Signal Processing for mmWave Communication for 5G and Beyond
Prof. Amit Kumar Dutta
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Module - 06
An antenna array processing concept
Lecture - 35
3D Concept of antenna many fold vector

Welcome. A many fold vector creation for different configure. Now, we will be talking some more example from the three-dimension point of view. At the same time, we will also be seeing what happens when I plug in the R x side also. So, this should be in the last example from the TX side.

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So, some of the things that will be covering.

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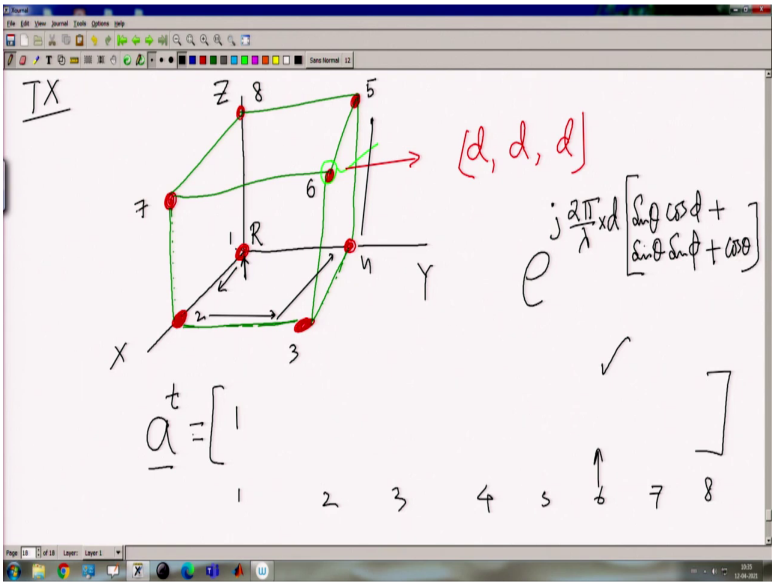

So, let us say I have Z, Y and X angle. These are the coordinates. So, this is completely TX side and my antennas are something like that. So, this is my first antenna. Let us say that is my reference point. My second antenna, this my third antenna and this is my fourth antenna which is the UPA.

But I make a small modification. I assume that there are here, so probably I will put it this way, something like that. So, which means that there are antennas like this; one antenna here, one antenna here, one antenna here, one antenna here and one antenna here ok. So, basically you can think this is more of a three-dimension kind of things.

So, let me just try to join it, see if I can create some sort of a safe here yeah. So, this is say in a box my antennas are there. So, this is 3D configuration. So, how can I? So, this is my TX side. This is how my antennas are there. So, how can I create the array many fold vector?

So, let us assume that this is my reference point ok. So, here just its an extension of what we have learnt, it just that the now the antennas will have X, Y, Z; some of the antennas will have X, Y and Z all non-zero. For example, this particular antenna, we will have X, Y and Z non-zero. Yeah, so this is one such antenna; so, rest of them at least 1 or 2 elements can be 0.

So, now you know. So, if I just take any one of them antennas, so let us take the for this particular antenna. So, ok. So, what it would be? So, let us say it is called what is. So, let us say everything is equally spaced. So, so that this the position of this particular antenna is d, d and d right.

You can you know for rest of the antennas, what are the different coordinates there right. So, what is my array many fold vector? So, how many elements in the array many fold array many fold vectors should be there? There are eight because there are eight antennas are there. So, a bar t we will have 8 component ok.

So, probably, this would be if I start from here, this is 1, 2, 3, 4 or this is 5, 6, 7, 8. This how say I am just proceeding. This is how I mean the numbering says that that is the; that is the point where it is the feed is touching; that is the order of touching the feeds electrical signals.

So, that means, for touching here first, then second, then third, then fourth and then it goes to you know fifth and in fact, you can if you are not so comfortable, you can change the coordinates here, where it is touching it here. You can say from 1, then it is touching 2, then it is touching 3, 4, then this can be my 5th, this can be my 6th, this can be my 7, this can be my 8 yeah.

So, that is the order in which the wiring is touching; I mean it is touching first here, then it goes to here, then it goes to here, then it goes to here, then it goes to here and so and so forth.

So, that is the order in which it is. So, I am talking of the sixth one. So, probably the 1, 2, 3, 4, 5, 6, 7 and 8; 8 position will be there.

For the first position, it is 1 because that is from there it is distributed. So, I am talking of the sixth one which is the one that is the most complex, that is not complex; but that is that has all the component, rest of them will be at least easy you know how to do it. For this one, it will be the j into 2 pi by lambda multiplied by this d you can take it common ok, that is the coordinate is d, d, d; 3 d's will be there.

So, this would be sin theta cos phi plus sin theta sin phi plus cos theta that is the term will be coming there and for rest, you can calculate ok. Got it? So, this is kind of a generic. Now, I can even further make it generic, what if the distances I can tweak it ok. So, if you understand how to do it, how to create the array many fold vectors from either transmitter receiver side; that is it. So, it does not really matter where my antennas are there; but you have been very careful, where the reference point is because from the reference point and then, the order of numbering.

Why the order of numbering is important? Because that shows in which order the electrical line is actually distributed. So, that means, when I say the R is my first order, then I say this is my second order; it means that from there the electrical line actually goes to the second antenna, from there it goes to the third antenna, from there it goes to the fourth antenna and so and so forth.

So, the way the electrical wiring is you know present, you have to number it in that order because that is the phase difference it is creating ok. So, you cannot say this is my first, this is my second; this won't work out ok. So, that ordering is important. Of course, that ordering has to be given to you; otherwise, the element propeller cannot be generated ok.

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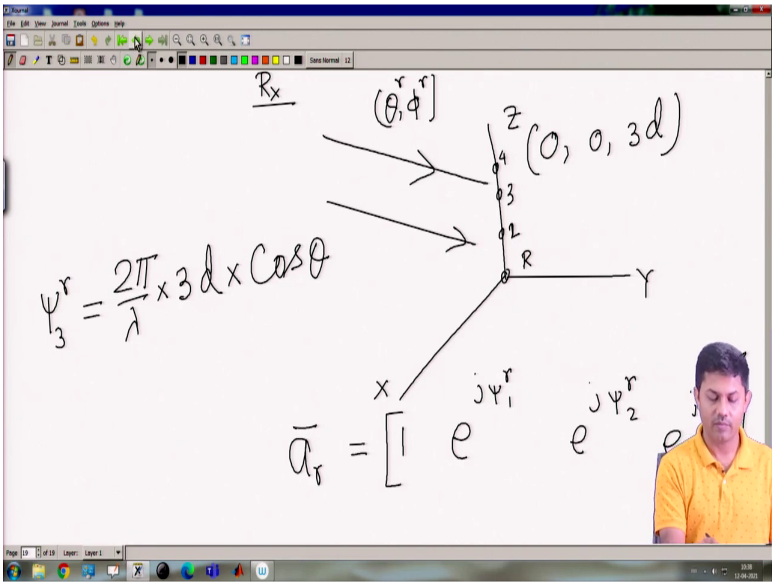

So, now, let us go to the R x side. Now, I am little relaxed from the R x side because I do not have to say anything extra from the R x side. Why? It is the same as TX side; just that in the TX side, it was an angle of departure; wherein, the R x side, it will be angle of arrival. But concept remains the same.

So, which means that same thing. This is my R x side. Suppose, I have my R x antennas like that and the rays are; rays can come at any direction, but I am considering because its R x right, so here angle of arrival is what is matter, what matters to me. So, let us say my angle of arrival is theta phi. This is the direction at my angle of arrival is coming ok.

How do I create the a r or the array many fold vector at the receiver side? Again, the same concept whatever you have learnt in the TX side, same thing you can extend it; just that only difference is that instead of angle of departure, now you have to take an angle of arrival side.

Now, here angle of arrival side means is theta r and phi i, phi r. Now, for the benefit of lot of notational issues, I may drop the r just to understand; but you can continue the r notation.

So, what is my a r? So, let us say this is my reference point, this is my 1st antenna, this is my; this is my 1st reference antenna, this is the 2nd antenna and probably, this is my 3rd antenna, this is my 4th antenna ok. How the a r will be constructed? This is the first antenna will be again 1, second antenna let us call it psi r 1 e to the power j psi r 2. I hope this does not require much explanation of how you can find out the psi each and individual psi.

So, how do you find out? Let us say I am drawing only for the third one, you know last one. So, how do I create this psi? So, this should be again the same formula 2 pi by lambda. So, that is the angle that it will be seen. So, that is the angular differences, it will be seeing multiplied by you know 2 pi by lambda multiplied by the coordinates. What is the coordinate for this gentleman? This would be 3d, 0, 0. I am talking of this gentleman right.

So, this should be 3 d and this is the Z part. So, I am let making little mistakes here. So, as it is in the Z direction, so it will be 0, 0, 3d ok. So, this should be 3d multiplied by as you know the direction vector remains the same. So, it is the same as what we have drawn earlier.

So, this will be cos theta. Now, if I have other combinations like whatever combinations I have drawn, whatever combinations I have drawn in the TX, same thing will be applied here except that the angles will now be referred as a angle of arrival. Now, what if I have multiple such reflectors, what would be the case ok?

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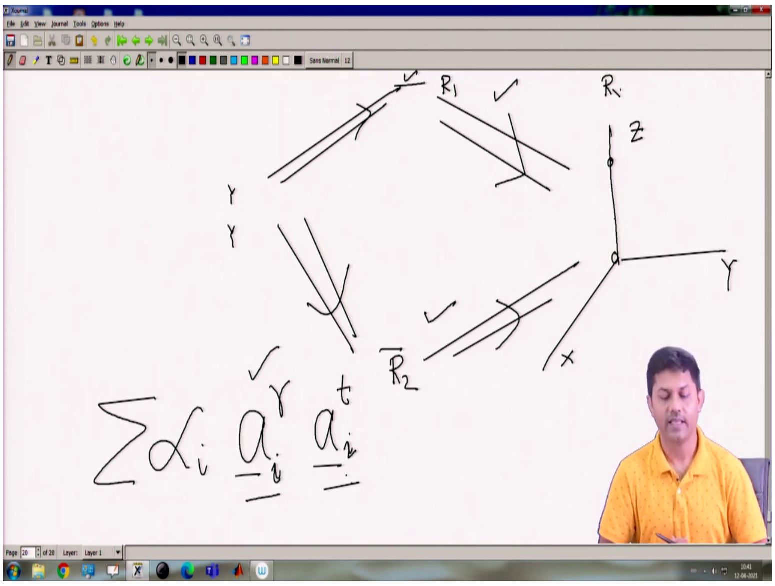

So, let us take that also. Let us say I have in the R x side, I have two antennas. So, this is my Z, this is my X, this is my Y. But now, I am saying that this particular system has two reflectors. So, which means that I may have one reflector here, I may have one reflector here ok; how do I do that? Now, as you know when you have a multiple reflector; for the multiple reflectors, you have multiple angle of arrival and obviously, multiple angle of departure.

Because there may be antenna system here, but this reflectors, the way he receives it and the way he receives it, it will have one set of angle of departure and the way he transmits it and the way he transmits it, it will have another angle of arrival here right. So, there are two different, it is as if like two systems are coming right.

So, how do I? So, it means that for the for this particular case, I have to know for each individual angle of arrival and angle of departure; then only, I can construct that. I mean you know the channel models there right. So, it was summation of alpha i a i r a t i vector right. So, now, this is i.

So, it is no longer just one constant angle of arrival and angle of departure, it is depends on how the reflectors, where the reflectors are. So, now, concept remains the same, is just that it will have in this particular case, it will have two angle of two array many fold vector array many fold vectors from the transmitter side and two array many fold vectors from the receiver side that is it ok. So, this does not change my complete configuration. It is just that it is an extension of it. Now, I hope you can extend it such kind of problems.

If I say in the exam, if I give you ok this particular configuration has these are the angle of arrival, these are the angle of departure for two cases, I hope you can construct the angle of arrival and angle of many fold vectors for transmitter side and receiver side right.

Because you know all the parameter, it is just matter of recreating the things ok. So, now, let us combine it. That is what we want. We want a i r a i t; we want to find it out separately and then, we want to combine it so that we can construct my H. So, let us take a simple example.

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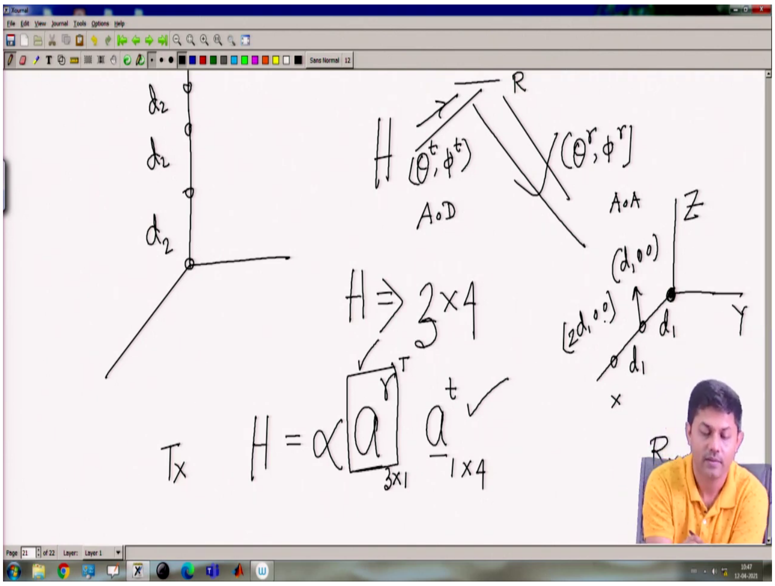

Let us say my transmitter side has configuration like this four antenna the Z direction; whereas, my receiver has configuration only in the X side and that only 3. This is my transmitter side, this is my receiver side ok, this is my receiver side. But mind it, in the receiver side, all my antennas in the X direction and there are only 3 antennas and in the transmit.

Now, I am combining it ok, because this view should be important to you. How exactly in a real system, they work ok? Now, there is a polarization problem and so on and so forth that will come into picture; but let us not get into that first, we just want to construct my many fold vectors and all.

So, here. So, what is my final channel? My channel final channel will be H into its H matrix right. So, what is the dimension? So, it is as if like there are 4 antenna from transmitter, there are 3 antenna at the receiver; but this is what the physical dimension ok. Just close your eye and try to visualize what exactly the scenarios; one in the transmitter side, I have Z direction 4 antennas; but in the receiver side, my antenna is already in the X direction.

So, I want to finally, create my channel here ok. Let us not worry about my reflectors and all let us say it is a simple reflectors. So, in this particular case, probably I can just introduce one, just to make my problem slightly complicated, let us I have only one reflector because otherwise, I cannot make my angle of arrival and angle of departure separate just to that.

So, let us say this is where the rays go from transmitter side, this is where my rays comes in the receiver side; this is how it is ok, just to make my understanding slightly better ok and there is no other reflector ok. So, here why do I do that? Because here I just want to make sure my angle of arrival and my angle of departure are separate; otherwise, I have to make my angle of arrival and departure, both same.

Because the direction at which I am sending and the direction at which I am receiving and I do not see a difference. So, I have just created a reflector so that that path is. So, I have theta t and phi t and here, I have theta r and phi r that is what I have given. So, under this scenario, I want to construct my channel here ok. So, what is given? AoD given. What is given? AoA is also given ok. 4 antennas in the Z direction, 3 antenna in the X direction for the R x, so I want to construct my H.

So, what is the dimension of H, that you should understand. What is the dimension of H? The dimension of H will be n r cross n t, so which will be it's simply a 2 cross; sorry 3 cross, 3 cross 4 vector matrix ok. So, which means my a r or receive many fold vector is a 3 cross 1 and transmit many fold vector is a 4 cross 1; that is all.

So, which means as there are there is only one reflector, the only thing I know is H is equal to now there should be gain. So, let us assume that generalized gain is alpha; I do not want to get into the gain problem here. Now, this a r, I need to construct which is a 3 cross 1, there is a T here.

And then, this would be mine a t vector which is a transmit many fold vector or transmitter side many fold vector and that is 1 cross 4, that is the only thing I need to make it. Now, this is T as a whole. So, this should be 1 cross 3 only. But just the transpose is there. So, this is what my job is. So, individually, I now need to create this one and I need to create this one ok.

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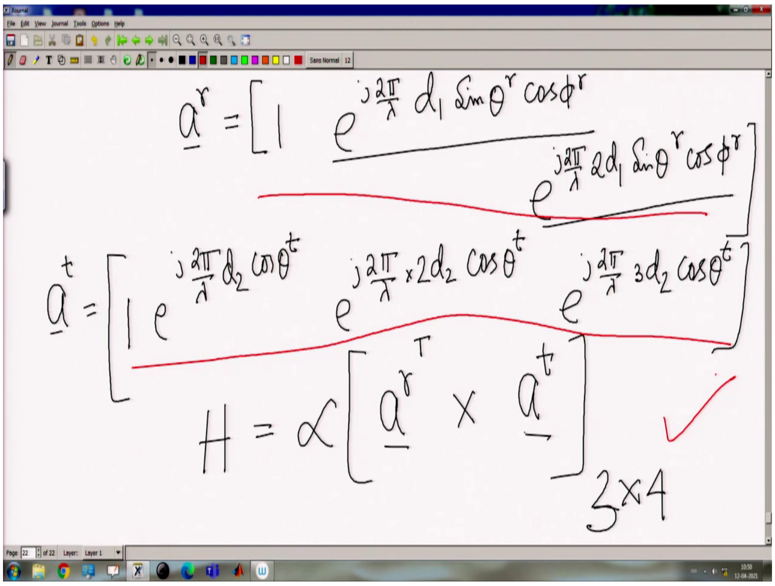

Now if it is a a r, how do I create it? Let us form a r vector. It is in the x direction ok. Let us see the distance among them is d 1, d 1. I try to make it slightly different which is more practical. Let us say this is d 2, this is d 2, this is d 2 that mean distance among the distance between two consecutive antenna is d 2 in the transmitter side; but d 1 in the receiver side.

There is no such relationship that you have to make them all equal, there is no relationship because I can have my own separation of antenna in the transmitter side, I can have own separation of my antenna at the receiver side; only thing that they have to be greater than lambda by 2. That is the only requirement; they need not be same ok. So, let us take a very general case ok.

Now, I am constructing here. So, what should I do? As it is as if like I am constructing a r in my receiver side and you know how you have already done it ok. So, for this is 0 0 0. So, that is simple enough. For this one, what is the coordinate? This will be d 1 0 0; this will be 2d 1 0 0. So, that is the coordinate right ok.

So, this should be 1 1 e to the power j 2 pi by lambda multiplied by the coordinate of the antenna which is this one, d 1 0 0. So, it will be d 1; but now direction vector is what? Direction vector does not change. So, that will be cos sorry this should be sin theta r will be there and cos phi r that is the point. That is it; nothing else will be there. The rest of the two points are 0.

Now the third point is I cannot fit everything. In the third point is e to the power j 2 pi by lambda, what is the distinction? It is 2 d 1; 2 d 1 sin theta r cos phi r. That is it; this is my third point. So, you see there are three elements are constructed element, this is many fold vector. What about my transmit many fold vector? There are four points here having a distance this d 2, d 2, d 2; but it is in the Z direction.

So, it will be easier to do that. So, 1 e to the power j 2 pi by you know lambda d 2 cos theta cos theta t e to the power j 2 pi by lambda into 2d 2 because it is the second antenna. So, distance will be 2d 2 0 0 that is the coordinate 0 0 2d 2. But this will be theta t e to the power j 2 pi by lambda 3d 2 cos theta.

Note that in a t, I do not need phi. That is ok because that is how the configuration is. Now, I got two vectors, multiply them you get H. So, your H will be multiply this alpha, alpha let us assume I know it; but I have somehow estimate it and then, this a r vector transpose multiplied by this a t vector whatever comes, so it will be a 2 cross 3; 2 cross 4 not 2 cross 3, it will be rather sorry not 3 cross 4.

This will be a 3 cross 4 matrix that will come. So, this is my one vector, this is one vector, this is a second vector right. Multiply whatever comes into picture; 3 cross 4, you can get some matrix there. So, that is my channel that is precisely my channel. I have not done beam steering; this is just the beam forming; in the sense that how ok. I have not still defined how exactly my beam properties will be there, but that is that I will be talking definitely next because this only shows how my channel is formed ok.

But what is the beam characteristics that I have not said ok. This is an array processing part that will come back next. So, this is exactly my channel is formed now. So, now, you can construct a channel. Suppose, if somebody gives you a configuration like that, now you will be confident enough to construct a channel.

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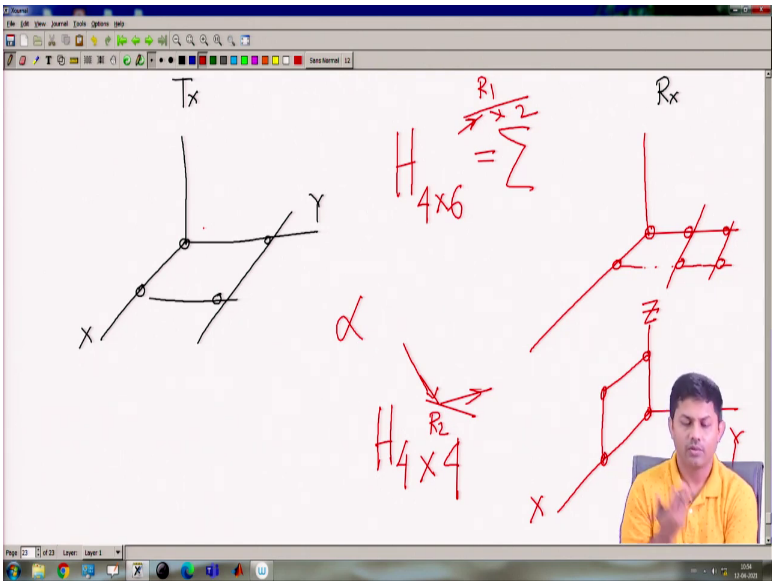

So, this is my R x side; let us say my antennas are placed like that 4 antennas. So, this is my X and this is my Y coordinate ok; but in the R x side, let us say I have antennas which are placed only like this. This is how my R x side antennas are placed ok. You know how to do it, you know how to solve it, you can number it, depends on how exactly the configuration is and you can construct your H.

So, there may be which questions like what is the dimension of such H; this is how physically it will be like that. It can even be like you can put the antennas in other direction as well. What if I put my antennas like this? It is X, Z plane I have place my antennas.

And in the TX side, I have put it on the X Y plane. So, can I construct my H? Yes, I can construct. For the first case, what is the dimension of my H? So, n r cross n t it is; n r is 6 and n t is 4. So, in the first case, it will be 4 cross 6. For the second case, H will be 4 cross 4 ok and you can construct your H very easily, but only thing you have to know what is alpha and this is with one reflectors. If I say same configuration I have one reflector here, I have one more reflector here, construct the channel?

Nothing; the same thing you just repeat it for two summation because for two reflectors, you need a two separate set of angle of arrival and angle of departure, as simple as that. For this, there is an angle of arrival and angle of departure, angle of departure angle of arrival with respect to the other one. So, I can say this is the angle of arrival for TX. From TX side, this angle of departure from that side and for this, here is one set of angle, one set of angles here ok.

So, that means, when it is reflecting angle of departure is acting as an angle of arrival here. For this reflector, angle of arrival is nothing but the angle of departure from the TX side. I mean if you can understand the concept that that is defined ok. So, that is it. So, this can be generalized to any concept.

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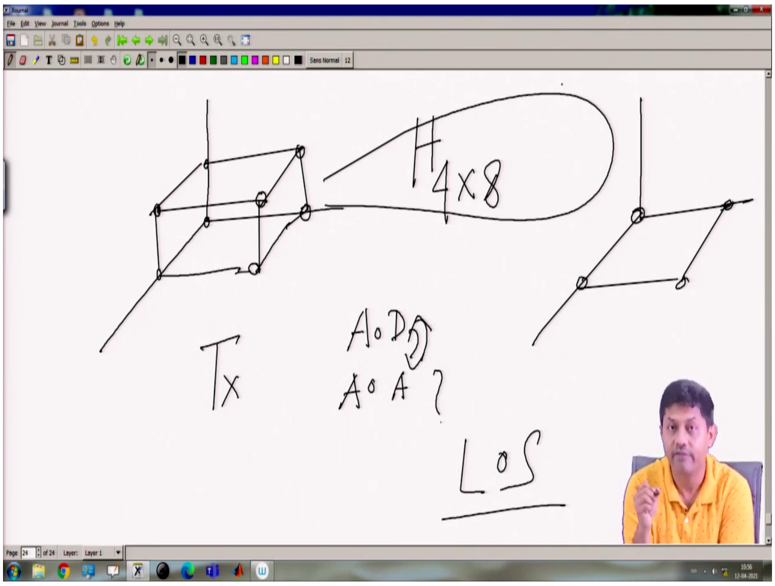

So, what if in the assignment somebody ask you something like that. In the T x side my antennas are configured like that three-dimension case; but whereas, in the R x, I have say UPA, can I construct my channel? Yes. So, what is the dimension of the channel first of all? It is just a matrix.

So, let us say only there is only one reflectors ok. So, what is the dimension of this matrix? So, it is n r cross n t. So, how many n r antennas are there? 4. How many T x antennas are there? 8. So, you will have to construct a 4 cross 8 antennas ok. Suppose, there is no reflectors in this case, so will it have two different set of angle of departure and angle of arrival? Suppose, no reflector, it is a line of sight; that means, if that is if at all a beam is created, the receiver is in the same path.

So, what should be the angle of departure and angle of arrival in this case? Will I have two different, do I have to give you two different AoD and AoA? No. Why? Because it is directly going; maybe it is in the beam form. That is ok. But whichever direction, the beam is going on the same path this is present here.

So, in this is a scenario, where the direction of beam, where there is no reflectors or in scatterer without it is just going and the reflector or the receiver is sitting inside it. So, it is as if like angle of arrival and angle of departure are all same or it will be a problem only if there is a reflector.

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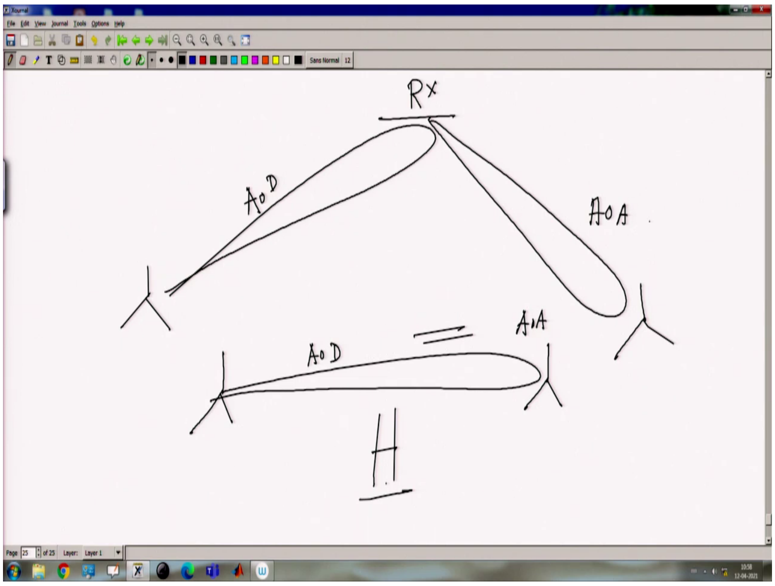

Because if there is a reflectors, what will happen? This beam will be going like this, but there is a reflector and with this reflection, the beam is getting reflected in this T x side. So, in this case the angle of departure, suppose there is only one reflector is present and angle of arrival at the receiver will be different because there is a reflector which changes the angle of arrival and angle of departure.

But if there is no such reflector, so then, what happens? The beam is going like this and my receiver is also sitting. So, whatever angle of departure he creates it, as because the receiver is also sitting in the same direction, in this case this will be equal ok. But in this case, angle of departure angle of arrival will be different; but these are the different scenarios.

I can create it from a simulation part parts or a practical; practical is how it will be right; you have a multiple set of AoD and AoA. But if it is an LOS transmission, it is the same AoD and AoA would be assuming the receiver is also sitting at the same point ok. So, with this, I hope, I make you understand how exactly the H matrix can be configured or can be constructed for different configuration of my antenna at the T x side and R x side with 1 d, 2 d and 3 d ok.

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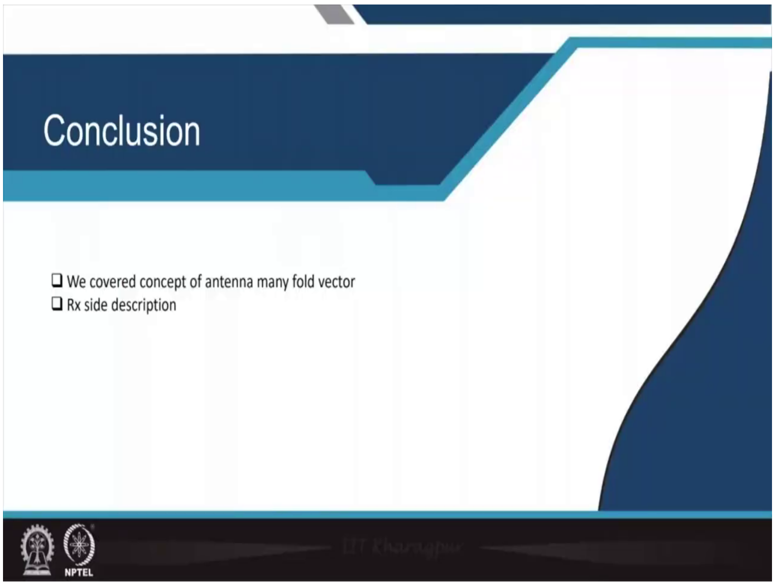

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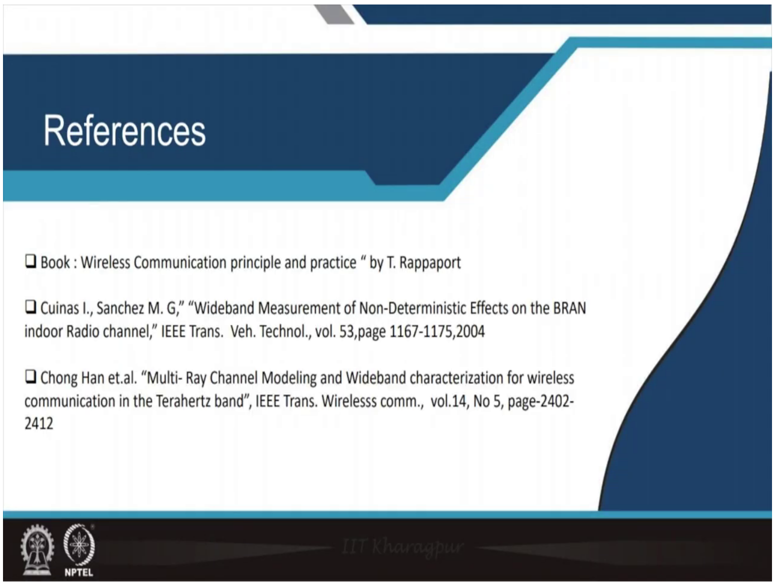

And these are the very much references. With this, I conclude the talk today.