Signal Processing for mmWave Communication for 5G and Beyond
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Module - 06
Beamforming and mmWave channel Model
Lecture - 33
Single Antenna beamforming

Welcome back. So, we were continuing the channel model so far the beam forming part, and then we were talking about the single antenna beams. When I say single antenna, always remember that it is a single data stream not single antenna to be very frank, but effectively it is like a single antenna.

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So, so far we have seen the following. So, these are the things that we will be covering.

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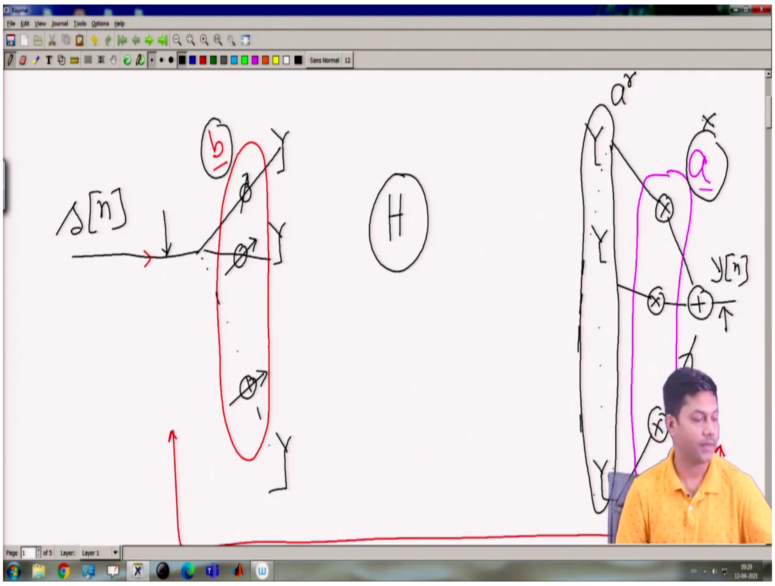

This is the data line. So, you have the data s n, and then you have an antenna here, you have an antenna here. Let us assume I am not creating any steering here. And this is where my received pattern ok. This was something, this was some multiplication some multiplication. It could be equalizer as well, or it could be m r c, we do not put much emphasis on that how exactly you design the last one. Now, here you can have a phase shifter ok.

So, and the channel in between was like this. This was the channel we characterized in the last class. So, from here if I stand and if I observe here so that is my point of viewing. So, my view is this, it is exactly before the bunch of the antennas and exactly after the bunch of the antennas where the single stream is split apart and the received streams are again getting merged. So, I am talking of that kind of view. So, in that view, what was the channel that we discussed last time?

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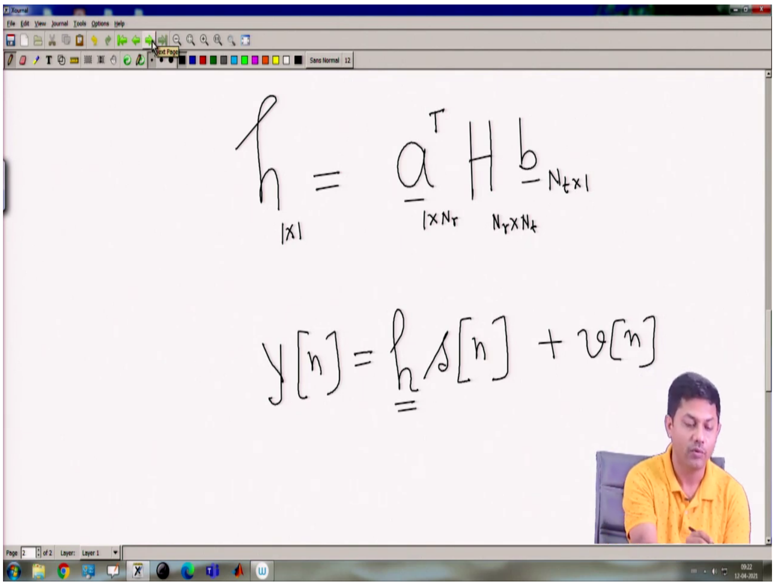

It was h is equal to some a vector, this was some H vector, this was some b vector probably, this is what it is ok. And the dimension of this channel matrix was, so this would be 1 cross 1 because if I view from s n to y n, it is like a one tap.

So, what is the data, final data model, final data model was like this right, single tap channel currently we are not going for it. And this is what it is as if like you have transmitted through a single antenna as if like that and this is my noise. So, this is what it was right.

So, but internally this small h will be going through multiple channel. And this is what we have decide we have seen yesterday in the last class. What is the dimension of this capital H? So, this capital H would be N r cross N t antenna, this b will be N t cross 1, and this could be 1 cross N r, this a bar T.

So, this b is here. So, this is what I can call it b, this vector I will call it, this whole vector I call it b. And the first one was more of a, so this is a sorry. So, this is the first one would be a, this would be a, and this should be your b. I will put a different color this is your b vector. So, this is what it is right. So, this is the b vector, and then you have a H, and then you have an a vector. If you multiply it, that becomes a single dimension point.

Now, last time we tried to characterize this particular one. Now, it will now we will be subsequently characterizing this b and a. So, what was H? H was more of a characterized from the a o a and a o d. If you remember in the last class, so there are two ways you can put the angle.

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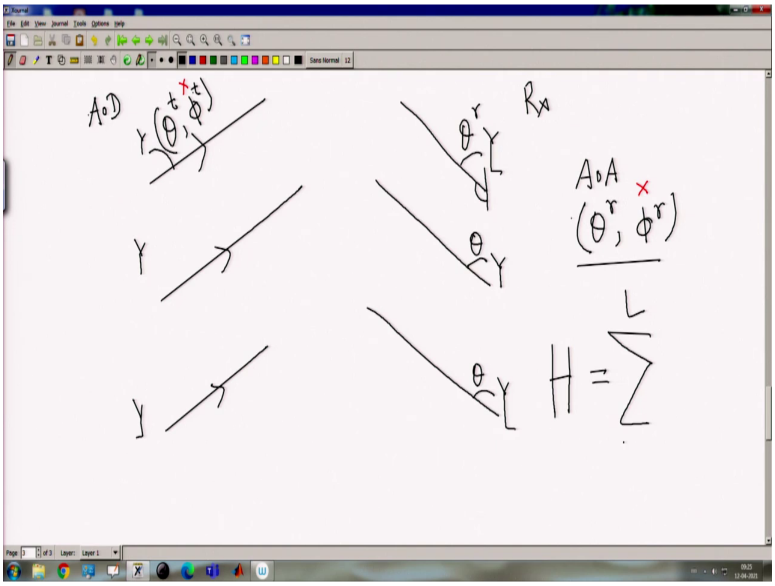

Either you can put the angle like that; either you can put the angle like this theta. If it is in the received side, I call it angle of arrival; or you can also put it like this angle also whatever I mean it really does not matter. If that is the case, then it will be sin theta; or if it is a theta, it is will be a cos theta.

If it is in the t x side, t x side angles if this is say all these rays are going like that. Rays will be going in all direction, but you are observing at a certain distance say theta probably I call it theta r, this should be theta t. Now, it is not that the theta t and theta r that will be coming as an A o A because it can be in three dimension space.

So, the angle of arrival will constitute will be constituted by two angles. One is the elevation angle, one is a azimuth angle. So, when I say theta r in principle there may be one more angle sitting inside. When I say theta t inherently, there may be one more angle sitting inside it. This is your angle of departure.

Now, we will be in this context, probably we will take some more example and try to understand what is this angle of arrival and angle of departure means in our case ok. So, this is if this is the case where you have both sort of angles, then we have seen last time how exactly the channel models would be.

Now, in the kind of configurations we have taken where this particular angle was not appearing your azimuth angle, but that is ok. It all depends on how the antennas are configured. Today, we will take some more example and we will try to generalize it the channel part. So, how exactly our channel was, the channel was h is equal to if there are you know L number of reflectors in our path or n number of reflectors rather.

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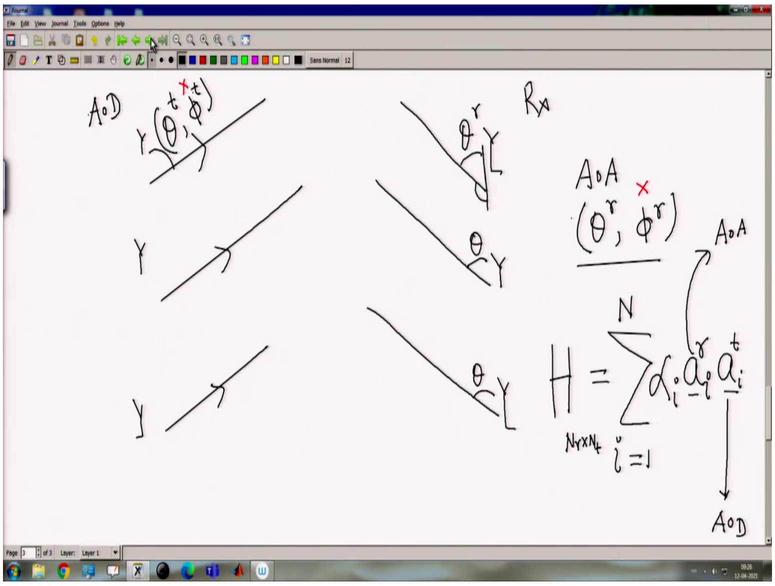

If there are N number of reflectors in our path. So, it was more of an alpha i, then you have a t and a r right. So, this would be your N r cross N t right. So, this would be your a r vector. And I put an i here to indicate that this could be for multiple reflectors, and then this is my a t i ok.

Now, this a r, a i r and a i t are receiver and transmitter manifold vectors which are completely dependent on your angle of arrival and angle of departure. This a t side it depends on angle of departure; and a i r side will be dependent on angle of arrival side. And how exactly this a i and a r are constituted?

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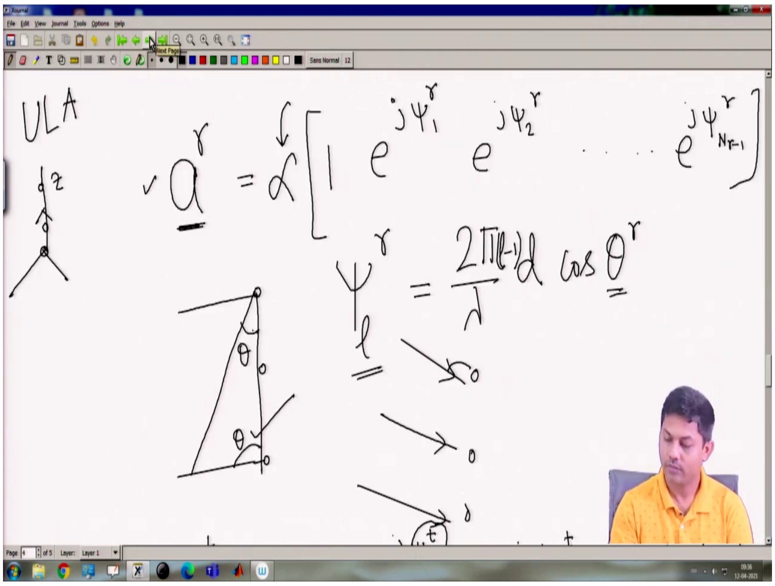

If you look at say let us say I am in the receiver side, so a r was, if it was an ULA, ULA, and specifically I was assuming that my antennas is in the z direction that mean all my antennas are in the z-direction. This is how my assumption was. If it is in some other configuration, we will see how exactly this whole scenario changes. This was more of an alpha then there is a common angle.

We do not want to introduce the common angle, everything is getting engulfed inside this alpha itself. Alpha will be a real as well as a imaginary part, then this will be 1 e to the power j phi if it is r side, then it will be receiver side r 1 e to the power j phi 2 r side and so on so forth. If there are N r number of receiver, this will be r.

So, how exactly this xi was dependent on? How you constitute this? Let us generalize it. Let us say I am taking the lth xi r. So, this is dependent on the angle of arrival side because I am in the receiver. So, this will be 2 pi by lambda d into cos of theta r. Now, this theta r you have to be very careful which angle of arrival you are talking because there are different way you can define this theta. And based on that, whether it is a cos theta or whether it is a phi theta.

For example, if you remember some cases I was drawing like this in the last class right. So, probably I was talking of this as a theta. It all depends how exactly you take it, but usually you can take that way or you can also take it like suppose angles it is coming like this. So, this was my angle.

You can think of it so which means that this becomes your theta ok. So, it is all depends how exactly you take your theta is. You can take this theta. If you take this theta, that will be sin theta. If you take this theta, it will be more of a cos theta. So, this cos theta r comes when you take this as a theta, so this is your angle of arrival that is coming into picture Now, this is for a ULA in a z-direction.

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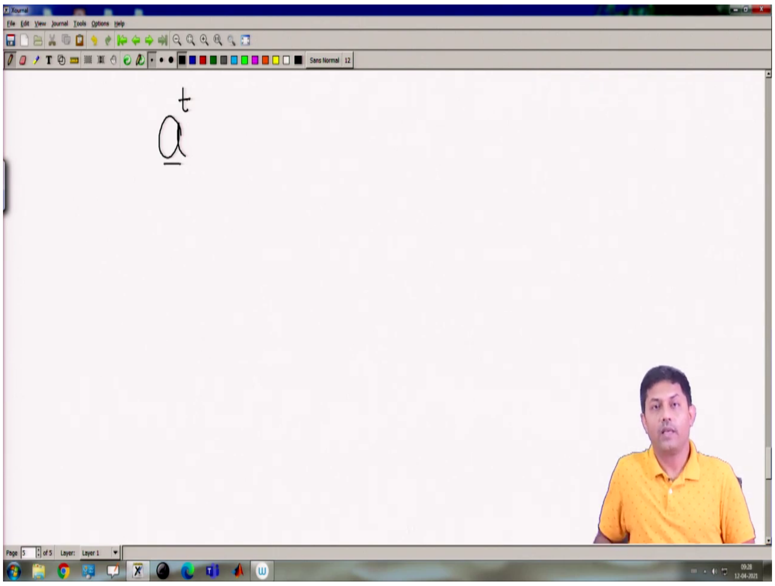

Similarly, if the same thing I do it in the transmitter side, a bar t ok. Now, this a bar t do not get mistake that this a bar or a r is not the same as what I have drawn here. They are different because I have not put any you know any t or r here, t or r, I have not put anything like that.

So, that means, this a, and this b is different ok. So, this a is more on this side, on this side, more on this side. Here this a r coming into picture, not this one, this is more on the equalizer side ok. So, I am removing this part just for making the diagram slightly cleaner ok, so, now ok.

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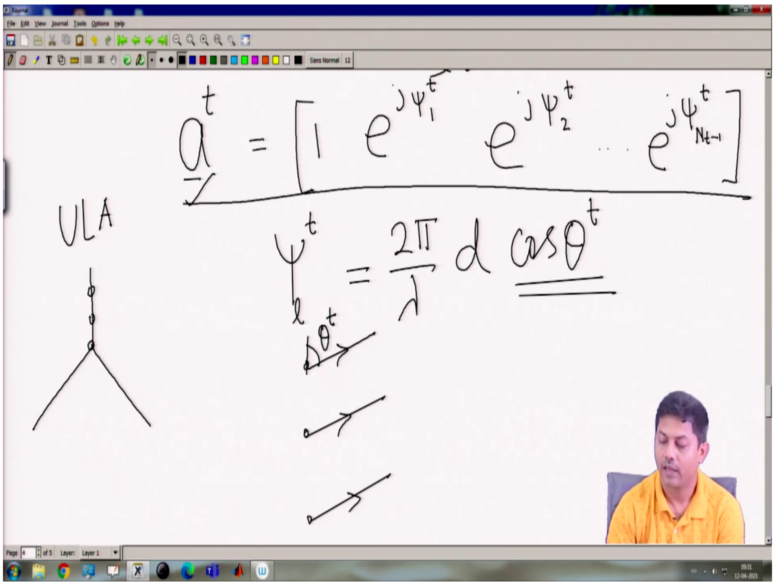

Now let us go to the a t side because I am just recap the whole thing a t side that means that is the array manifold vector in the transmitter side this would be again it depends 1 e to the power j xi this will be let us call it 1 e to the power j phi t 2 and so on e to the power j phi t, this should be N t because I have N t number of transmitting antenna. So, this is what it is right, where I generalize it say I take the lth xi l which will be again 2 pi by lambda into d into cos of theta t.

Now, is it only cos theta t? What is cos theta t? Cos theta t was if I if you look at the earlier class, it is the angle of departure at which angle all of them leave. Now, it can leave in all direction, it is basically to indicate at which angle you are looking at the data that is what it is right. So, if it if you look at the data, at an angle theta t, so now as you know along with theta t, there is a phi t also. But if I am assuming it is a ULA, and if it is only in the z-direction, this theta and all these things will not come.

So, today I will discuss that how exactly this part will be structured ok. So, this was the more of a summary of what we have learnt it. So, this is your transmit manifold vector. And this side, this is your receive manifold vector, receiver side manifold vector. And this is your, sorry, this is your transmit side manifold vector ok.

And so this is your receiver side manifold vector, and this is your transmit side manifold vector. So, transmit side manifold vector depends on angle of departure, and receive side manifold vector depends on angle of arrival. Now, I will discuss more and take some more example what happens when I take a different configuration of antenna, how exactly my you know my, this xi angles will change ok.

So, now let us take a different sort of configurations. So, so far what type of configuration we have taken? The z, on the z-axis the antennas are actually placed ok. So, can we generalize it? Like for example, can the antenna be on x y plane, can the antenna be on a y z plane, can the antenna be on a three-dimensional plane or any other geometrical shape, what happens to this particular xi angle? So, let us try to understand.

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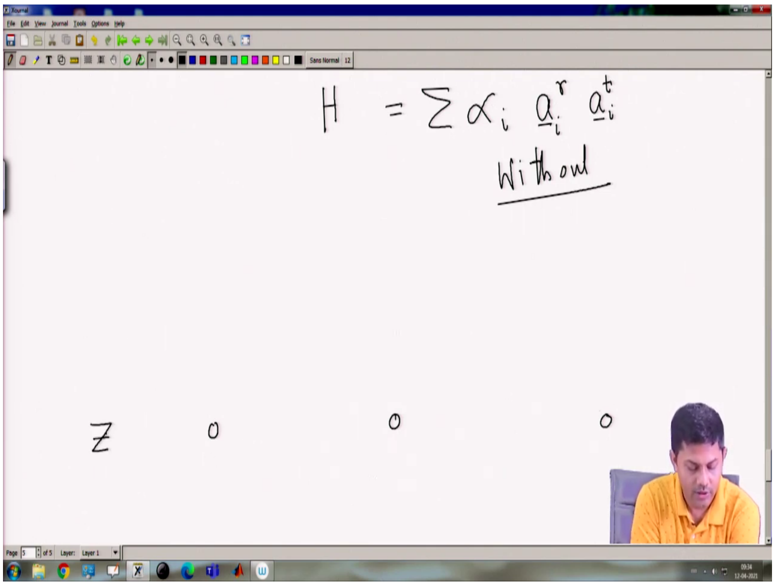

So, the conceptually your channel do not change, conceptual conceptually this remains same your H is equal to summation of alpha i a i r vector a t i vector. So, this two remains same. There is no difference. Now, this is without, this is without the steering as well as in the equalizer part because that part again a vector and another b vector comes into picture. So, those two vectors are separate, so that I will also explain.

But today I am considering more on the channel side, H side, how exactly my transmitter and receiver manifold vectors depend on the configuration of my antenna. So, these are the I will take more and more example. So, let us take some other way of understanding it ok. Now, let us say how exactly this angle comes into picture here ok.

So, let us take the receiver side first because it will be easy to understand, then we can get into the transmitter side. So, let us say this is my antenna these are just my two antennas, but probably I can draw one more antenna. Let us say this is in my z-axis, probably I will put in a vertical form, so that it will be easy for you to understand.

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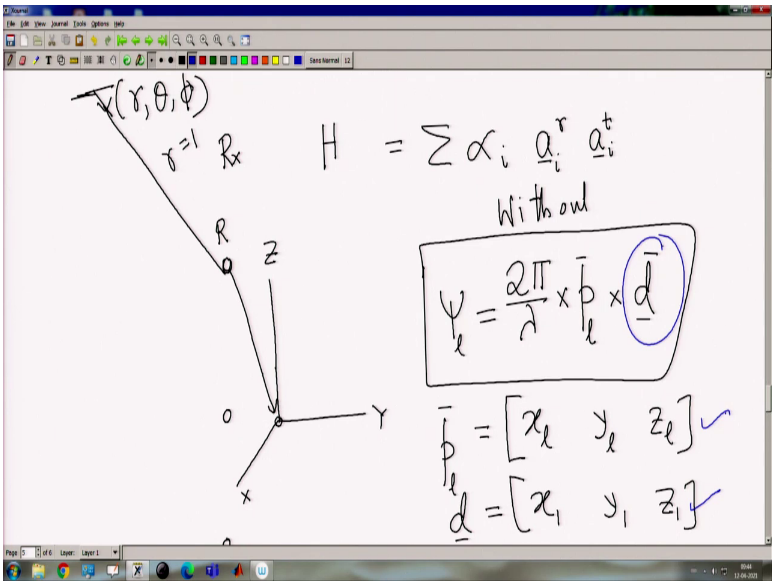

So, let us say I have this, this, and this ok, ok. So, let us say I am in the receiver side ok. Now, let us say z. This is my z-axis, this is my x-axis, and this is my y-axis. So, this is my y-axis, this is my x-axis, and whatever I have drawn that is my z-axis my antennas are placed. And this point let us call this point as my reference point. So, this is my reference point ok.

Now, reference can be anywhere, I mean you can either put central point as your reference point, that means, this antenna is somehow placed here that is what it means ok, so that is my reference point ok. So, that particular angle this xi, this xi, it actually depends on position of the angle position of the antenna and the direction at which the rays are coming ok. So, basically the formula for this particular xi is the following 2 pi by lambda ok, multiplied by the position vector of the antenna.

So, let us call it p if this is say n I am talking of for the nth antenna what exactly that phi. So, basically I am talking of any one this lth, lth or probably let us take it lth antenna, so that it will be consistent ok, so yeah. So, here, there is a small mistake here. So, there is a small l minus 1 would be appearing yeah.

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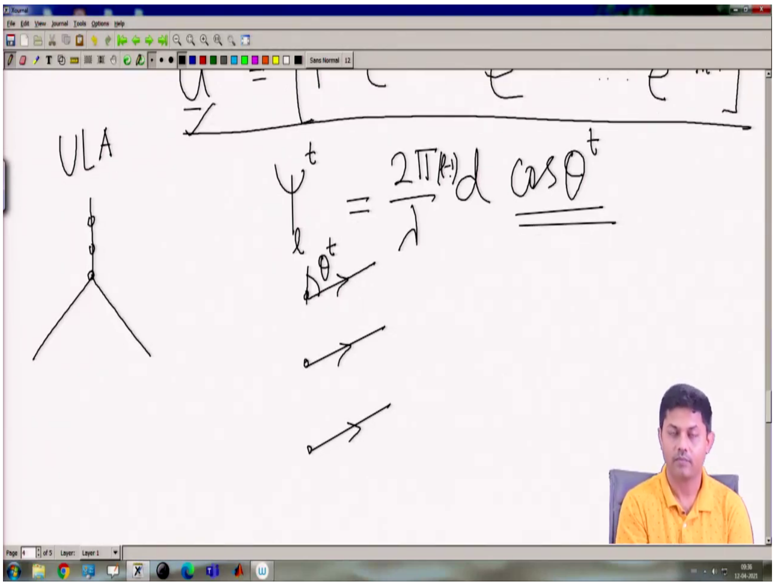

So, this should be slightly l minus 1 because it is the lth meaning there should be some weightage for the lth side ok. So, let us call this as a vector p l. I will explain it what exactly this p l vector means it is a position vector multiplied by the direction at which you are viewing your signal.

So, if it is a receiver side, you cannot view it because you are in the receiver. So, your it is mainly because it is mainly something like you are which direction the signal is coming. If it is a t x side, it can be at which direction you are standing there right. So, this is the direction at which the signal is arriving or at which you are standing from the t x side.

If it is a receiver side, this d vector is the direction at which the signal is coming from. So, this is basically the formula that comes into picture. So, what, 2 pi by lambda into p l vector multiplied by a d l vector d vector. Now, what is exactly this p l vector? p l vector is basically it corresponds to the vector position of the antenna.

Now, if you look at any antenna is in the 3d dimension right three-dimensional case, so obviously, it will have three coordinate so which means p l, p l vector for any antenna will have three component. One is x l component, another is a y l component, another is a z l component ok, three components will be that is a position.

So, any position, it is basically a position vector, so that is what I am trying to understand. So, that mean if the antenna is somewhere in this z direction, you can even calculate what is the x coordinate, y coordinate, and z coordinates ok. And d vector is the normalized direction of my antenna now of my ray that is coming or getting ray.

So, in this particular R x, d is basically the direction at which the signal is coming. So, which direction it is as if like I am standing at a unit distance somewhere here from this reference position, r is equal to 1. And I am standing at this location it is as if like that. So, basically it is the location at it is the location through which the rays is coming on the R x side ok.

So, d vector is also denoting a position vector. But here position of what? Position is of not the ray, ray cannot have a position. It is as if like you are standing I mean its if the ray comes from the theta and phi direction because rays is coming in theta and phi direction because there may be a reflector here.

And through that reflector the ray may be coming here and the direction of arrival would be theta and phi. And r i just take it one it is as if like from a unit distance maybe 1 meter or 1 kilometer whatever with an unit distance which direction my ray is coming. So, this d vector is more of a direction of my ray ok, so that will also have x, y, z coordinate because if the something is coming in a three dimension that will also have an x y z coordinate.

But now deliberately I will make the d in a polar coordinates ok. So, let us for the timing let us say the ray that is coming it can be say x 1, it will have a y 1, it will have a z 1 direction. In that direction say the ray is coming. Now, what is the polar equivalent of such x 1, y 1 and z 1, so that needs to be expressed it here.

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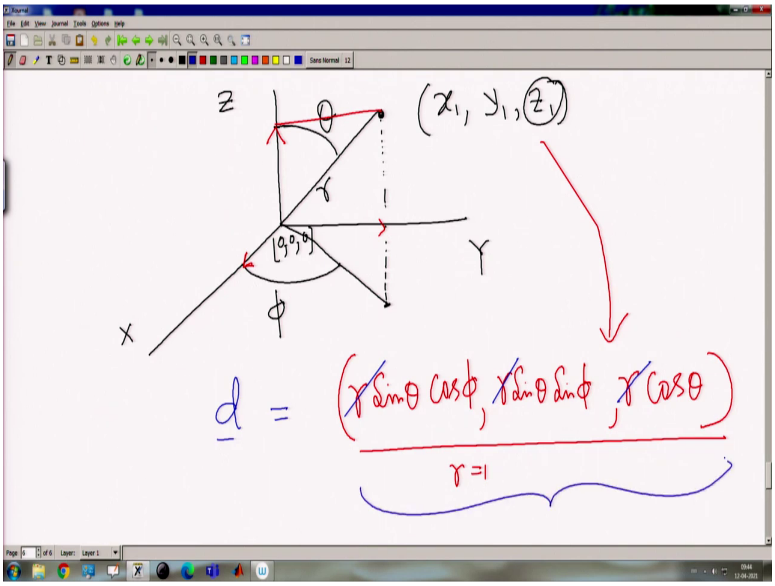

So, here if you logically think here suppose this is my Z coordinate, this is my X coordinate, this is my Y coordinate here right. And let us assume that a particular point is here whose direction is say x 1, y 1, and z 1. This is the position vector. How do I express it in terms of a polar coordinate?

So, let us assume that the same thing has a distance r from 0, 0, 0, which is the coordinate part or which is the origin part. And let us say it has an angle theta which is the elevation part. And let us say this is having an angle phi which is the azimuth path, this is how it is right. I will put it this way, so that is how a theta would phi would be. So, how do I express x 1, y 1, and z 1 in terms of r, theta, and phi. So, that is a polar coordinate Cartesian to polar coordinate conversion.

So, if you take it, what is my z? z is nothing, but this part right, z is this. So, z 1, z 1 will be r into cos theta right. So, what about x and y? So, this is x part, this is say y part ok. So, again it is relative how which one you call it x, which one you call it y, which one you call it, sometime you may find that the way I have written x in some book the x is in the other direction.

So, with respect to what you define you should consider that. So, that is why this cos theta sin theta many such terms you may find slightly different from other papers and books. It is all depends on who is considering x, y and z ok. So, if I draw this is my x, this is my y, and this is my z coordinate, and this is what my diagram would be.

So, what is my x equivalent? x would be this side. So, that is nothing but r sin theta, it is a projection on the x y plane multiplied by cos phi. Similarly, the rest y will be sin theta sin phi. So, this is the classic conversion from Cartesian to polar conversion. This is a very standard conversion ok.

Now, I am actually denoting this part. Now, here you make r equal to 1, because r does not change your direction ok. I am more interested in the direction rather than the absolute distance. So, you can take this fellow off, take this r off, r off, just make it r equal to 1. And this will represent your d vector. This is your d vector, that means, sin theta cos phi sin theta sin phi and cos theta. So, this is exactly your direction vector.

Now, when I say when I say I am multiplying I am multiplying p with d, I am essentially targeting d as a coordinate of the ray incoming ray if it is in r x side the polar coordinate form. So, if the ray is has is having elevation angle theta and azimuth angle phi, I express the direction of the ray in a polar coordinate so that is my d. So, my d is nothing but this particular one. This is my d vector which can be a standard vector. And this p is more of a you know the antenna coordinates part. So, this is my antenna coordinate part.

Now, I will show you what happens when I place my antenna in different, different position how exactly my phi will change in my case ok. So, let us take few examples then the concept will be clearer. Let us say I am in the r x side ok. Now, the same concept I will hold it for transmitter side also. The concept remains same, it just that the way you perceive the configuration.

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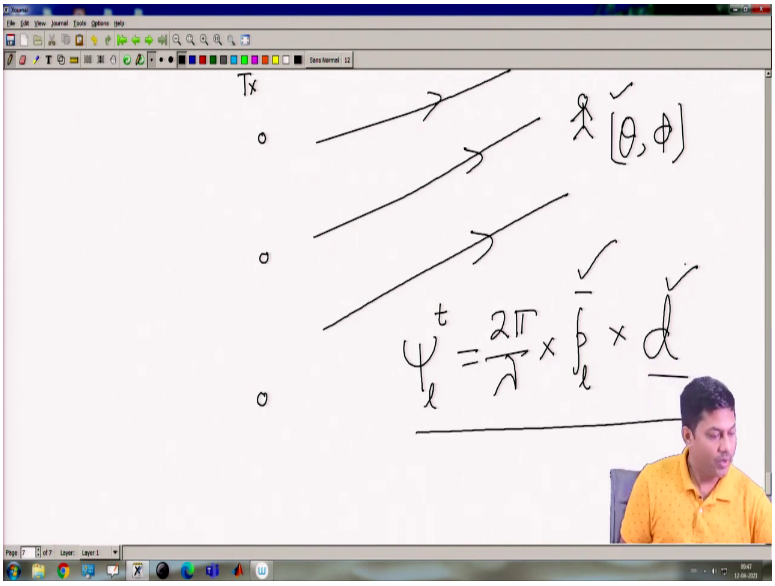

Now, this is I am in the t x side right ok. Now, here what is meant by theta, phi and all? So, here basically it is as if like you are standing somewhere here, you are standing somewhere here. And your position is defined by theta and phi. I am interested in the position I mean the direction not exactly the r.

So, you can assume r equal to 1. So, it is as if like I am receiving the ray in theta phi direction because I am that is the direction I am observing. So, this is my general theta phi direction, so that is my angle of direction, and angle of departure because that is the you know that is the position I am standing it right. So, I am observing as if like in a theta and phi, and I am trying to understand how exactly my pattern would be right.

So, in this case, in when I transmitter side, it will be the angle of departure at which the rays is going as if like I am standing somewhere in the angle of director, direction, angle of departure and I am observing all my incoming rays there so it is as if like. The same concept there also.

So, here also the xi t will also be 2 pi by lambda multiplied by the direction of if it is l direction of the lth antenna multiplied by the direction at which I am standing. See if I am it is a general theta and phi I am standing in, so that is the direction I am standing again the d will be remaining the same, d does not change because that is sin theta cos theta everything is together because that is a three-dimensional spacing observing the pattern here.

So, this makes the difference how exactly my phi will be looking like ok. So, let us few, so this is what my this is at the again transmitter side. So, equation wise they remain same. It is just that now this p l represents the transmitter side antenna configuration, this d represents the transmitter side is as if like observing the data from the transmitter side ok. So, let us take a few examples so the concept will be clear.

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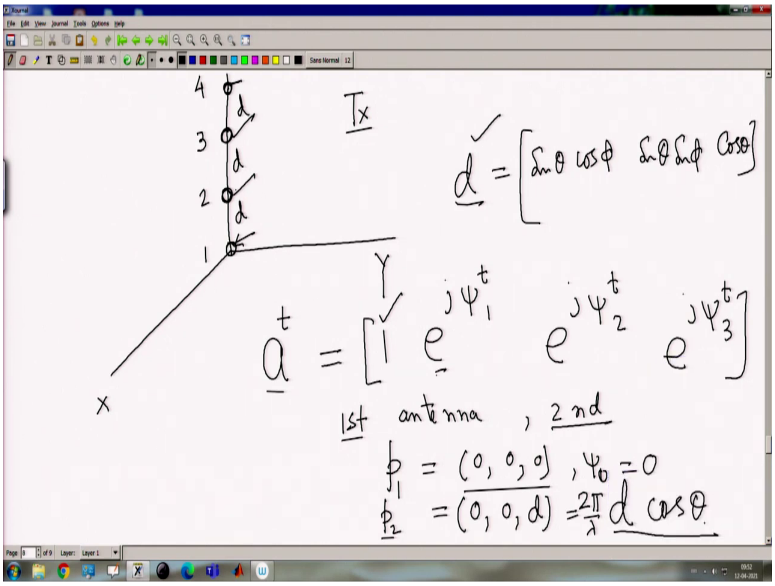

So, let us say again I take the old example where I have an x I have an y. And let us say this is my reference antenna. This is my first antenna. This is my second antenna ok. And this probably I may have one more antenna here ok. So, antenna number 1, antenna number 2, antenna number 3, antenna number 4 ok, this precisely how things will be going on ok.

So, how do you form the transmitter side? So, this is say I am in the transmitter side ok. And I would like to know what is my transmitter side manifold vectors right. So, transmitter side manifold vector is a bar t. So, this is my transmitter side whatever I have drawn ok. So, this will be 1 e to the power j xi t 1, xi t 2 and so on. So, this will be to the power 1 xi t 1 e to the power j xi t 2 e to the power j xi t 3. So, this is my transmitter side manifold vectors right because that will be used for my channel creation.

Now, what is the first ones? The first, what is the first one ok for the first antenna? So, this is basically for the first antenna; this one is for the second antenna and so on. So, so there are 4 antennas, so 4 elements will be present there right. So, for the first antenna, for the first antenna, so for 1st antenna, let us say the distance among all the antennas is d, so that is the distance I am taking it and it is in the z-direction ok.

So, what is the position? So, that mean what is my p 1? My p 1 would be 0 0 0. It is, it is a location, this is the initial position ok, this is my reference point so that is the 0 0 0. So, no matter what my d.

Now, what is my d vector? d vector is always constant, d vector meaning it is the direction at which I am observing the data, so that is constant. What is that? That is you can go back to your equation sin theta cos phi sin theta sin phi and cos phi so that is the one ok, that mean sin theta cos phi sin theta sin phi and cos theta that is my d vector ok.

So, for the first antenna what is my data as because all are 0, my phi 1, my phi 0 I should say because this is the first one will be 0 there. So, e to the power j 0 is equal to 1 fine. For the 2nd antenna, for the 2nd case, what is my p 2 position of this antenna, what is that? That should be 0, 0, d, d is the difference between that antenna ok.

So, what will happen? Multiply this p 2 vector with this d vector ok So, what should be the case? So, it will be 0, 0, d. So, it will be d cos theta.

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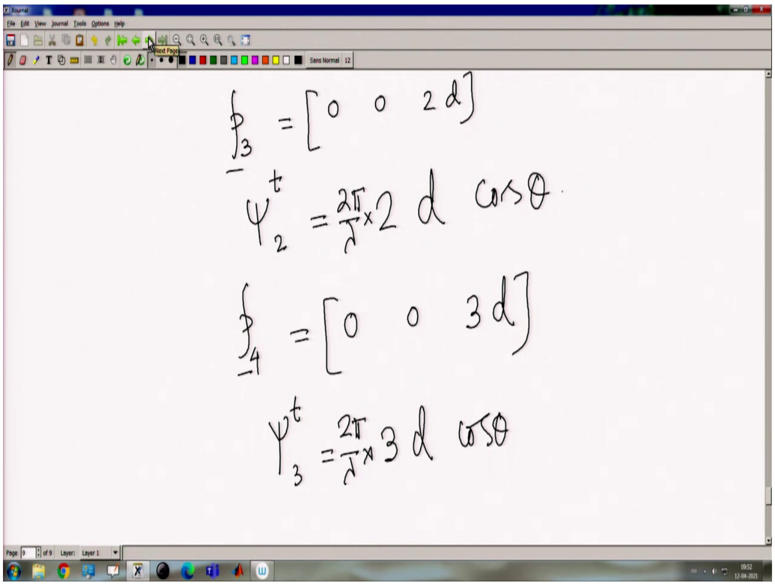

Go for the third antenna. What is the position, the third antenna? So, p 3 vector ok. So, this would be 0 0 2 d. This 2d height it is remaining. So, what is the xi corresponding to say this is xi 2 corresponding to 2 0 1 2 second case. So, it will be 2 d cos theta. What about p 4, p 4 vector? p 4 vector meaning the position of the 4th antenna, this antenna. So, what is that? So, this should be 0 0 3d ok. Then what is the angle corresponding to it? This will be 3d cos theta.

Now, you understand how why we exactly draw 1 e to the power 2 pi by lambda into and all these things. Now, this is my 3d, but of course, there is a 2 pi by lambda will be coming. So, I am not drawing that part. So, you can multiply that part. That part is common for all of them, that part will be common for all of them. So, that is my angular. So, that is the reason why it was 2 pi by lambda into l minus 1 cos theta because it is the distance that is coming.

So, let us take some more example in the next class. And we will see for different configuration how exactly this phi will be different. Now, this is all transmitter side. Similarly, you can draw it in the receiver side also. Let us take some example on the transmitter side. I will not draw any reference on the receiver side because it is the same there is no difference. One is the angle of departure, another is the angle of arrival, but equation remains the same ok.

So, in the next class, we will be taking some more example of different, different configuration, and try to understand how exactly this array manifold vectors can be constituted because once that is done I am done with my channel configuration or channel creation ok.

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This is what we have just concluded.

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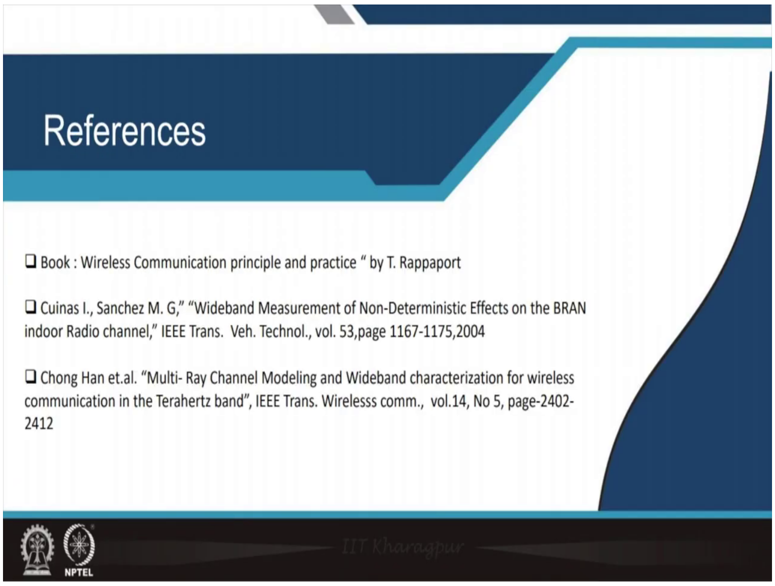

These are the very first references. With this I conclude the talk today.