

Signal Processing for mmWave Communication for 5G and Beyond
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Module - 05
MmWave channel model
Lecture - 29
MmWave channel model (continued)

Welcome. Welcome to Signal processing for millimeter Wave Communication for 5G and Beyond. So, we will be continuing the millimeter wave channel models from antenna to antenna.

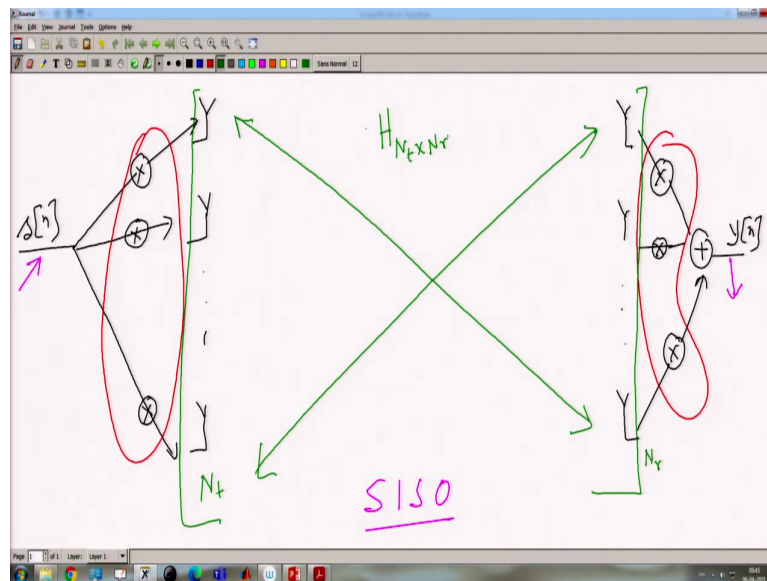
So, this is more of a multi-antenna channel model not the MIMO channel model ok as we explained it. This is still a single antenna; Single Input to Single Output system, SISO system, but internally the antennas will be more and more, I think we have explained that part. We will continue the channel model here.

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So, we will be continuing the millimeter wave channel model and various approximation that we started and as you know, the kind of approximation you have done earlier was the distance wise the gain was almost same, then we made some approximation on the angle of arrival. We will continue doing that what are the other approximations and what is the consequence of these approximations on the channel model finally, ok.

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So, what we started with? We started with a multiple antenna system here not the MIMO system because this is the difference between multiple antenna and MIMO, we are talking of a SISO system meaning inputs are same just only one input and you are observing in one output stream, but internally, there will be multiple antennas for the millimeter wave part and because we want to do a beams and we have explained it that only a multiple antenna can create those beams.

So, you have multiple such antennas here, at the receiver also you have multiple such antennas. So, here, the data that we are sending is still single data so that is why it is a single input single output. So, here also, I am receiving $y[n]$ so, obviously, there is an adder here which where I will multiply it and from here, as we explained it that we try to design these components, who are these multiplication component, this is what our final goal would be.

From here also, we want to design this multiplicative components such that my addition of these signals are optimal because I would like to have a beams at transmitter side as well as. So, this is our final goal, how to design this component, how to design these components, but it still a single antenna system because this is one data stream, and I am receiving one data stream.

So, I would call it; it is still a single input single output system from a signal processing point of view, but internally, it will have a multiple antenna so, it is actually a multi-antenna system, but not definitely a MIMO ok. So, now, what were modeling it?

We are modeling the channel between this part first because we are first interested to model the channel this pattern and we have explained it what happens when I put the gains in the left side, when I put the gains in the right side, we have not designed it, but we have given a small time, small kind of overview that what should be our goal, we will talk about that, but our current focus is to design this part, this channel.

Now, as you know understand, this is nothing, but a simple MIMO system. In the traditional MIMO system, if you do not consider anything beyond anything before this transmit antenna, anything beyond the receiver antenna, it is nothing like a it is nothing, but a MIMO system kind of things, but if it is a MIMO system so, you know we know how to model it, it is like a channel matrix will come, if there are N_t matrixes, in N_t antenna here, if there are N_r antenna at the receiver so, this channel matrix would be some N_t cross N_r dimension matrix. So, this is this we know, we know the end goal.

But the point that we are trying to model here is that each and every component of that matrix which is N_t cross N_r antenna. From earlier discussion, it is nothing, but some random numbers, it can be from a Rayleigh distribution or from complex Gaussian distribution, some random numbers it can be, but we try to; we try to approximate this channel because that motivates the beam forming that is actually creates the beam forming activity much simpler.

Had not been the case, then the beam forming itself would be very complex job ok. And this aspect will also be covered what if I remove all these assumptions, what happens to my beam forming case, what happens to the millimeter wave communication and that will be covered at one of the lectures towards the end where we will talk about the impairments and all these things ok. So, this is our end goal.

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The screenshot shows a presentation slide with the following content:

Approximations in AoA

- The angle of arrival (AoA) at each Rx-antennas is almost same provided the source point is same i.e single Tx without any reflection, scattering.
- Assume this to be θ .

The slide contains two diagrams illustrating ray incidence on four antennas. The top diagram, labeled 'Actual incidence of rays on Antennas', shows four rays originating from a single source point and hitting four antennas at different distances and angles. The bottom diagram, labeled 'Approximate incidence of rays on Antennas', shows the same four antennas but with rays that are parallel to each other, all hitting the antennas at the same angle θ .

Figure: Same Angle of Arrival (AoA) for all rays at Rx-antennas

The slide is part of a presentation titled 'Rx Beamforming in mmWave/THz' by Amit Kumar, DuttA, dated May 11, 2020, slide 10/16. A video feed of the presenter is visible in the bottom right corner.

So, now, we continue that. So, this is where we stop last times that we would like to have a approximation at the angle of arrival.

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Channel Angle ϕ_i Determination

- Let us now approximate the channel angle ϕ_i i.e. $h_i = \alpha e^{-j\phi_i}$.
- Let us assume $D_i - D_{i+1} = \Delta_i$ for $i = 1, 2, \dots, N-1$.
- In mmWave/THz, this Δ_i is very small as $D_i \approx D_{i+1}$, but $D_i \neq D_{i+1}$.
- This Δ_i will not have impact on α , but it will have impact on ϕ_i .
- We can write any $D_i = \bar{D} + \Delta_{i-1}$, where \bar{D} is the common part among D_i .
- Hence, we can write

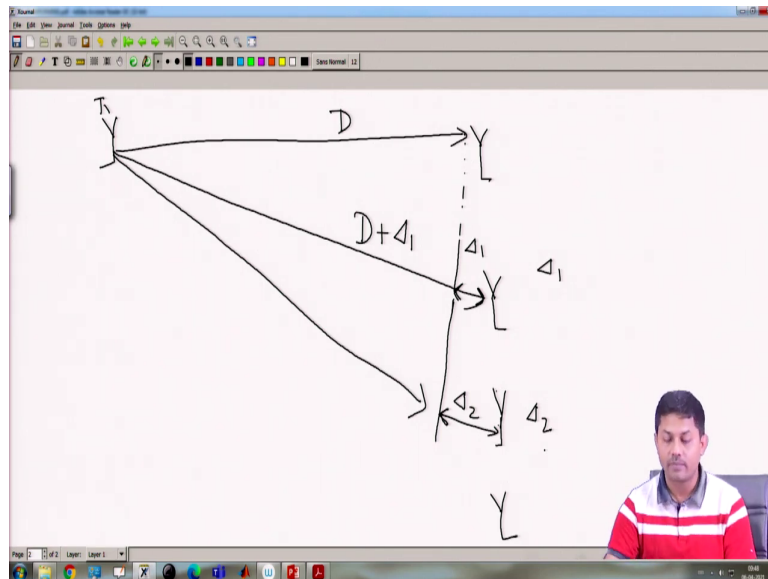
$$\begin{aligned}\phi_i &= 2\pi \frac{\bar{D} + \Delta_{i-1}}{\lambda} \\ &= \phi + \Delta_{\phi_i},\end{aligned}\quad (3)$$

where ϕ is the common phase part of channel TAPs and $\Delta_{\phi_i} = 2\pi \frac{\Delta_{i-1}}{\lambda}$.

Amit Kumar Dutta | Re-Beamforming in mmWave/THz | May 11, 2020 | 11 / 16

And then, we said that if this is an angle of arrival approximation so, what we said earlier that we would like to have the delta part of my delay element here right. So, in this case, what we want is that the phase difference from path to path. For example, if I go here, if you take the let me go to the diagram here.

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So, what I mean to say here so, this is my receive antenna, this is my receive antenna and this is say some other receive antenna, I am at the rx side. So, the distance I am only assuming here currently that it is sent from one antenna so, this is going here, see if this length is D , this length will be D plus as we said some notation I have used here, some Δ_i we have used that notation.

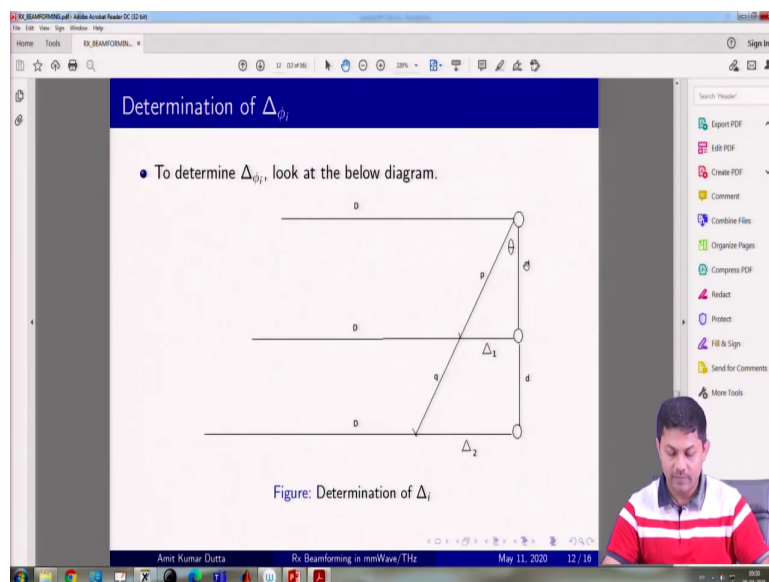
So, let me just. So, this would be Δ_1 I can say. So, that mean this is my D , this extra part is only in the distance and that distance will be small enough to ignore for the gain, but not for the phase, this is what we have discussed. So, this will be, this extra part I am talking, this extra part will be Δ_1 .

Similarly, when I go to the third antenna, this should be the D , same D plus an extra addition right that extra part which will be coming as that. So, let us call that Δ_2 and so and so

forth ok; and so and so forth. So, that mean antenna to antenna, the only thing I am incurring is that there is a small distance increment.

So, here, the distance from the transmitter antenna to the receiver antenna is increasing a small one. So, this is 1 so, this would be delta 2 and so and so forth ok this is what we have seen and then, we have also explained that what happens of arrival? Approximation is made same so, that means, we are assuming that the rays are entering parallelly.

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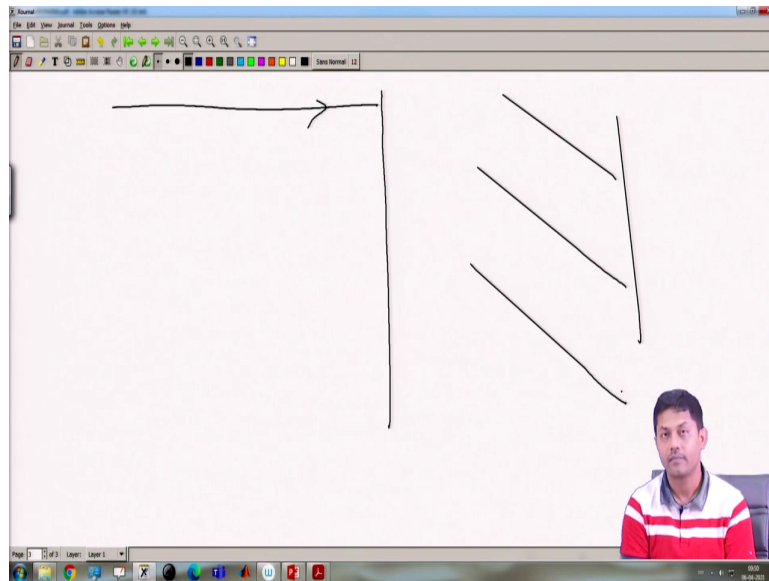
So, what is the consequence of such kind of assumption? The consequence of such kind of assumption is that the delta 1 and delta 2 will have an interesting property when I say that angle of arrival is same.

So, if the angle of arrival is same, what does it mean that each and every antenna if you look at the first antenna; if you look at the first antenna, this antenna, then if you look at the second antenna, if you look at the third antenna so, every antenna will be receiving the data parallelly that is the assumption of making angle of arrival same right.

So, it is as if like all the antennas at the receiver is receiving the data parallelly instead of a in different inclination. So, that mean, for everybody the angle of arrival in the elevation as well as from the azimuth both I am talking, they all be same, that is a very crucial assumption. So, what is the consequence? Like as if like the rays are coming parallelly ok.

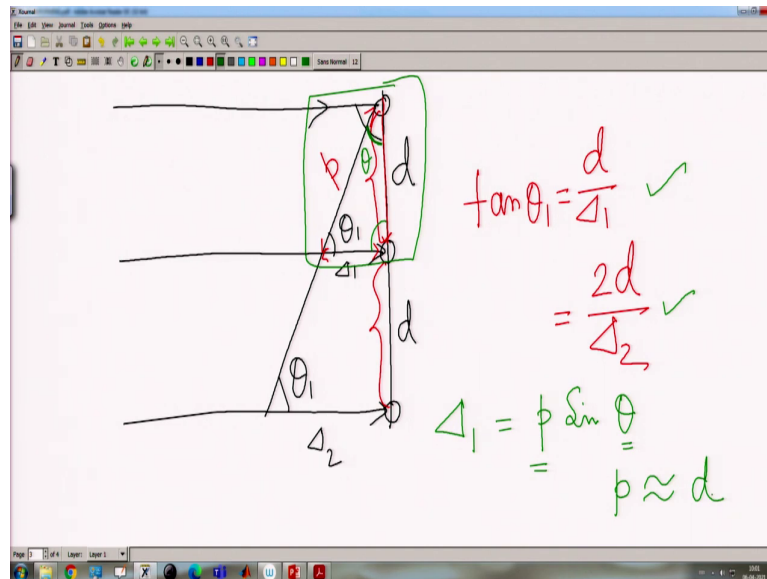
Now, if I make that assumption, what next ok? So, this is a consequence that rays become parallel. So, now, consequence is that as if like if you look at this particular diagram whatever has been you know shown here so, let me just if you look at it means that it is something like a kind of a triangle ok, very nice shape triangle and this particular theta whatever has been shown there, it remains same for all antenna because why? Because rays are coming parallelly ok.

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So, which means that which means the angle which means if I now come back to my diagram part which means it is something like this kind of things coming, as if like I am just drawing little bit horizontally because just for my ease of diagram, but it can be even a inclined.

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So, for example, it can be like this parallelly ok, just for my easiness I am drawing it slightly parallel just all are there so, this is my first antenna, this is my second, this is my third antenna right and so and so forth ok, and their inclination like this angle though I have drawn like a right angle, but it is not exactly 90 degree, it can be inclined just for my easiness.

So, the what is the consequence of it if it is parallel? Parallel is that this angle would be same for all of them right. See, if I call it say theta 1, this will also be theta 1. So, that is the consequence of making angle of arrival same because all are coming parallelly, this is the first assumption. Now, this one delta 1, this one delta 2, this is the; this is natural distance, but I have not made my fourth assumption.

Now, what is my fourth assumption ok? The fourth assumption is the following; if the distance between the antennas were assumed to be d, this is d, I mean these distance I am

talking, these distance d and d right. So, what will be the case here? What is my \tan with respect to the θ_1 here $\tan \theta_1$? What is my $\tan \theta_1$ here? $\tan \theta_1$ will be d by Δ_1 right. What is my $\tan \theta_2$ in the second case θ_1 ? It will be $2d$ into Δ_2 , this what is coming into picture here ok.

Now, think again from a dimension point of view, will the Δ_1 , this Δ_1 be very large? It will never be ok, it will never be so large because I really do not know what is this Δ_1 because this is one of the point that I am trying I am struggling to find out because unless I know Δ_1 , I will never know what is the phase difference from antenna to antenna because the Δ_1 is very important for me. So, this is what we are trying to find out here.

If you look at these equations trying to find out here that this particular one equation number 3. So, what we are trying to find out? Basically, we are trying to find out the phase of phase difference due to that particular extra Δ that we are trying to find out. So, what will be the case? It will be like if I say at the i th antenna, the ray that is entering it will incur a extra phase, what is that extra phase, what does it depend on?

That extra phase depends on this see Δ_i , this particular one, Δ_i ok because D is assumed to be equal for all of them, the extra Δ_i is getting incurred. So, this $\Delta \phi$ is what matters to me, that extra phase due to the path extra path that we are trying to understand here ok. So, this is making my life slightly easier if I make another good assumption and what is that particular assumption ok. So, that assumption is the following ok.

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Determination of $\Delta\phi_i$ (Contd...)

- As per diagram, we can approximate the AoA to be same for all incident rays. Assume it to be θ .
- $\Delta_1 = p \sin(\theta) \approx d \sin(\theta)$.
- $\Delta_2 = (p + q) \sin(\theta) \approx 2d \sin(\theta)$.
- Hence, as generalization, $\Delta_i \approx id \sin(\theta)$.
- Hence, $\Delta\phi_i = 2\pi \frac{(i-1)d \sin(\theta)}{\lambda}$ for $i = 1, 2, \dots, N$.
- The channel vector at Rx-antennas i.e. \mathbf{h} can be written as
$$\mathbf{h} = \alpha e^{j\phi} [1 e^{j\Delta\phi_1} \dots e^{j\Delta\phi_N}]^T \quad (4)$$
- Assume $\alpha^0 \triangleq \alpha e^{j\phi}$ for notational simplicity.

Amit Kumar Dutta Rx Beamforming in mmWave/THz May 11, 2020 13 / 16

So, what is that particular assumption? So, let us understand that ok. Now, let us go to the diagram again and try to see what is my assumption here. So, this is my tan theta right ok. So, what I am trying to say here? This is my tan theta here and also there is one more point that I trying to make here is that what exactly my delta 1 will be dependent on the angle that I am trying to consider here right so, that can be d of sin theta.

If I say this is my d ok, this is this angle and I do not know that angle right. So, this is what one of the point that I am trying to make here. Now, what if let us say this distance is p ok, this distance is first decision let us take; let us take a taking to consideration only the first triangle here, the first triangle here ok.

Now, tan theta I know here, some way it does not help me much because there is a theta 1 and I do not know the theta 1 because p itself is not known to me. So, I make another way of

proceeding. So, this angle I am talking, this theta angle where that inclination is coming into picture there.

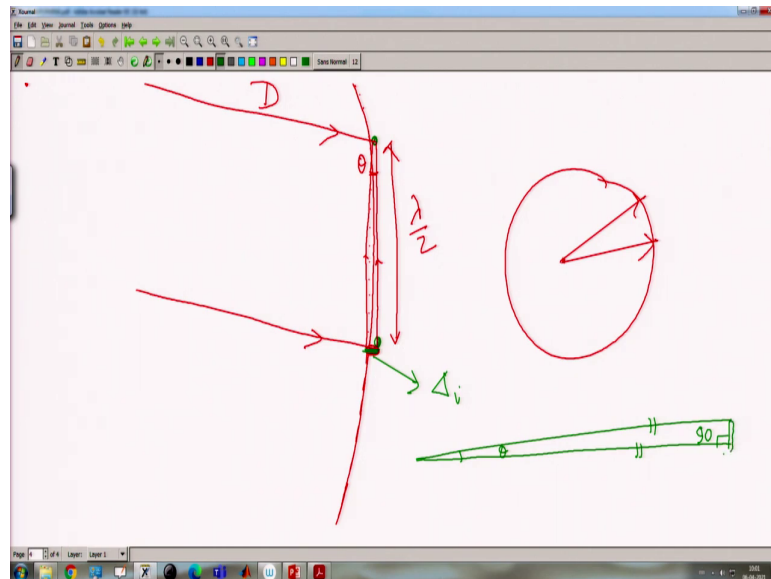
So, if this angle is theta so, what assumption I can make? I can say this particular delta 1 will be $p \sin \theta$ right, this is what I am what is my ultimate goal? Ultimate goal is to find out this delta i that is my fundamental goal because that gives me the phi extra delta phi.

So, to do that, what I say? I started with theta 1, then I said ok theta 1 is also not very known to me because that is also function of p, delta and all these things. So, what if I take the other angle which is basically the inclination angle, but of course, where my d is equal assume to be equal from that point of view. So, basically, I am talking of this particular here so, this angle I am talking of ok.

So, this angle is easy to find out because that is mode of a if I know the inclination and if I know one of the distance, I know the second point so, I can find out what the theta would be so, that is not an issue, but the point here is that from there, what I know is that I am still not clear because there is a p there.

So, this delta 1 if you look at this particular triangle, this delta 1 is nothing, but $p \sin \theta$ not theta 1 it should be $\sin \theta$, but still I am not done because I still do not know p and this is where my fourth approximation. If this theta is small, why should I make that assumption? Let us assume this theta is small ok, why this assumption is? Look at this.

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Suppose this is my antenna point, this is my antenna number, let us take a very simple two case, first antenna second antenna ok. Let us say simple general case my ray of inclination is this because of angle of arrival, I have assumed that ok. Now, somewhere virtually in the left side somewhere your single point is there, from there I am say drawing one circle because it is like a only when it is a circle, everywhere in the circle the distance is same right.

So, it is as if like this parallel two lines are coming from exactly some central circle. So, if I draw one circle here, touching the first one and touching the first one, but you notice I cannot touch the second one, this circle and the radius of the circle is D . So, this distance, this particular tiny distance, will it be very large? It will be very large right, why it will not be large? Because the radius, this D is very very large compared to the distance between this two antenna.

This antenna is how much distance? $\lambda/2$, it is in the order of millimeter. If it is in the order of millimeter and this D I am talking almost in the range of meter say 50 meters, 10 meters and it is a millimeter so, the radius if I touch the first one and create a radius it create a circle touching the first antenna and then, I am crossing second, third and fourth, the only distance I will see this small tiny distance, it is extremely small tiny distance.

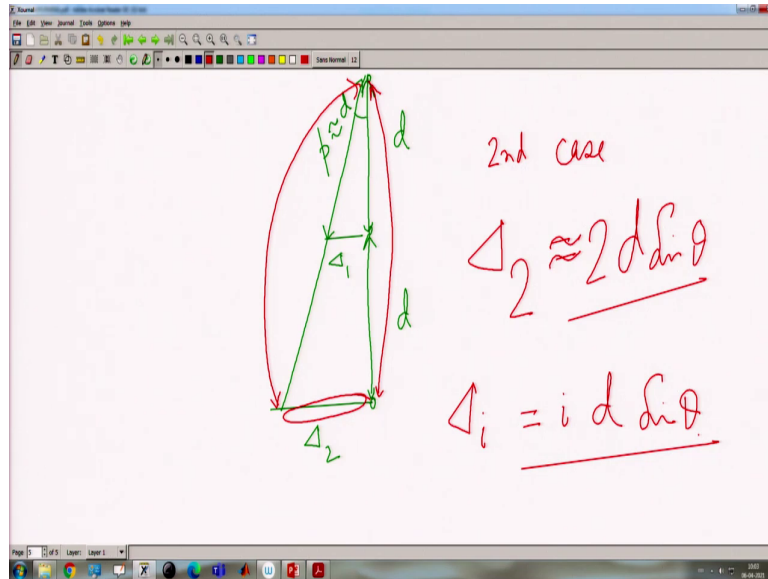
Now, I am talking of the angle where this is this angle I am talking, though I am not able to it is so small in fact, I am not able to draw it in a very proper manner, it is so small, I am talking of this angle, this angle I am talking, will that be very large? It will never be large because this D is large, this $\lambda/2$ is extremely small, this angle I am talking of as a θ ok so, I am talking of θ and that is not a easy I mean that is not so difficult one you know find out. So, that angle will be extremely small ok.

So, what does it depends on? That depends on all these distances, this distances and all these things. So, this distance, this particular small one whichever I have drawn and that is just very small distance because this angle θ will also be small because I am drawing a large radius circle. If I touch the first one, then the second one will not be touched only a small distance Δl , this is what my Δl right, this is finally, my Δl , this is my Δl . So, Δl will be very small.

If the Δl is very small, this angle of θ will also be small. So, which means that if I go back to my earlier diagram, if this Δl actually now, I have enlarged this diagram, put it like in microscope that this Δl is equal to $p \sin \theta$ so, where p is basically the if I think this is a right angle, this p I know what how to find out this p . Now, this Δl equal to $p \sin \theta$, but because this θ is extremely small ok so, I make all the second assumptions that if this p is small, what is an eligible assumption that I can make is that what if this p is almost equal to d , this is my fourth assumption. So, it is as if like I have a angle, let this be a right angle, this is θ I am talking ok, if this θ is very small, this length and this length will also be very equal almost because this is; this itself is very small right.

So, this is a fourth assumption which is very crucial for us ok. So, which means now I can make a interesting point here. So, which means that this p is nothing, but my d, this is my fourth assumption.

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What happens to the second case that mean when I go to the next one, go to the first with respect to the first and the third antenna which means if I go to a cleaner diagram, this is the one, this is the one, this is the one and this assumptions are very important to know. So, this is my d, this is my d so, this is my connecting point, this is another antenna connecting point ok, I am making slightly larger for your easy understanding.

So, what have we said? This is delta 1; this is delta 1 and this is delta 2 ok. Now, this p is equal to d that means, this length, fourth assumption. Naturally, what happens to the second

case when I go to the second antenna? If this theta is small obviously, this distance and this distance will also be same, almost approximately.

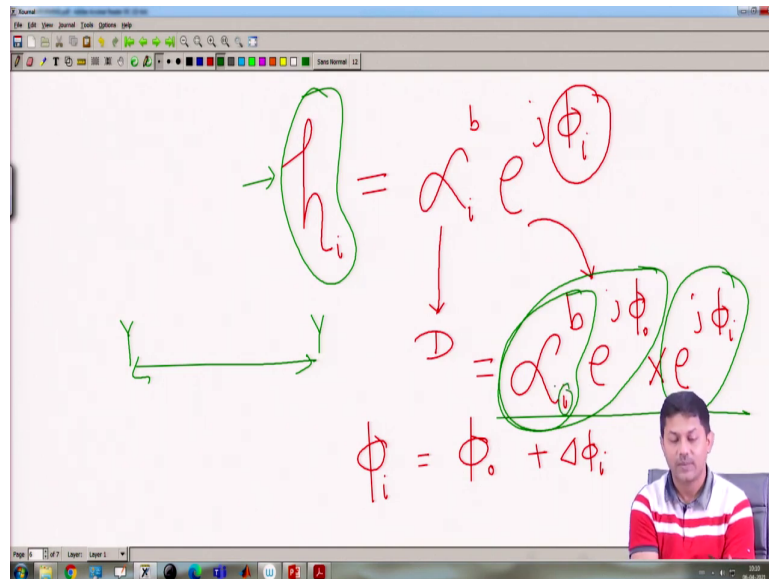
So, which means that for the 2nd case, this delta 2 will also be what? This whole, this first red line and the second red line are also equal so, that mean it will also be almost approximately I would say $2d \sin \theta$ ok. Can I generalize this? Yes, you can generalize it, you can say $\Delta_i = d \sin \theta$, this is a interesting approximation ok.

And now, let us see what is the consequences, what are the consequences of such kind of approximation ok? Let us go to that diagram again, let us see that here ok. So, if you look at the second line here so, the first line is fine, the second one is the extra $\Delta_1 = d \sin \theta$. Second; third line, third bullet would be $\Delta_2 = 2d \sin \theta$ so, I can generalize it hence, I can generalize $d \sin \theta$ ok.

So, I know delta. See if I now, if I know delta, I know ϕ_i , extra $\Delta \phi_i$ and that is precisely my goal is so, that is what I have written here. So, if I know Δ_i so, the next one which is this part, $\Delta \phi_i$, this $\Delta \phi_i$ I am talking, this $\Delta \phi_i$, it will also be known to us because that is nothing, but $2\pi \frac{\Delta_i}{\lambda} = 2\pi \frac{d \sin \theta}{\lambda}$ that is my final goal here ok. So, that mean now, I know what is my phase, the extra phase, the extra you know the delta phase.

So, I know total distance, I am assuming that all the distances are same, then for the phase purpose that extra delta I have determined it making another approximation ok. If I do; if I do know all these parameters, what will be my equivalent channel now? This is what we are struggling to find out right, what will be my equivalent channel ok. So, let us see that what is my equivalent channel. So, now, the 4th equation would represent my equivalent channel. Let us explain from a diagram point of view.

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If we go back to our, if we go back to some of the earlier classes, what was the h that we have assumed here? h will be some α obviously, that α was coming from that baseband gain that we have talked about ok and then, what will be the case? Then, all your this is the angle part and then, there will be phase part. If it is a i th one, this probably i th one, this i th one, no confusion here because this is my channel of course, that is itself is my channel. So, it will be gain into a some phase.

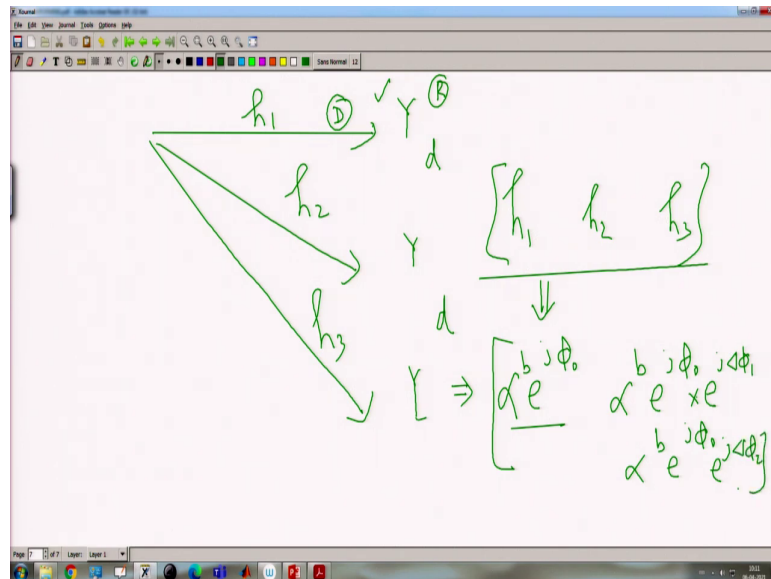
This we know from distance D that is easy to find out, this is what we have also tentatively found, how? You know this ϕ , but we are struggling to find out, what is the phase that will be coming into my extra data. Now, this ϕ will have one common ϕ because of the common D plus the $\Delta\phi_i$, Δ of ϕ_i right, this is final conclusion.

So, if I put it, what will I get? Some $\alpha e^{j\phi}$ and then, $e^{j\phi}$ you can call it 0 multiplied by $e^{j\phi}$, this is what is coming, this precisely what is coming here ok. This is what my final channel, one part of my channel that mean one antenna to one antenna. If it is one antenna to one antenna, this is the point I am talking just that.

So, earlier, when you studied the 6 gigahertz, we just said this is one complex number and how is what how was it coming? With some you know interpolations and all these things. Here also the same thing is true here, but again, we are making some assumption and make our you know life slightly simpler. So, we are saying that the same thing holds here, but we make some assumption or some approximation.

Now, instead of making it just like a one complex number now, I am splitting into a gain as well as in the phase. Earlier, it was not so important for us, we just know it is a complex number, here also it is a complex number, but we are splitting the gain part and we are splitting the phase part separately ok because we are, because we will show in a subsequent classes what is the advantage of such kind of assumptions. So, this extra part will be added up.

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Now, if somebody ask you, I have two antenna or three antenna, who are you know equally $\lambda/2$ space or d space apart so, this is my h_1 , this will be my h_2 , this will be my h_3 and this is my point of reference, then from there I am mix, I am making rest of the assumptions right.

So, this is my D distance, no reflectors in this case, plain vanilla you know no reflector, no scatterer, just I am assuming plane you can see think of its a free space, we will subsequently introduce what if there is a reflector and scatterer.

So, now, the channel was it was h_1, h_2, h_3 , this is just like a vector right, we have seen it in our earlier classes. Now, we are saying that hey, it is also a vector, but because I know some of the components of h_1, h_2 and h_3 , take out the common parts. Now, this part you can think of, it is common for all of them and in fact, this is equal for all of them, this is the gain

part right and the gain part is same for all of them why? Because the distance is same that is our key assumptions. So, this i will no longer exist in that case ok.

So, this, this is what we have seen it right, the gain is same for all of them only the phase. So, that mean from here, I am saying this will be $\alpha b e$ to the power $j \phi_0$ common part into e to the power you know $j \Delta \phi$ now, $\Delta \phi_0$ will not exist for the first one because that is the reference point so, this will be more of a.

So, this is what it is because this is the reference point from here with respect to that I am measuring. So, second one will be $\alpha b e$ to the power $j \phi_0$, but now, for this, this one will be coming and the third one $\alpha b e$ to the power $j \phi_0 e$ to the power $j \Delta \phi_2$.

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$$\Rightarrow \alpha e^{j\phi_0} \left[1 \quad e^{j\Delta\phi_1} \quad e^{j\Delta\phi_2} \quad \dots \quad e^{j\Delta\phi_N} \right]$$

$$\Rightarrow \alpha \left[1 \quad e^{j\frac{2\pi d}{\lambda}} \quad e^{j\frac{2\pi d}{\lambda} 2} \quad \dots \quad e^{j\frac{2\pi d}{\lambda} (N-1)} \right]$$

R_x - ARRAY MANY FOLD VECTOR

See, if I take the common part so, it is a channel vector, this will be $\alpha e^{j\phi_0}$ $e^{j\phi_1}$ $e^{j\phi_2}$, you generalize it $e^{j\phi_1 - 1}$ something like this. So, this is now your channel, same channel if you look at earlier also, instead of three complex number as a general three complex number now, here also I am making three or four whatever the number of antennas are same thing.

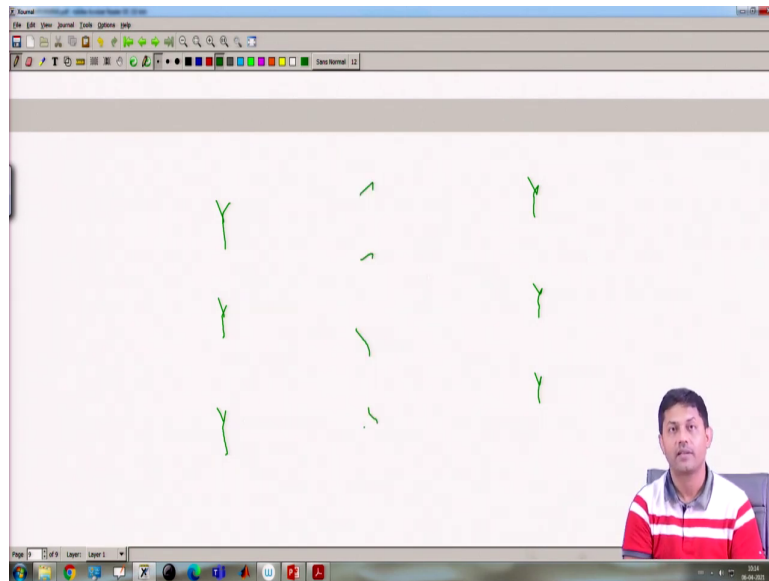
But now, I am interested move here now, this is like I can box it ok. So, let us call it α_0 , I will just box it because it is common for all of them, and this will be the case. Now, if I know the their equation also so, we can even put the exact value $2\pi d$ by $\lambda e^{j2\pi d}$ and so and so forth.

Now, this d is known to you, this λ is known to you so, this is my new channel model. You see whatever we have taught in 6 gigahertz, there is not much difference, only thing the some approximation lead to some better form of my channel so, this is my channel vector.

So, that mean if I look at; if I look at my channel here, it is as if like I am looking at simo case, something like that. So, earlier it was like just like a vector, here also it is a vector, but with this basic approximation and this basics structure. Now, this particular vector, this extra vector is called array manifold vector.

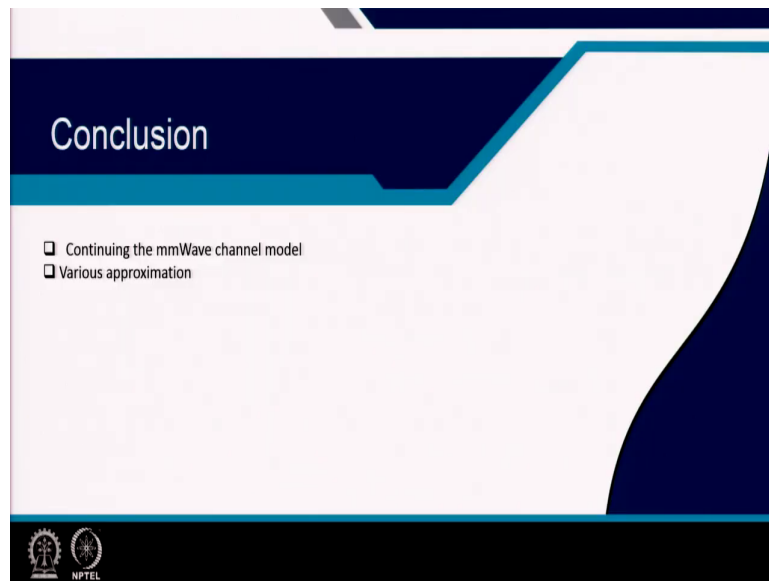
The term comes from array signal processing, array manifold vector, but it is at the receiver side so, this is the receiver array manifold vector right. So, this is how the now, you see now, the channel is to me it is dependent on the antenna distance because of that phase difference ok. So, this is my new channel model from one.

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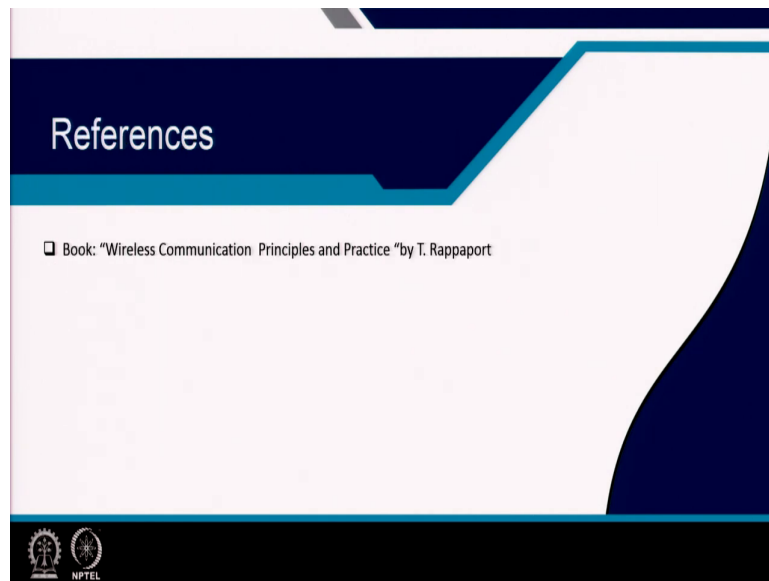
Now, here, it is only from one antenna to multiple antenna, our ultimate goal is to find out what happens when I introduce multiple antenna here? What happens I introduce other reflectors and so on here? So, that we will be talking in the subsequent classes. So, this is the basic channel model approximation when I go from milli; when I go from sub 6 gigahertz to millimeter wave and this based on that, we will build the next step. So, with this, I conclude the session of channel model approximation for millimeter wave.

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So, if I go here, if I go back to my conclusion part so, the conclusion wise I say that we have just covered the various approximation and this is good enough for all my approximation that is one more level of approximation because it is all about approx unless you heard, the various approximation and this is good enough for all my approximation that is one more level of approximation because it is all about approx unless you do approximation, this channel model will be very ; difficult to be realized for beam forming. So, we continue the channel model, millimeter wave channel model.

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So, this is one of the book, but this particular one whichever I am following you just follow the class note that I am trying to do, but this book is more of a guideline.

Thank you.