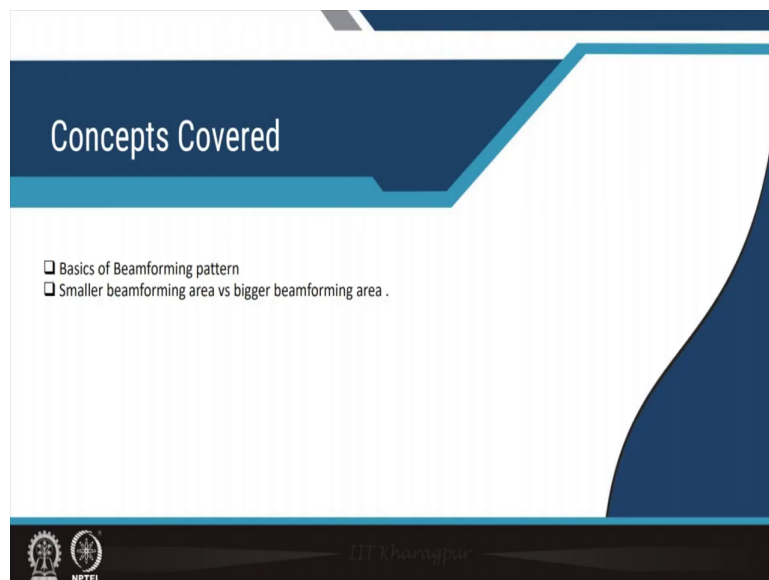


**Signal Processing for mmWave Communication for 5G and Beyond**  
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**Module - 07**  
**Details of Beamforming in mmWave**  
**Lecture - 37**  
**Basics of Beamforming pattern (part-I)**

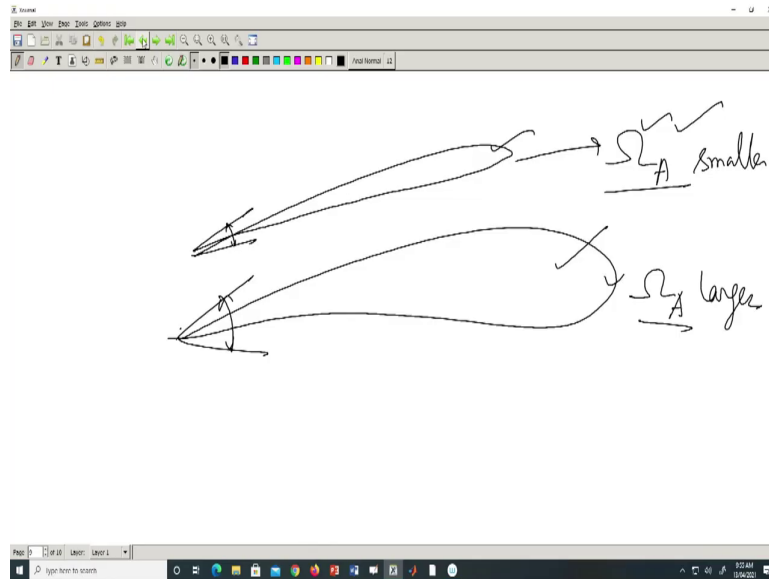
Ok welcome back. So, we will be continuing the other part.

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So, today we will be covering the like things that will be covering are the following. So, so far we have discussed the basics of the beam patterns and the solid beam angle.

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So, if you look at our earlier classes here, so this is where we left it where you have a smaller beam patterns, smaller beam area versus a bigger beam area, ok.

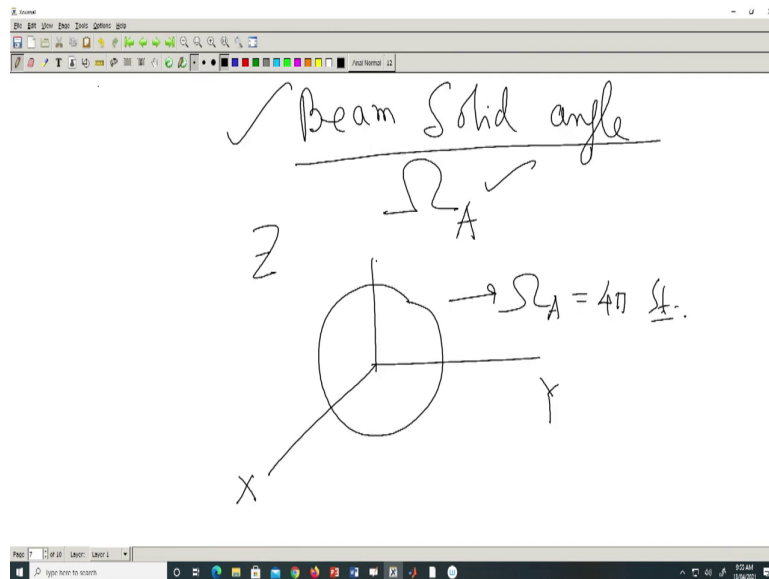
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$$U_n = \frac{U}{U_{max}}$$

$$d\Omega_A = \sin\theta d\theta d\phi$$

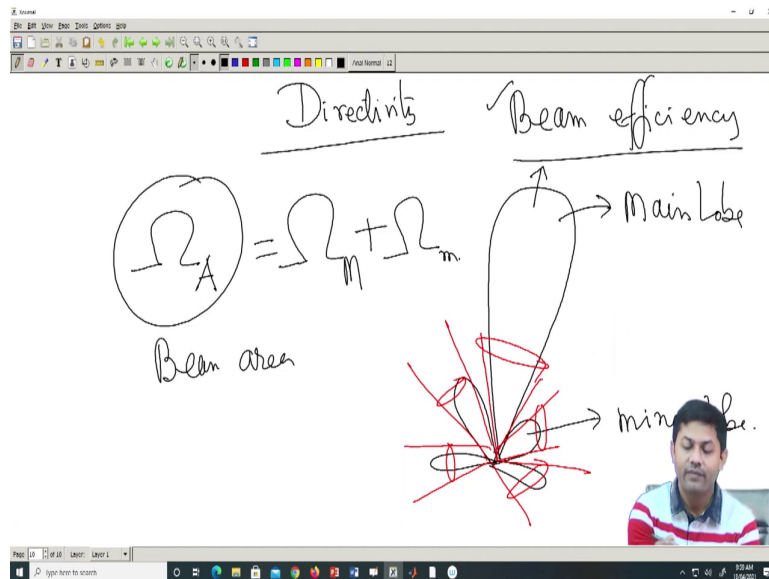
$$\int U_n \sin\theta d\theta d\phi$$

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So, this is beam solid angle. So, now we will be continuing that some more examples will be definitely doing when I complete the definitions of all the parameters. So, now we will be talking about the directivity, ok.

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So, what is this directivity? Directivity is mainly it represents how sharper the beam should be, but before I get into directivity, there is one more parameter called your beam efficiency ok. Now, in the beam area or the beam solid angle area that is sigma A; beam area sometime people call it beam solid angle or beam area. Both are correct or beam solid angle area ok whatever.

Now, as you know that if you have studied your antenna theories, no beam is exactly the perfect balloon set, ok so. Why? Because it may be like that, but along with it, it has other parts right. So, this is your main lobe and these are all your minor lobe right. So, this major lobe and minor lobe right. Now we need to consider that aspect.

So, now this minor lobes are not something I will be definitely interested in, because this is again I will say this is kind of wastage of power right. Because, your main intention is to send

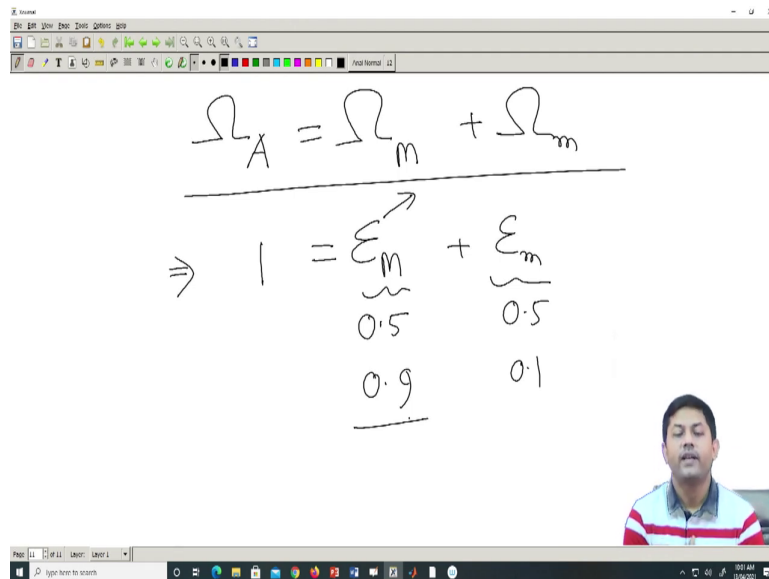
the power in certain direction and this side lobes or the minor lobes are basically kind of wastage.

So, obviously there are antenna techniques and others how to minimize the minor lobes, but I am not getting into that, but from a design point of view from a electrical engineering point of view, we need to know how much power is getting wasted in the minor lobes, right. That needs to be quantified and that somehow will be defined by the beam efficiency, ok.

So, how do you define the total sigma As. Total sigma A represents the total beam area. It is the beam solid angle how much you know, how much solid angle is involved for creating the beams here. So, how much it would be? See if I just draw this diagram, one solid angle will be dedicated for this main lobe. This is one cone you can think of, there will be one more cone here, there will be one more cone here and so and so forth. One more cone, this is one more. So many cones are here right.

So, this there is A. So, there is a bigger one and there are small small minor ones also, ok. So, this sigma A will be constituted by this main is it will be constituted by the main lobe as well as the minor lobe. So, sigma is what how much solid angle is involved in the power; in the power you know power movement. So, I can have one sigma which is coming from the main lobe. So, I call it sigma capital M plus another sigma which is coming from the minor lobe sigma M and sigma small m.

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The screenshot shows a presentation slide with handwritten mathematical equations. The first equation is  $\Omega_A = \Omega_M + \Omega_m$ . Below it, the number 1 is equated to the sum of two terms:  $\underbrace{\epsilon_M}_{0.5}$  and  $\underbrace{\epsilon_m}_{0.1}$ . The value 0.9 is written below the 0.5 term. A video feed of a presenter is visible in the bottom right corner of the slide.

$$\Omega_A = \Omega_M + \Omega_m$$
$$\Rightarrow 1 = \underbrace{\epsilon_M}_{0.5} + \underbrace{\epsilon_m}_{0.1}$$

0.9

Now, the question is that how much would be my efficiency? So, how do I define that? See if I normalize it see if I normalize this one, so this is my solid angle. You can precisely quantify it this sigma capital M and sigma small m provided you know the radiation intensity. If you know the radiation intensity or the pointing vector, those equations if you know or electric field if you know those equations, you can you know what is the solid angle.

Now, you just normalize it by sigma A. So, what will I get? This will be 1. So, let us say this is my xi capital M, this is my xi small m, ok. Now, these two quantities will be less than 1, but closer the value of sigma m to 1, I would say my minor lobes are getting reduced. So, this one this capital sigma is all your efficiency.

So, this will be representing how much efficient your beams are. So, if this is 0.5, this is also 0.5, right. So, the efficiency is 50 percent. Because, 50 percent power is wasted in the minor

lobes. If it is say 0.9 and this is 0.1, so 90 percent is power in the main lobe, 10 percent will be in my side lobes ok.

Now, this exact equation maths will be part of antenna theory. We do not get into that, but these are the some of the parameters that we should be aware of that. There is something called a efficiency. There is something called a main lobe, minor lobe and how they are how the how they look like kind of ok and some of this reduction of minor lobes, you may not have any control over it because everything that we try to do is from the signal processing point of view.

Now, not everything I can do it from signal processing. So, several things can happen from the structure of the antenna. The distance of the antennas, those kind of things will come I mean of course they are not in our hand, ok. So, now let us have the definition of my directivity. Now, directivity what does what does it mean? The directivity represents how sharper my beams are.



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Directivity  
(D) =  $\frac{4\pi}{\Omega_A}$  ✓

If isotropic  
 $\Omega_A = 4\pi$

Now, obviously if you have a  $\Omega_A$ , smaller the value what does it represents? Smaller the value, it represents that it is more sharper beams because that is the small solid angle where my maximum power is situated. So, that is what it represents, but can I define the so it is somewhere the directivity come. So, the directivity definition is basically from that defined by  $d$ , it is  $4\pi$  just few definitions a bunch of definition this is that definition.

So, this is how directivity is defined. So, that mean what is the maximum value of  $\Omega_A$  it is  $4\pi$  because in all  $4\pi$  solid angle, my whole power is equally distributed. So, then the directivity will be 1. In that case, I should not write it like that if antenna is isotropic. If it is an isotropic antenna, what is that for  $\Omega_A$ ? It will be  $4\pi$ , then directivity will be 1.

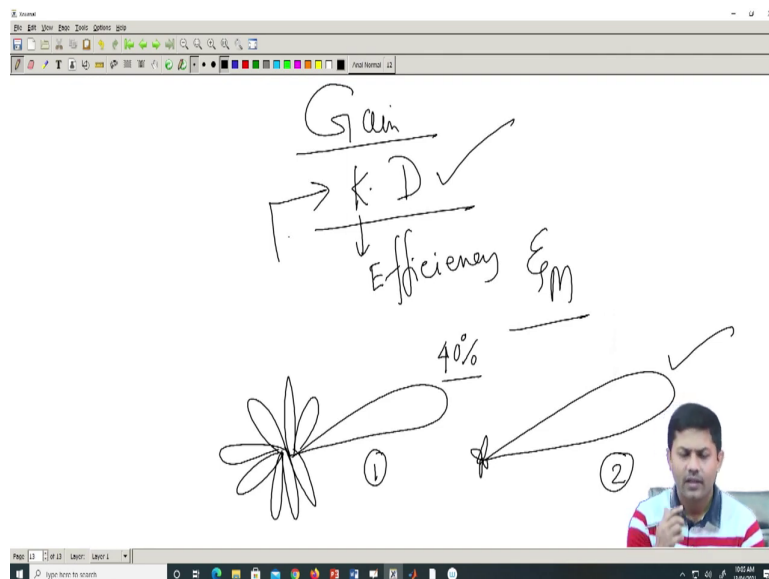
So, that is why so far if you look at the sub 6 gigahertz channel model when you consider the path loss, we never considered the directivity there because we assume it is a isotropic

antenna directivity is 1, but in fact directivity need not be 1. There directivity is 1 definitely, but it has to come up with the efficiency because we need to know in that same directivity how much is in the main lobe, how much is in the minor lobes.

Now, for isotropic antenna there is nothing called a minor lobe. Everything is one main lobe, right. So, their efficiencies is like a 1 because everything is in my major lobe major lobe, but not anything in the minor lobe, but their directivity is just in this case 1. Now, smaller this sigma A, you can see the directivity increases quite naturally, ok.

We will take some examples later on because this is more of a antenna kind of topics. So, I would like to avoid some of this parameter, but I will try to restrict as much discussion as possible just required for our context ok. Now this is more of the definition. Second definition was kind of a gain.

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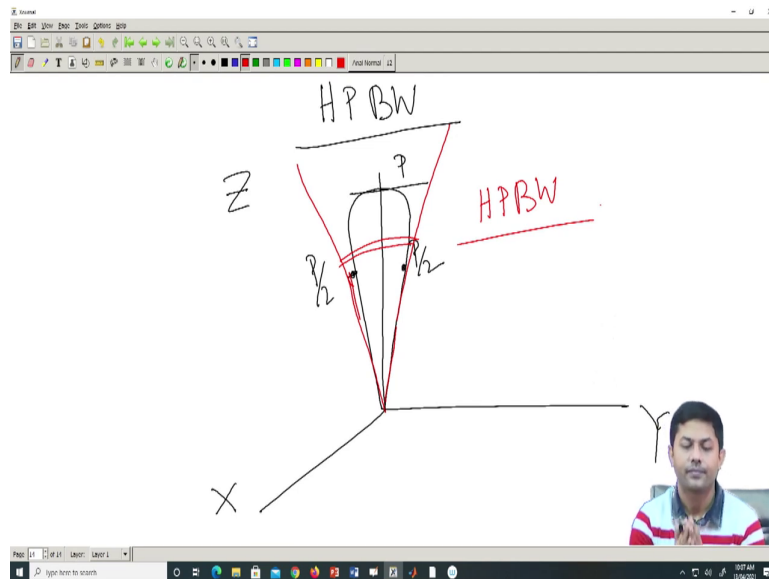
So, gain is defined by  $K$  into  $D$ , ok. So,  $D$  is your directivity and  $K$  is the efficiency. So, that was my sigma capital  $M$ . So, that was my major lobe efficiency right. So, that is the gain. So, now sometimes this part is very important because even if you have a directivity, it does not mean that you have a good gain on that because it may happen that you have a major lobe like that, but I have lot of lot of powers wasted in the minor lobes ok.

So, there is no point of having this kind of beam also right because a major part of my power is also wasted in the minor lobe. So, probably its efficiency is just 40 percent. So, even if it has a high directivity, I need to understand how much the gain is coming. So, this is the same thing. What if I have a this kind of things?

So, you see this very small amount of power, only a small small cone can be in the minor lobes. So, probably this is a more preferred one where so how do this move is between this case 1 and case 2. So, directivity probably the same for both of them, but what is not same for both of them is the efficiency. So, that is why this gain part is important for us  $K$  into  $D$ , ok.

So, these are the basic definitions and we will move on because this is not something mathematically we will be going so details into it, but given some of the equation probably we will be showing it.

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Next one, the HPBW; Half Power Beam Width. Now, if you look at the beams. So, let us say ok and this is my Z direction let us say my beam let us assume the beam is in the Z direction only ok probably that is not a good beam structure, ok. Now, this is the total power say whatever power or intensity does not matter power let us say power  $P_{max}$ .

So, at some point of time, at some point physical point I may have a power which is kind of you know kind of half power. So, for example, here power is  $P/2$  physical location I am talking here power is  $P/2$ .

So, I am drawing an angle connecting that point, ok. Now, this one HPBW, this is your half power bandwidth. So, that much it is basically it is kind of a solid angle where my it occupies

the half of the power. So, sometimes this is also representing my you know representing my beams structure kind of things.

So, now to be very frank some of the points to summarize. What are the points that we I would like to have it? Is the gain naturally, the directivity and most important is this part, beam area or beam solid angle area whatever. So, they represents the how good or how bad my beams would be ok. Now, for us (Refer Time: 13:59) a non-isotropic antenna. For us everything is a isotropic antenna. So, what does it mean? If it is an isotropic antenna for us sigma A is nothing, but 4 pi steradian is fixed for us, ok.

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One antenna

$$\left\{ \begin{array}{l} \Omega_A = 4\pi \\ D = \frac{4\pi}{\Omega_A} \\ k = 1 \end{array} \right.$$

Antenns Array

Directivity

4 pi

= 1

So, what is my directivity then? That will be just 1. What is my gain then? It is also 1 because there is no main lobe. That is only main lobe; there is no side lobe. So, my gain is completely 1. So, this is the configuration we start with for us.

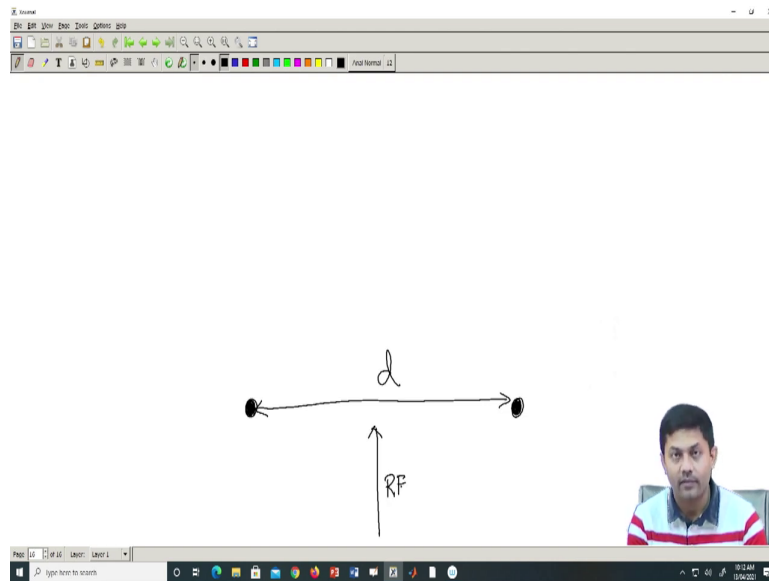
So, for us every antenna is like that characteristic, then what beams are we talking about right. Because, if we know for sure that the kind of antenna will be dealing with has this fixed parameter, we will never deal with non-isotropic antenna. We always deal with isotropic antenna.

If we always deal with isotropic antenna, then why we talk about the beams because anyway we are not using any beamed antenna ok. So, that means from isotropic antenna can I create a beamed antenna that is the focus of the beam forming. That is the whole about this beam forming because it is a beam forming, right.

I am forming a beam not by using a beamed antenna rather I am using the same isotropic antenna in different ways such that it forms a beam. So, that is that particular you know that field is your array processing, antenna array processing. So, this is what we are interested in, ok. So, for us everything is this three parameters are fixed for us;  $K$  equal to 1,  $D$  equal to 1,  $\sigma$  is equal to  $4\pi$  ok, but we are not dealing with one antenna.

This is for one antenna; this is our communication antenna. I mean I am not talking of a base station where in base station you may have a parabolic antenna. There you know directivity is different, beam area is different and all things are different, but I am talking of in a mobile phone where I do not use parabolic antenna. I completely use the isotropic small point source antenna. So, I will be more interested in the antenna array processing. In that context does it for maybe that is my main focus ok. So, let us think about it.

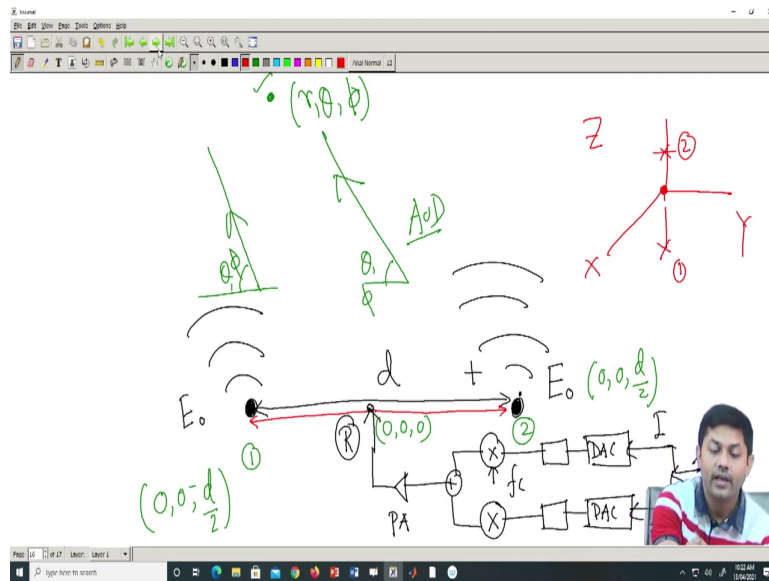
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So, let us say I have one antenna here, I have one more antenna here. Let us start with very basic two, ok. So, this is my first antenna, this is my second antenna. Let us say the separation between these two antenna is  $D$  and these two antenna individually they have this characteristic. Whatever I have drawn here, both the antennas have the same characteristic ok correct. Now let us say when I do the, so this is my physically on my you know say mobile phone, these two antennas are placed there, ok.

And physically what I do here is that I am feeding an electrical RF line here. What does it mean? What is this feeding of RF line means? Let me draw it little carefully here, so that you can understand.

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So, this is say I have a power amplifier here, ok and this is what my IQ modulation and these are some bunch of filters front end filters and then these are my DAC, this is simplified diagram of course and this is my this is my digitization. So, this is I this part is Q. Real and imaginary part goes through DAC and this is my FC, this is what my RF, this is  $\cos \omega t$ , another is a  $\sin \omega t$ .

So, I am just writing it an  $f_c \sin \omega t$ , another is a  $\cos \omega t$ , this is my feed line. So, physically it is feeding at a particular this whatever is drawn on this screen whatever you are seeing just midpoint, it is there and from there one physical connection goes to one antenna another physical connection goes to another antenna, ok.

This is what it is. So, let us say this is what my physical diagram, ok. Now let us say each and every antenna is a isotropic antenna. Now, let us say when they transmit it how exactly my



pattern would be, how exactly my electric field would be ok? Now, from here this side let us say this is my from there one side is going to right side, another side going to the left side. Now, let us assume that from this midpoint, this is my reference point, ok.

If that midpoint is a reference point and one side is going in one direction, another side is going another direction. So, from here this will be radiated from here, this will be radiated like that the radiation will happen, ok.

Now the radiation which is coming here, so let us call it  $E_0$  amount of radiation is going there, here also  $E_0$  amount of radiation which is going there, ok. Now, I am standing let us say I am standing somewhere here at a distance  $R$ ,  $\theta$  and  $\phi$ , ok and last two last classes I have enough explained how exactly my radiation phase difference should be, right.

So, it is as if like now you think from electrical angle point of view, do not think from an electromagnetic point of view because it is just a visualization and imagination. Literally it will be electromagnetic. You first you think that two rays are coming, two electrical rays are coming to me because it will be easier for you now one ray.

So, if it is in  $\theta$  and  $\phi$  direction, all direction this particular first antenna will be you know radiating and the second antenna will also be radiating in all directions. But you are standing at a distance say  $R$  with  $\theta$  and  $\phi$  polar coordinates and it is a far field. Of course, everything I am talking is a far field.

So, what does it mean? As if like it is like one radiation will be all are parallel, right. This will this will also be coming like a parallel to you. So, two radiations are coming to you like a parallel with the you know because you are direct, you are in the direction of  $\theta$  and  $\phi$ .

So, that is my angle of departure because you are standing at a particular  $\theta$  and  $\phi$  direction. So, this is my angle of departure  $\theta$  and there is of course a  $\phi$  is also present.  $\phi$  is also present because that is the angle at which you are receiving it because you are standing at that point, right.

So, these two or that particular receiver what is the phase difference. He will see between the first antenna array and the second antenna array. I think we have enough discussed that yesterday, ok. So, if you look at the phase because now you think just from a receive signal point of view, the two rays are coming to me because when you say beam, it is basically a perception that you are seeing how much power you are receiving at your end.

What power pattern you are seeing it or what electric intensity you are seeing it or what radiation intensity you are seeing. So, it is in the context of your receiver point of view. Now, you are standing at  $R$  theta phi direction and then, two antennas is radiating in isotropic waves.

I mean it is all direction, it is radiating, but you are seeing a phase difference between the two; because, why? Because it is as if like you are receiving them parallelly and assume this  $D$  is much smaller than this particular  $R$ , where you are standing in far field.

It is as if like two rays are coming to you parallelly, but there is a phase difference. Why there is a phase difference? Because they too come from two different physical locations. Obviously, there will be phase difference and we have discussed it enough how much the phase difference should be that depends on who should be your reference. Now with respect to the reference point, how much the phase difference that will be seeing it right.

So, now you can again you know you can see how much the phase difference. So, with respect to reference point let us say my reference point is  $0\ 0\ 0$ , ok. So, this is my reference point  $0\ 0\ 0$ . That is my reference point, ok. So, what is the coordinate of the first antenna if you can think of if it is in the  $Z$  direction? So, this is my  $Z$  direction. So, it will be  $0\ 0\ d$  by  $2$ . You can say it is minus if left side is and this is in the positive direction of my  $Z$ .

So, this is  $0\ 0\ d$  by  $2$ , that is the coordinate of this two antenna with respect to my reference point. Again I am repeating reference point is very important because that is the point from where the distribution is happening. So, that is my reference point. Reference point is my  $0\ 0\ 0$ . So, this is how the coordinates are, ok.

You are standing at that point. So, how much electric field now you are we receiving it? So, if we say  $e$  is representing the electric field intensity, not the power intensity if it is a power just  $e$  square will be there nothing else, but let us say it is electric field that I am dealing with ok.

So, how much electric field intensity the first antenna is giving, how much electric field intensity the second antenna giving at the position  $R$  theta phi because both the ray will be coming to me.

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$$\begin{aligned}
 \psi \quad E_r &= E_0 e^{-j\psi} + E_0 e^{+j\psi} \\
 \rightarrow \frac{d \cos \theta}{2} &= E_0 [2 \cos \psi] \\
 \rightarrow \frac{2\pi}{\lambda} p \times d &= 2E_0 \cos \left( \frac{\pi r d}{\lambda x 2} \cos \theta \right)
 \end{aligned}$$

So, the for the first case from antenna one, you will be receiving a electric field  $E_0$  with a phase difference right because that is the amount of phase difference with respect to the because I would like to know the phase difference between the two. So, what is the total phase difference?

Let us assume total phase difference is  $\phi$  or  $\psi$  that we have calculated last time, right.  $\psi$  that is the total phase difference among them, but now the way I have architected my you know antennas, it is like one midpoint and then, it is just a positive side negative side distributed, but you can have your own configuration. So, all you have to be careful is that with respect to reference point what is my phase difference among all these rays, that's all.

So, in the first ray is coming to me with a phase difference of this one, another one is coming to me with a phase difference of this one. What is  $\psi$ ?  $\psi$  is it is a position of the antenna multiplied by the positions of my own point, where I am standing right. See if it is that case, then it will be nothing, but  $d \cos \theta$  that is my position of the antenna.

It is the position vector multiplied by my antenna position vector multiplied by my own position vectors, right. So, this is the 1 which I will be receiving it. So, this is my total received electric field at a distance  $R$ . Now, I am not so much interested in  $E_0$ .  $E_0$  is the field intensity at the receiver at the  $R$ , ok.

So,  $E_0$  you just do not bother about it. Now take this two point, how much it would be?  $2 \cos$  whatever  $\psi$  right is what it is. So, it would be do not worry about this two and all this factor. That is not something I am interested as I said the exact magnitude of the field or power is not of important to me. It is what is important to us is the shape. This will be  $\cos$  put this value here  $d$  by  $2 \cos \theta$ .

This is my received field at a distance  $R$  at an angle  $\theta$ , at an angle  $\phi$ . This is my received field. Now, this  $E_0$  is how much  $E_0$  will be dependent on your distance those part is ok. As I said I am not interested in the exact magnitude of  $\theta$ . I am interested in the shape of that  $E_0$  does not determine my shape. Shape determined by this  $\cos \cos$  and all this parameter is this parameter is my set parameter.

Now, this particular configuration whatever I have drawn here which is like in a  $Z$  direction, this is my midpoint is a positive, this is my negative side. It was as if like this, if you are

slightly not confident of how the configuration. As if like this is my reference point one antenna is here, another antenna is here as simple as that.

Now do you understand that? This is my first antenna, this is my second antenna, this is how it is. Same thing I have just drawn in a flat horizontal way, so that it will be easy for me to explain. Anyway now this is my field intensity at a distance  $R$ , this is precisely the array processing.

So, it does not matter it really does not matter what my  $E_0$  is. What matters is the what is my phase difference is because  $E_0$  is more of an isotropic phase. What is  $E_0$ ? It is nothing, but some constant because it is an isotropic antenna.

So,  $E_0$  at a particular  $R$  has no I mean it has is a mod of  $E_0$  is like a one just one value. It is a isotropic antenna. So, what matters is that as both the rays are coming from two different sources, I would like to know what is the phase difference, absolute phase difference among the two. If there are multiple I would like to know the phase difference among them. So, that matters to me because that finally determines what is my resultant electric field.

Now, this  $E_R$ , this equation is my resultant. Now, if you plot it what will you get? Do not worry about this part. This is of no importance. Just plot it now with respect to what should I plot it. Plot it with respect to  $\theta$ . Let us say what happens when  $\theta$  equal to 0. We will see that ok now this is the interesting point.

Now, now next class we will be plotting and will be taking some more antennas and see what exactly my radiation patterns would be, ok. So, this is what the kind of equation that I will get, but there is a small here one more parameter that I will be drawing it here  $2\pi$  by  $\lambda$ . So, this is the ultimate equations that you will be getting it at the end of the day. So, that is the equation you will be getting it for the received electric field, ok.

Now, how do you get this equation, that is we have discussed it in some of the earlier classes where you know how much I mean if it if you are standing at an angle  $\theta$  and  $\phi$ . What is the phase difference that will be getting it? It was  $2\pi$  by  $\lambda$  if you remember the vector

P, that is the position of a particular antenna multiplied by you know the vector of your own position. So, the similar things I will be doing it here as well, ok.

Now, in the next class I will be showing you how exactly this equation will translate to in terms of the received electric field because that we have to plot it now in this particular case as I have shown only Z direction antennas. You can also have UPA kind of you know antennas, but currently we are sticking to the ULA, so that the beams can be easily understood. And we will show you that how this particular ER, this electric field at the received electric field how it will be with respect to the theta.

And that is what we exactly want in our array processing activities.

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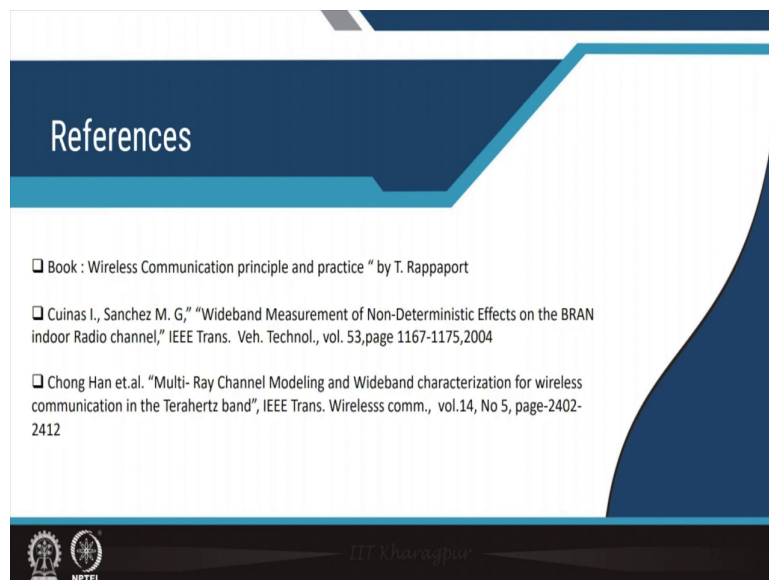
**Conclusion**

- We covered Basics of Beamforming pattern
- Smaller beamforming area vs bigger beamforming area

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NPTEL

So, with this I conclude the session and so, in conclusion we have kind of completed the yeah.

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And the reference will be same as what we have. I will show you in the next class the plotting of the resultant electric field with respect to various angles, ok.