Signal Processing for mmWave Communication for 5G and Beyond
Prof. Amit Kumar Dutta
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Module - 06
An antenna array processing concept
Lecture - 34
Concept of antenna many fold vector

Welcome, so now, we will start the array many fold vector creation for different configuration at the Tx side and Rx side. So, we will be talking only for Tx side, Rx side is just the mirror image of what we talk for the Tx side ok, it does not really matter which side I am talking of.

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So, these are the things that I will be covering. So, I am creating the array many fold vector, let us take some more example.

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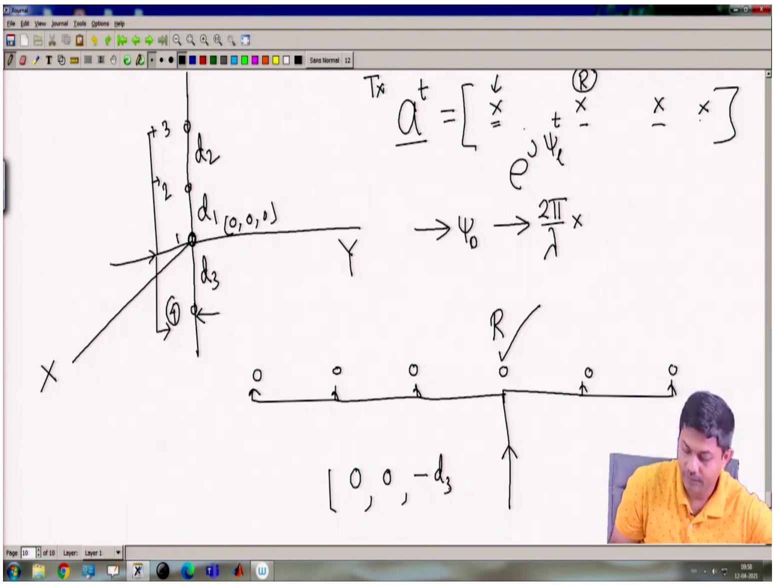

Let us say my configuration is now this configuration ok. Let us say my antenna is d, my second antenna is 2d between this distance between 2d or rather I will put it little generic so, let us say d 1 and I have a second antenna which is the distance between this is 1, this is 2, this is 3 ok.

Let us say the distance between 1st and 2nd antenna is d 1, the distance between 2nd and 3rd antenna is d 2 and probably, I may have one more antenna in the below bottom side of z angle so, that is my 4th antenna ok and the distance from my reference point is d 3, this is what my configuration of my antenna. This is my X; this is my Y and I am in the Tx side.

So, I would like to construct the Tx side array many fold vector ok that mean I would like to construct a t by the way I am not considering multiple reflectors here, only one reflector because if I change my reflector, number of reflectors, same thing will be repeated for different different angle of departure that is the only difference should be otherwise, there is no other difference. So, it is just a one reflectors.

Now, I would construct, I would like to construct the Tx side array many fold vectors ok. Now, this is my configuration, whatever I have drawn here. Now, for each and every antenna so, what is my reference point? My reference point is this one, this is my reference point ok and the reference point should be my 0, 0 so, that is the reference point ok. So, reference is my coordinate 0, 0, 0 why? Because I am feeding my data electrically, I am feeding that point first from here, I am physically feeding the rest of the antennas, this is how the feed is going.

So, you can always ask right why, what so special about the reference point? Reference point is the one where the first electrically the signal will reach that is my reference point ok because that is the point where I am first feeding it from transmitter side. So, that mean if I have say one, two, three, four, five, six antenna, I suddenly say this is my reference point, what is the big deal about it?

So, here, the big deal is that electrically, I will be connecting that antenna first nearest after here, electrically I will be connecting like that, electrically I will be connecting like this; this is how it is ok. So, in two direction, the electrically it will be going there. So, that mean the direction of phase can be totally differ, one side will be positive another side could be negative with respect to my reference point. So, reference point mean that is the point I am feeding it first ok.

So, now, this is what my configuration ok so, I want to create the array many fold vector, how do I do that? So; again so, how many so, this is a vector a bar t so, how many points will be there? Four points will be there, four points will be there. What the format? e to the power j phi l that is the format, e to the power j phi l and let me put it t just to ensure it is in the transmitter side.

Now, the point here is that l equal to 0 meaning the first one, second one, third one, fourth one and l equal to 0 does not mean that it is the reference point, l equal to 0 simply means this point ok. So, you can assume who will be the, it does not really matter so, what exactly, but that is my first because that is where it is starting and then, probably it is going in the up direction.

So, this is my reference point in that case right, that is my reference point ok. So, how will I construct that? So, for the first case, that means this is the antenna for the first case, how I construct my xi ok? That xi is 2 pi by lambda so, for the first case so, let us call it xi 1 or you can call it xi 0 also just to make a consistency with respect to the other. So, the first antenna which is this antenna, fourth antenna in this particular case.

So, I am drawing from first, second, third and fourth like that. So, it does not matter how you number it, matter how you exactly place it ok 2 pi by lambda multiplied by what is the position of the fourth antenna? The position of the fourth antenna is 0, 0, minus d 3 right opposite side right ok.

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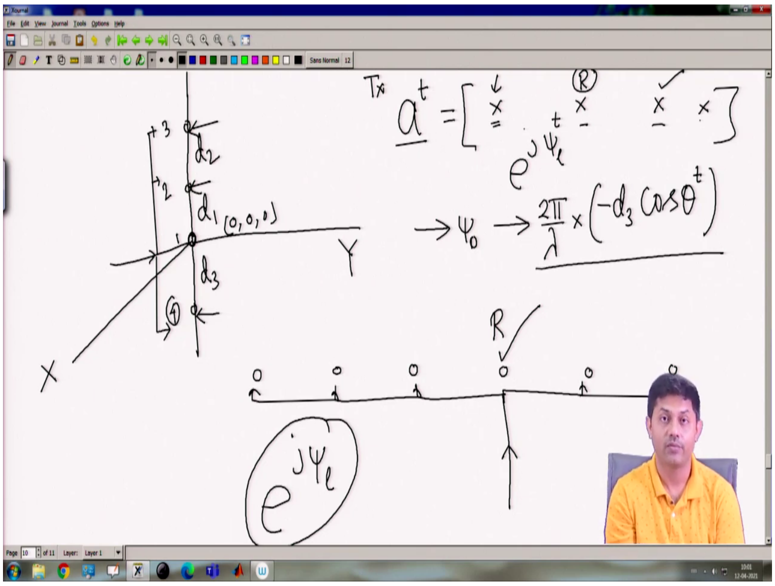

So, it would be minus of d 3, but as it is in the z direction so, for d vector only the third point will be coming so, that will be cos theta t will be coming into picture here, that is a first angle.

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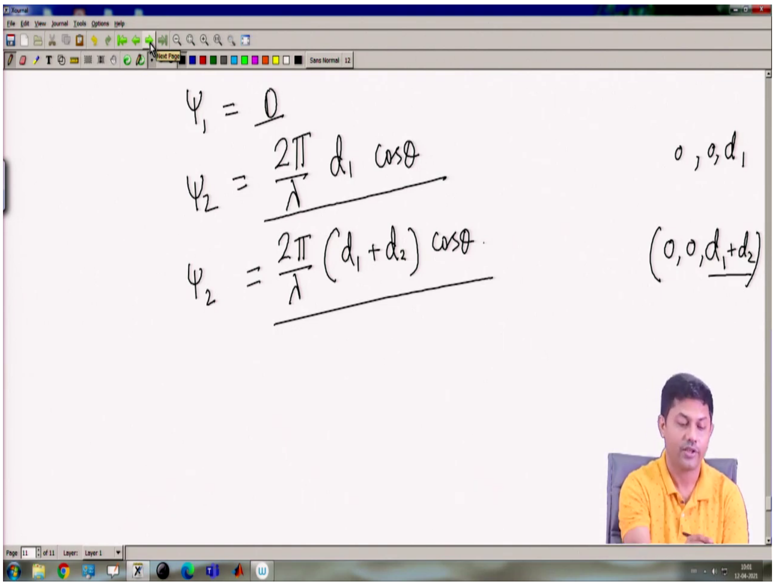

For the second case, for xi 1 case, probably I take the for the xi 1 case meaning the second antenna where that is the reference point every coordinate is a 0, 0, 0, 0, but that is the first point so, it will be 1 only; so, it will be 0 only e to the power j 0 will be 1. Then what about xi 2, second one probably this antenna which corresponds to this antenna ok. So, how do I constitute that? So, it will be 2 pi by lambda. What is the; what is the coordinate of that particular antenna? 0, 0, d; d 1 if I am is it d 1? d 1, this is d 1. So, it will be d 1 cos theta ok.

xi 2 meaning the this antenna case, everything with respect to the reference ok, what would be the case? 2 pi by lambda. So, what is the coordinate of it? 0, 0, but be careful here with respect to reference, it is d 1 plus d 2, it is d 1 plus d 2 right that is the coordinate because this distance is d 1, this distance is d 2 so, the total distance. So, the coordinate of the third of whatever I have drawn there that angle, that coordinate is d 1 plus d 2. So, this would be d 1 plus d 2. So, it will be d 1 plus d 2 cos theta ok.

So, what was my first angle? This one, second angle 0, third angle here, fourth angle here. Now, e to the power j of that you can construct it now. So, you can now construct this one, what is that? It will be e to the power j xi l. Now, individual xi l you can get it and put it what exactly my array many fold vector will be. So, there is an e to the power j term will be you have to ensure that because I have only calculate the phase.

But you have to put e to the power j into phase ok so, that you got it. Let us take some more example. So, this is just in a linear fashion.

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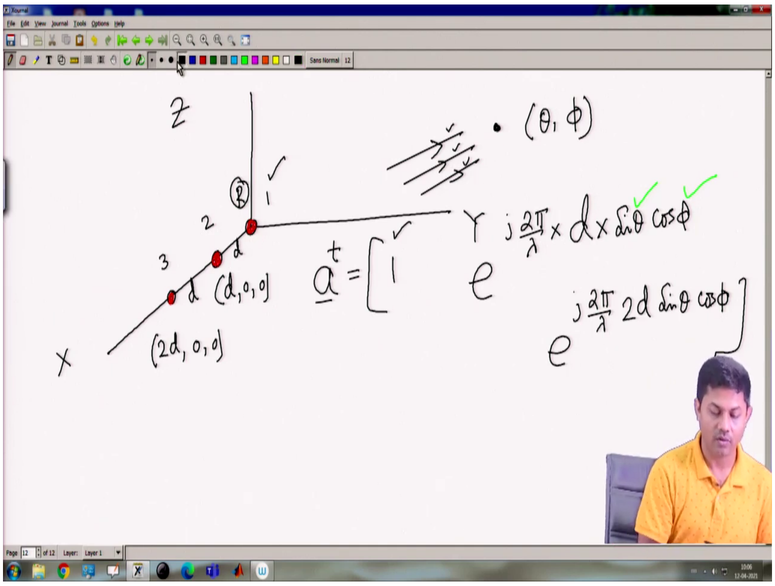

So, let us say I take a little interesting case. So, let us say I take Y, X, this is my Z ok. This is my now, let us say I have one antenna here ok, but I have one antenna in the X direction, I have one antenna in this direction or yeah, let it be ok.

This is my 1st antenna, this is my 2nd antenna, this is my 3rd antenna, this is how the corresponding antennas are placed here and I am observing in a theta phi direction so, how my. So, this is how my antennas in the Tx side are placed. So, I would like to know how exactly my Tx side array many fold vector will be created there ok.

So, for the 1st case obviously, so, what is my a bar t I want to calculate, for the first case it is easy because as it is in the 0, 0, 0 direction that is my reference point let us assume, it will be 1 always. The reference point meaning that is the point I am first feeding it ok. So, that is let us call it 1.

Then, go to the 2nd one. Now, here be careful here. For the 2nd one for this antenna, what is the position here? It is X, Y, Z coordinate, but only X exists, here there is no Z so, it will be X.

So, let us say the distance is d, this distance is another d, I mean antenna to antenna spacing is d so what is the coordinate for this particular antenna, this particular antenna? This is my antenna, this is my antenna and these are my antennas ok. So, the coordinate for the second antenna would be d, 0, 0. Coordinate for the third antenna would be 2d, 0, 0.

But now, be careful here. Here, it is in the X direction ok. Now, you multiply with the d vector, d vector that is direction vector, direction vector meaning it is a general vector I am standing something somewhere here whose this is where I am standing as if like that. So, I am observing it here and all the rays will be coming to me; it is as if like that ok.

So, with respect to that, how do I see the phase difference among each and individual rays that is basically the point here, that is basically what the array many fold vector represents right, it is the; it is the phase difference of individual ray coming out of the antennas at a particular distance r equal to 1 with an angle theta and phi as simple as that ok.

So, now a t I am calculating, how do I calculate a t? The first reference point it is 1 because its standing at 0, 0. What about the 2nd one ok? So, e to the power I am just directly taking it now, you might be knowing how quite how it should be calculated e to the power 2 pi lambda ok multiplied by the direction vector is only d, 0, 0 so, it should be multiplied by d multiplied by who else is there in the X; X?

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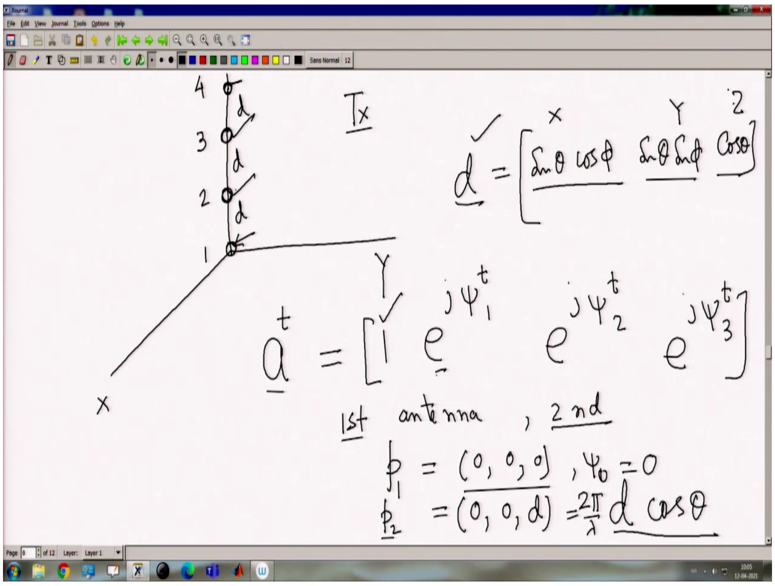

If you look at what is there in X? This is the X coordinate part, this is the Y coordinate part, this is the Z coordinate part; this is my X coordinate part, Y coordinate part, Z coordinate part of my direction of departure or the angle of departure. So, that mean as if like I am standing at theta phi direction that X, Y, Z is that particular general coordinate so, it is sin theta cos phi. So, it will be sin theta cos phi now you see.

So far, we are only dealing with one angle, now there are two angles appearing, just because I have changed my antenna configuration and the way I am standing. What will be my third point because there are three antenna, it will be e to the power j 2 pi by lambda. Now, there are 2d here right that is the thing so, instead of d, it will be the same 2d sin theta cos phi. So, that is my transmitter side angle of array many fold vector.

Now, you see there are two angles appearing here, this angle both the angles are appearing here ok. So, this is interesting point. So, depends on how exactly I put my antenna, the array many fold factor will bring in either azimuth or elevation or both ok.

Let us take some more example probably that make the concept more clearer ok.

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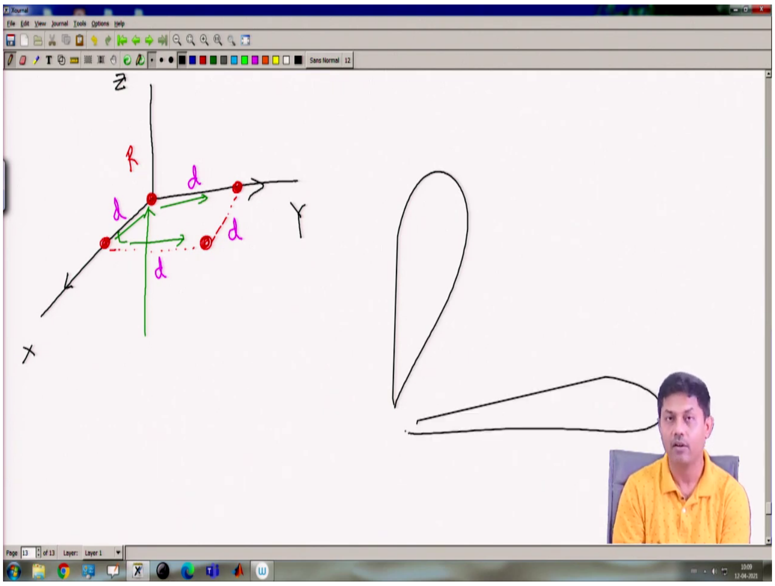

Now, let us say I in the transmitter side, I am still in the transmitter side because receiver side is just the reflection, there is no other difference. Instead of a o; a o d, it will be a o a, but equations remain the same so, it really does not matter which side I am talking ok. So, what should be my array many fold vector for a configuration as I have drawn here ok?

Let us say one of my antenna is placed here, second antenna is placed here, third antenna is placed somewhere here and fourth antenna is placed somewhere here, it is in the XY plane ok let us. So, these are this is my antenna.

So, it is on a XY plane ok. Let us say this is my reference point, what does it mean? It is as if like from here, I am feeding this; feeding that antenna first, from there its distributed, this is how it will be created ok. Third distribution will not be like that, third distribution will be from here. From there it goes and then, it is going like that ok. So, this is how the reference is created.

Now, let us say the distance among each and every antenna is same I just take it d for simplicity, all four directions ok. Now, for the four antennas, I would like to create the array many fold vectors at the transmitter side. Now, this concept is so important, the same thing I can do it for receiver side and most importantly, the same concept is extended when you try to create the beam pattern.

See so far, I have not talked about the beam pattern for example, this four antennas are placed like that, how exactly the beam will look because that is the intention of doing beam forming right, we have not even talked about that how exactly the beam will look like, this is more of the antenna that is getting configured. So, this is more of the array processing I am doing, but how exactly the beam will finally come, will the beam look like this, will the beam look like that, all these things we have to ; we have to mathematically formulate.

And when we try to do that, we will see that this same concept is applied there, the way I am drawing the phase, how exactly my phase will look like so, the same 2 pi by lambda into position vector multiplied by the direction vector of the ray. So, that is the phase difference which will be seen even for the beam pattern also. So, this is more of the how exactly my array many fold vector from the channel point of view ok. So, this is how it is right. So, this concept is extremely important.

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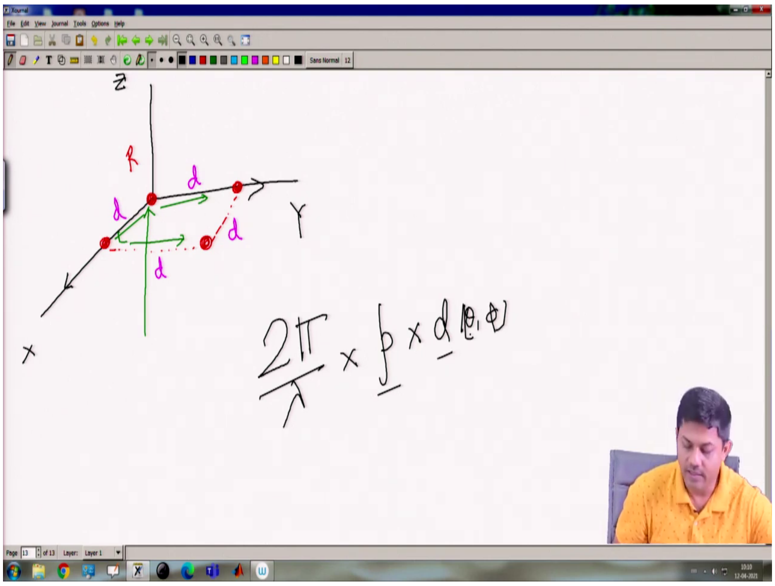

So, this will be applied so many places, this concept, 2 pi by lambda just understand the phase difference among these antennas are nothing but 2 pi by lambda into antenna position vector multiplied by the direction of either arrival or direction of departure or it is a position where you stand it, the general position ok which is a function of theta and phi. So, this is the crucial part ok. So, I am just trying to show you various applications on that. So, let me remove this part ok.

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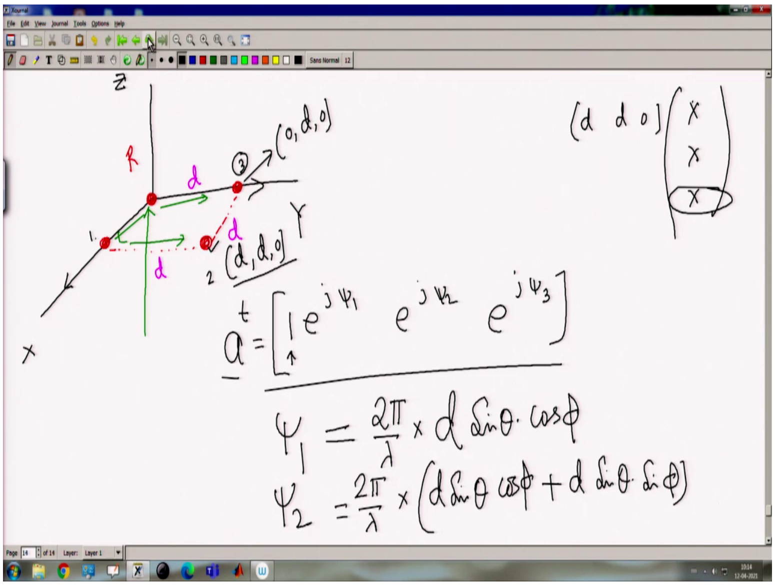

So, now, let us say this is my configuration. Now, so, this is my reference, my 1st antenna, this is my 2nd antenna, this is my 3rd antenna let us assume this is how I am trying to get the array many fold vector.

So, what is my array many fold vector? a vector t right. It is good enough to understand the angle, I need not to create phi because this will be same as 1 e to the power j phi 1 e to the power j phi 2 now, why 1? 1 would be basically where reference point is where the where place is 0, 0, 0 so, it will be always 1, the angle because the position is 0, 0, 0 so, I put a 1 here. So, it all depends on where you start as a reference.

If you say that is my reference point well, that should be 1, I will show that what if that is not a reference point something else is a reference point, but that is also there we will show. So, blindly, four angles will be there. So, here also there is a phi 0, but phi 0 is 0, 0 because the reference point is 0, 0. So, these are the four things that I have to calculate phi 1, phi 2, phi 3. So, what is phi 1, phi 2, phi 2, phi 3 so, these four things I have to calculate.

So, what is fine is xi 1? So, what is xi 1? Xi 1 is the position of this antenna, 1st antenna multiplied by the direction vector. So, what is the position of this angle antenna, 1st antenna? So, this particular antenna whatever I have drawn, it is on the X axis so, what is that? So, this 2 pi by lambda it would be multiplied by what is the position of the antenna? That is d, 0, 0 so, only d will be appearing so, this will be cos theta cos phi sorry sin theta cos phi; sin theta into cos phi because d we know, what is d? D is a standard vector so, that is my xi 1.

For the 2nd one, be careful here. What is the direction? So, here you can see for the 2nd antenna, the coordinate is what is the coordinate for the second antenna? It is d, d and 0 right that is the point. It has X, it has Y, also d and there is a 0. Now, this should be multiplied with my original direction vector so, what it should be? Again, 2 pi by lambda. Now, the position vector is d, d, 0 multiplied by that particular d vector which I have drawn here, here this one.

So, always remember that sin theta cos phi, sin theta sin phi, cos theta. Now, multiply that vector so, what it should be? So, it is as if like you are multiplying d, d, 0 with another vector ok so, that is the direction vector. Now, this will not appear because this is 0 anyway, but the second two terms will be appearing with a summation ok. So, this will be d sin theta cos phi plus d sin theta sin phi so, that is the xi t ok.

Similarly, what happens when it is the 3rd one, 3rd case is a simpler case, 3rd case what is the coordinate 0, d, 0 that is the coordinate vector ok.

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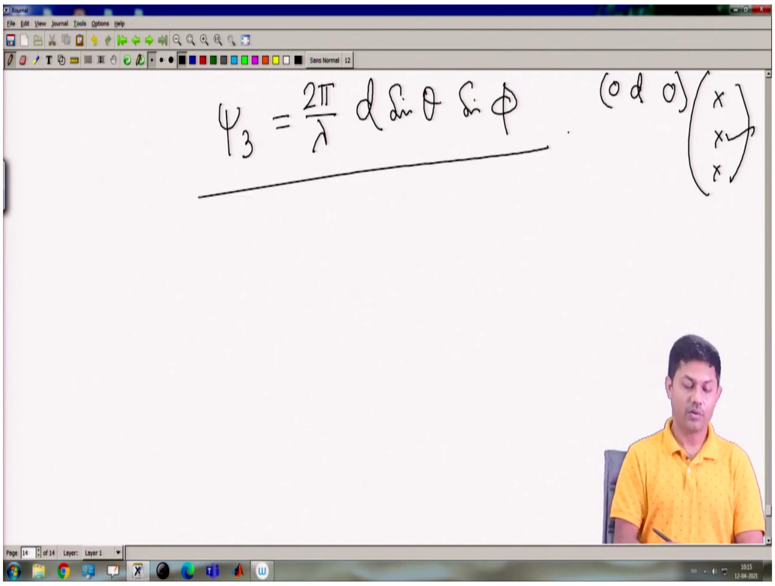

So, in that case, what will be my xi 3? It is multiplied with 0, d, 0 and direction vector, only this will second term will appear right. So, this will be again 2 pi by lambda into d into sin theta sin phi that is my angle so, which means that now you can see how complicated the angle should be, it just that there are theta and phi.

So, you should not remember the general (Refer Time: 22:23) always l minus 1 d cos theta that was only if it is a linear in the Z direction, but if I change my configuration, it all depends how exactly my array many fold vector will be constituted in this case ok.

Now, let us takes now, we can generalize it, I am not making any more generalization so, you can extend the number of antennas; you can extend the number of antennas and for that you can really create different array many fold vectors ok.

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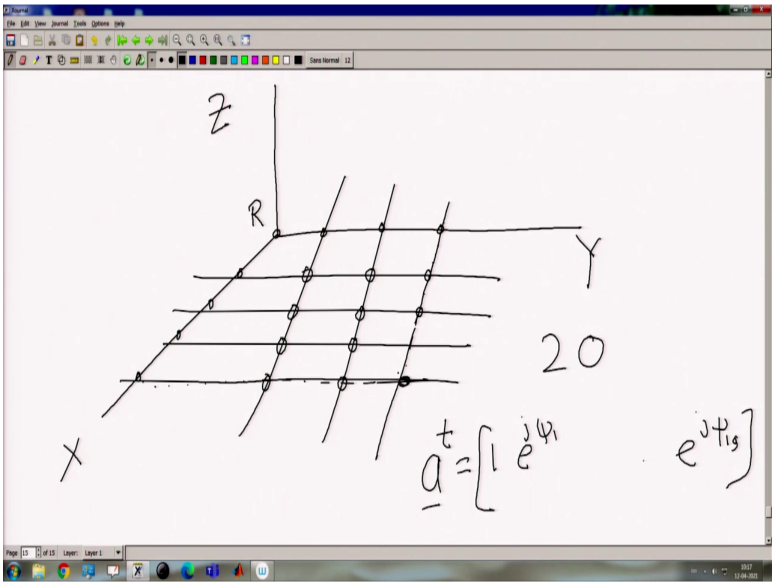

Solet us take another interesting direction probably I will not solve that problem, but I can only tell you how exactly the process should be. Let us take a case where you have a three-dimensional case. So, this is the whatever example I have just shown you that was a UPA uniform planar array.

So, I can even take more and more examples, you can have antennas like this, I am not doing the example really because if you understand the concept, it is just a matter of extension right. So, you can have as many as antenna you want, I am just writing a grid so that it will be easy for me to place antennas.

So, now let us say this is my antenna, it is like a grid structure, it is on XY plane, and this is what my antennas are, this is how my transmitter side is how my antennas are. So, let us say this is my reference point. So, you can say the first row, then second row, third row, fourth row and so and so forth as many as you possibly. Can I have some random configurations? Now, you know how to do it right.

So, how many antennas are here? Here there are 20 antennas are there right. So, the a t will be something like 1 e to the power j 1 to e to the power j xi 19 and you know how to control it. For each and every antenna, you can first calculate the what is the position vector multiplied by the direction vector that is it, over. So, that is my array many fold vector from the transmitter side ok.

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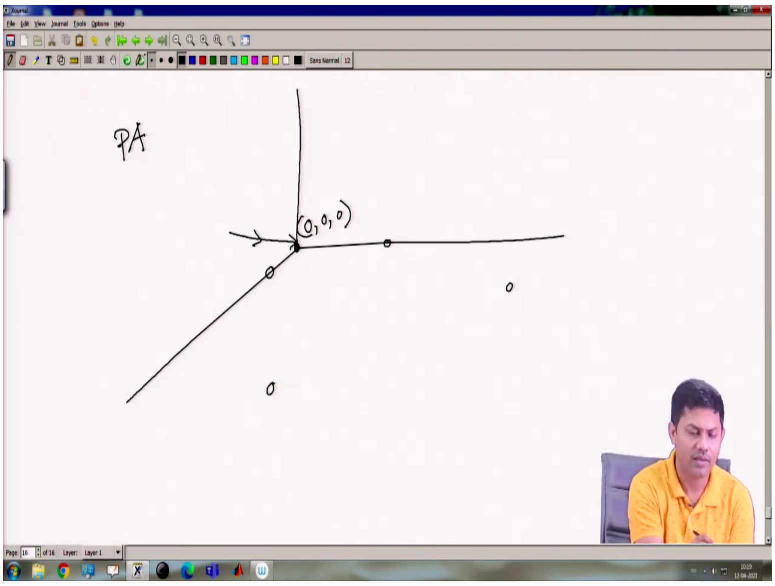

Take some more, take some more configuration, let us I take some random configuration ok one antenna is here, another antenna is here, another antenna is here, another antenna is here, some random configuration. It is on XY plane, still planar array, but not uniform planar array so, it is a PA, it is a nonuniform planar array ok because they are not uniformly spaced equally. So, how many antennas are there? Four antennas are there right. So, array many fold vector will have four component.

Now, here, who is the reference point? Just see I have not drawn any reference point here ok. Now, I consider that as my reference point let this be, there is no point here because it is as if like I am first coming here and from there, I am distributing physically so, let us assume that is my reference point, but here there is a catch, I am not making any of the antenna as a reference point so, who should be the reference point?

A reference point is fixed, 0, 0, 0, but the feed line is not coming to the reference point directly so, what does it mean? It means that electrically the signal will first come there, from there it get distributed. So, that is not the first antenna to touch it ok. So, how do you solve it I mean just generalize it.

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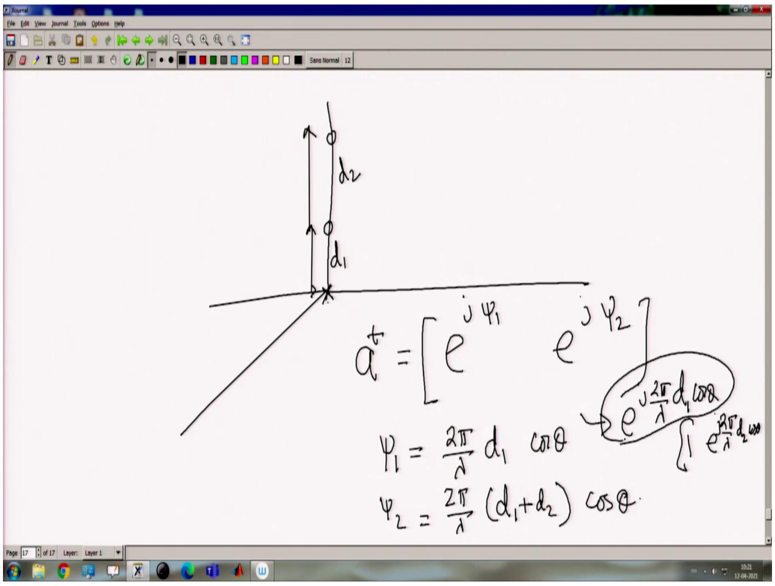

So, for example, before solving such issue, you can think of this kind of scenarios let us say I have one antenna here, another antenna here, but my reference point is this and there is no antenna there so, what does it mean? Physically my feed line comes here, from there it get distributed.

So, you just proceed so, what is my a t? E to the power j xi 1, e to the power j xi 2. You do not care about how they are configured, you just see from reference point where my antennas are that means, that is a feed point from that it is getting distributed.

So, now, as there is no reference point, as there is no antenna in the reference point so, it will be like a general structure. So, let us say my distance from here it is d 1, my distance from here it is d 2 so, what is xi 1? Xi 1 is nothing, but 2 pi by lambda d 1 cos theta. What is xi 2? 2 pi by lambda because it is in Z direction, d 1 plus d 2 cos theta that is the only thing will happen.

But now, you catch here as because the reference point you have assumed something else so, there is a common d 1 cos theta term is there so, that is ok. So, from here, you can just take one part e to the power j 2 pi by lambda d 1 cos theta 1 e to the power j 2 pi by lambda d 2 cos theta something like that you can do it as if like you have shifted the reference point to that one of the angle; one of the antenna, but this phase will be incurred there, but that is ok, I mean this is just a general point here ok.

So, with this, I conclude this particular class; let me take some more example with 3D configuration.

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This is what we have just concluded.

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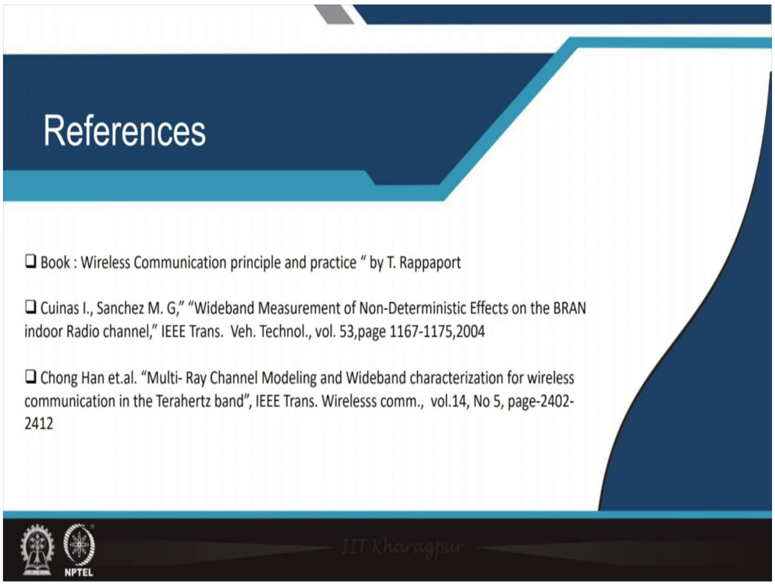

These are the; very much references. With this, I conclude the talk today ok yeah.