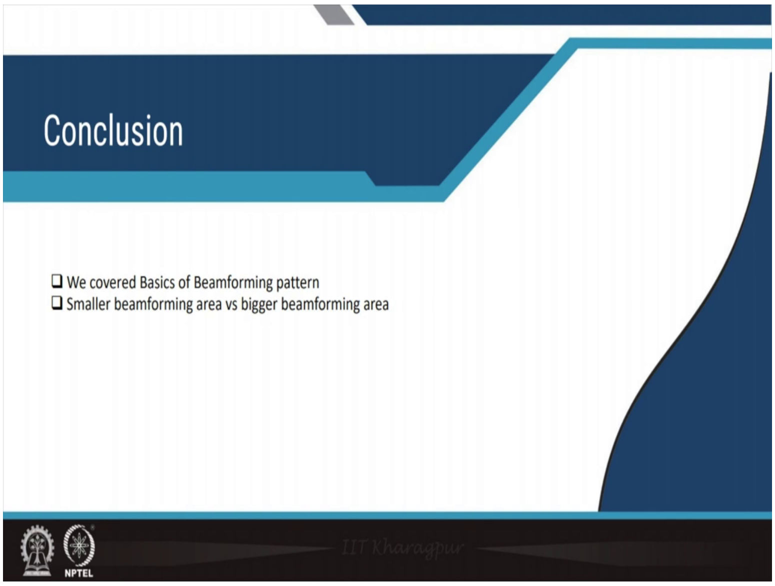

So, in conclusion, we have kind of completed the.

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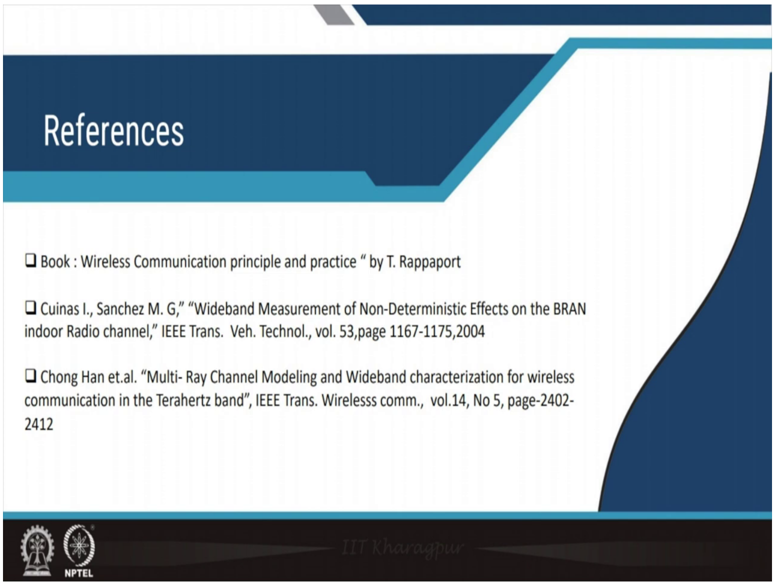

And the reference will be same as what we have.