

**Signal Processing for mmWave Communication for 5G and Beyond**  
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**Module - 05**  
**MmWave channel model**  
**Lecture - 28**  
**MmWave channel model with Rx beamforming(continuation)**

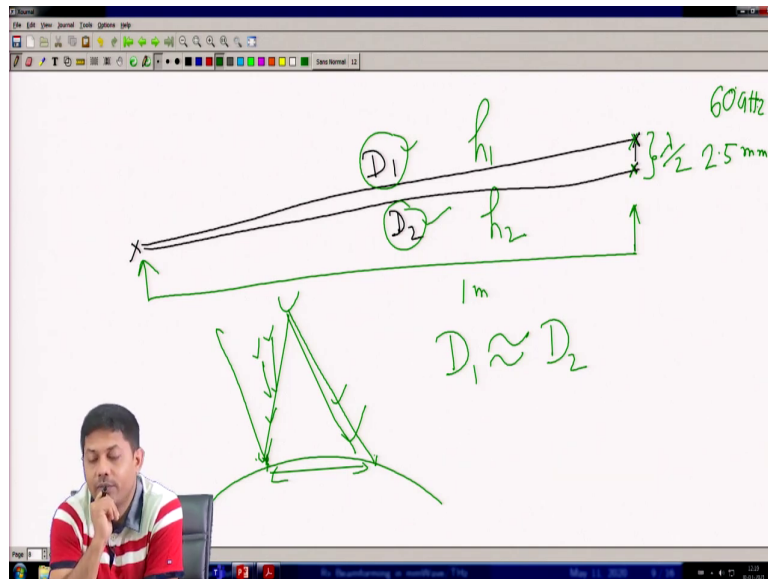
Welcome, welcome to Single Processing for millimeter Wave Communications for 5G and Beyond. So, we will be continuing that Millimeter Wave channel model with Rx beam forming first and that is the continuation.

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And we will be covering some of the approximations involved that we have already started in the last class and we will continue for the characterization with the multiple channel.

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So, this is where we left it if you look at the diagrams so, this is where we left it. So, we said that if the if we are in a far field and in the distance between the antennas are very small which is the case for millimeter wave we can make such assumptions.

Now is this assumption true for even subset guards, of course true, because I have not said that this is only applicable for millimeter wave. The only thing that has been said is that the dist the dimension of the distance and the order of the distance and the distance between the antennas what should be the what is the relationship.

If you look at this is 2.5 millimeter per sixty gigahertz and I am assuming 1 meter if it is a 6 gigahertz if it is a 6 gigahertz what would have happened. This probably would have been even larger and then then I have do consider even a larger distance that is all. Suppose I assume a 1 kilometer distance or 2 kilometer distance in that case and then even if the

antenna separation is say 1 feet or 10 meter not 10 meter 10 centimeter probably I would have taken the same approximation ok.

So, this is general approximation. So, its the order of the distance and the order of the antenna spacing ok. So, now, let us get into the ok.

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Approximation to Channel Model (mmWave/THz aspect)

- So far, I have not written anything new. Just simple equalizer design.
- But, now think of mmWave/THz, where  $\lambda$  is in the range of millimeter or smaller than this. Whole story on channel will be different now.
- We can come up with a simple approximation of channel.
- Remember Rx-antenna spacing of consecutive ones is  $d > \lambda/2$
- Hence,  $D_i \gg \lambda$ .
- Now, each  $D_i$  will be almost same with respect to Rx-antennas. i.e.  $D_1 \approx D_2 \approx D_3 \dots \approx D_N$  if  $\lambda$  is very small compared to each  $D_i$ .

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So, this was the last assumption that all the distances have to be almost equal ok. So, now, each  $D_i$  whatever the distance is will be almost same with respect to Rx antennas that mean  $D_1$  almost equal to  $D_2$  and so on  $D_n$  if  $\lambda$  is very small with respect to each  $D_i$ . So, that we have explained it.

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Approximation to Channel Model (mmWave/THz aspect)

- Don't assume that each  $h_i$  is almost equal. This can't be true.
- Then what will be almost equal ?
- The magnitude gain of each  $h_i$  is almost same as Friis equation is dependent on distance.
- Each  $h_i$  can be expressed as  $h_i = \alpha e^{j\phi_i}$ , where  $\alpha$  is almost same for all paths.
- $\phi_i$  is the phase corresponding to the  $i^{\text{th}}$  path channel TAP.

Another important approximation happens. Look at the next slide.

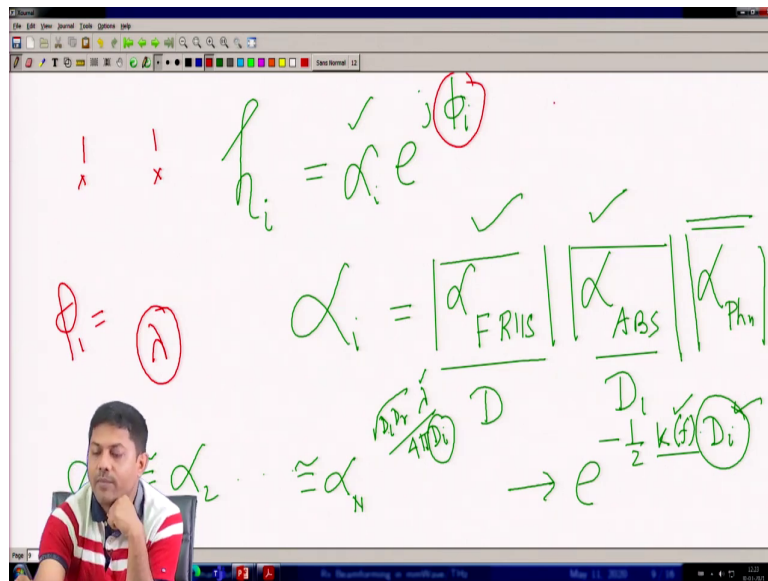
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Let us see ok, let us see the what are the other assumptions. So, distance is same, does it mean my does it mean that  $h_i$  is same? I am talking of the channel because distance is one contributor for the channel, what does it contribute to distance because based on distance you have path loss based on distance you have a phase all these things will come into picture.

So, when I say my distance is equal does it mean that the channel itself will be equal ok let us understand that ok. So, in the channel  $h_i$  the channel gain that may not be equal so and why that is different when my distance is still same. So, can I create some sort of a what exactly creates the inequality ok, that we need to need to understand that. Now from a channel point of view what are the components that is I mean at ADC motor that mean I am completely a single tap channel.

So, at I can think of this just like a complex number. So, this is say D 1 D 2 I will say my channel is say h 1, h 2, distance is almost equal, and let us see the channel part. Now when it is a h 1 and h 2 it will be complex number because I am I probably viewing it from the ADC point of view after the ADC from the digital point of view.

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So, my h 1 and h 2 is a complex number. So, that mean each h i, I would say if it is a complex number what is what does it mean, some gain will be there and some phase term will be there, I can think of alright that is the quite general assumption here. Alpha i e to the power j phi i for every h i. What is the alpha i?

Alpha is your the actual path loss gain or absorption gain everything is there now I am not making any distinct discrimination that it should be only path loss it can have everything because we are dealing with millimeter wave everything. So, alpha i is everything it is a gain

coming from you know Friis that is the P of path loss it is a gain coming from absorptions, its a gain coming from different phenomena like reflected everything is there.

Now, in this particular case we are not assuming any phenomena right like we are assuming no reflector and scatterer. So, probably this may not appear, but at least these two will appear in a millimeter wave context there is no difference ok.

So, this alpha will have that ok. Now here the question comes, will the alpha i be different for different path, because if you look at Friis what does it depends on. It depends on distance D, what does the absorption depend on? Of course once in the frequency component at the same time it will also depends on i.

So, if you look at alpha it was e to the power minus half into k into f into r order the D i will say in this case D i distances. So, k f is your absorption coefficient. So, it is dependent on D i and you know the Friis part ok. It will be root over of  $D_t D_r \lambda^2 / 4 \pi D^2$ , the D i is compulsory everywhere ok.

Now we have assumed what? D i is same almost same right every D i same. So, which means that apart from anything else that mean apart from D i the other components are assumed to be same that mean I am assuming a frequency I am talking of the same frequency here.

So, these are frequency dependence if I do not bring the frequency dependency there is a frequency of course, but if I do not consider my bandwidth to be very large I would say my that part is taken care.

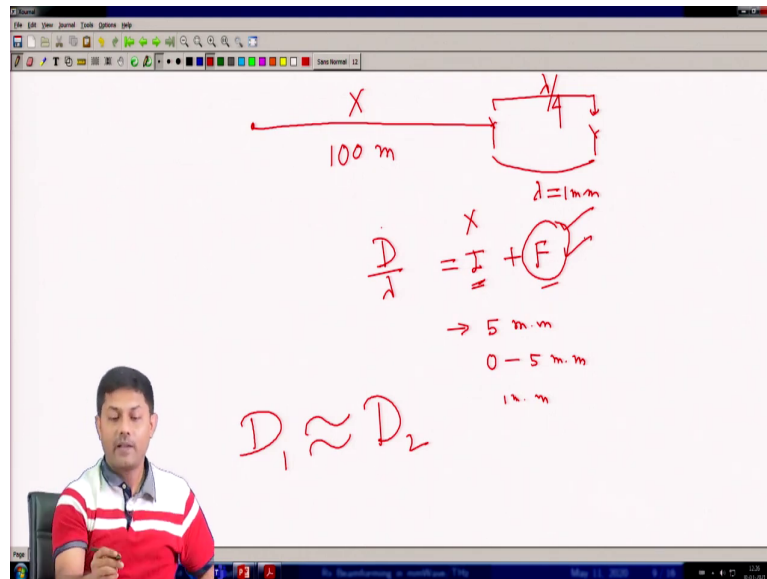
So, it is a distance that matters now if the distance is equal, this part is equal, this part is also equal, that is the consequence what does it mean? That mean because I have assumed the distance to be almost equal my  $\alpha_1 \alpha_2 \dots \alpha_N$  if there are N paths they will be equal almost I should not say they are equal there that is the an approximation careful.

So, which means that I am assume there the magnitude part of the channel is constant for all the paths and that is a very critical assumptions. So, this was not the case when you consider the 6 gigahertz channel model right, this is also 6 gig I mean you can equally apply the whole thing here as well. We are not deviating from what we have learnt, it is just an approximation. Now we are saying that hey we have a channel vector  $h_1$  to  $h_n$ , but the magnitude part of the channel is actually almost equal because the path is almost equal.

Now, that part is taken care what about the phase part, will the phase be same because you can always say hey what is phase. Phase is nothing, but what is phase. So, what does the phase depend on? Phase depends on your distance right phase depends on your distance. Now if the distance is equal will the on the phase be same natural questions right.

Now, here we are making a small distinction guessing the no, phase depends on your distance definitely, but it also depends on your lambda. So, that mean if the distance between two point is lambda apart I would not feel any phase difference, suppose I have a point here I have a another point suppose I am making a transmission like this.

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And there is a data here and there is another antenna sitting lambda apart, will I see a phase difference suppose this is 100 meter. But from here this is just lambda is equal to say 1 millimeter it is the timing will I see a phase difference, I not see a phase difference. Suppose this is lambda by 4 even time here will I see a phase difference, yes now I see a phase difference.

So, it is not the distance that matters; obviously, distance finally, comes into picture, but it is with respect to the lambda how much fraction see if I divide this  $D$  by you know lambda. So, there is an integer part and there is a fractional part. So, it is this fraction that determines how the phase is not this integer.

So, this  $D$  is actually material to some extent it is this fraction that matters and what is this fraction. If this lambda is say 5 millimeter that mean a distance between 0 to 5 millimeter will



create a phase difference, if the distance difference of some two objects is just say 1 millimeter. I will say phase difference, will I see alpha difference, the gain difference, no I will not see alpha difference because 1 millimeter is nothing almost right.

But this phase is sensitive to that fractional part not this integer part, that is the reason even if the D is same almost same I have not said equal it is a almost same. So, that tiny difference is good enough so; that means, this D 1 is almost equal to D 2 it is not equal, why it is not equal. It cannot be because there are these are two different physical point it need not be same it can be different, but this almost same is good enough for making the gain partition.

But this is not good enough to make the phase partition because phase depends on the D by lambdas fractional part not the integer part. So, which means even if the D is 10 kilometer and lambda is just a 5 millimeter, a 5 millimeter difference is good enough to create a phase change, but that would not be reflected in your alpha and that is the key part in this millimeter wave.

So, in a summary if I say this channel from 1 T x to every R x alpha is same for all of them which I comfortably can assume, but this phi will not be same for all of them, because this phi is hell and heaven difference. In fact, it can even be a negative phase difference based on the distance and that can be a very tiny difference and this is the key point that I am trying to say here ok.

So; that means, if you look at the 4th point here this this one 4th point here each h i; each h i can be expressed as  $h_i = \alpha \cdot P_j \cdot \phi$  where alpha is almost same for all. But this phi the next point is basically the phase and that is basically your different.

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### Approximations in AoA

- The angle of arrival (AoA) at each Rx-antennas is almost same provided the source point is same i.e single Tx without any reflection, scattering.
- Assume this to be  $\theta$ .

Figure: Same Angle of Arrival (AoA) for all rays at Rx-antennas

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But and if you look at this diagram which will make the point clear ok. So, before I go to the next diagram. So, this is the key assumption here. So, this  $\phi$  is basically your phase differentiation ok. So, what is the consequence of it? So, consequence is that I have taps single tap having the same magnitude, but different phase that is all that is the consequence correct.

Now, because if  $h_i$  is equal to  $\alpha e^{j\phi}$  this one  $h_i$  is equal to  $\alpha$  I am not putting  $\alpha$  here because  $\alpha$  is now same for all of them almost the tiny difference is not an issue, but this phase will be different for all of them ok. So, this is a very very key difference this is a good approximation ok. So, this is my first good approximation or the consequence of assuming my distances being same ok. So, what is the next approximation?

What is the next approximation that comes into our picture, ok. Next approximation that comes to our picture is the angle of arrival ok. So, what was the angle of arrival, angle of

arrival is nothing, but at which angle the ray is incident on a particular antenna. And what are the two components for an angle of arrival, it will have two components of elevation and azimuth both will be there ok. So, that is the point here.

Now, for our simplicity we are just taking one angle for our case because this is kind of a you know uniform linear array. So, one angle is good enough, but if you take UPA probably elevation and azimuth both will come into picture. So, here we assume that my probably my azimuth angle is same for all of them. So, you know that is what happens. So, when you have a ULA the way the beam is gone is that you can play with the elevation angle not with the azimuth angle.

So, the steering that I was talking of you can play with only the azimuth angle elevation angle. So, if it has a by default azimuth angle you have to live with it, if it has a elevation angle you can play with the elevation angle provided the antennas are in ULA position, but if it is an UPA position you can play with elevation angle and azimuth angle, we will explain that later because this is not part of that context, but I am just telling you that this is what is going to happen.

So, in I am in a ULA configuration meaning my antennas are in a z direction single simple z directions x y z if you have you can just think of say x y and z and you are just thinking of z direction.

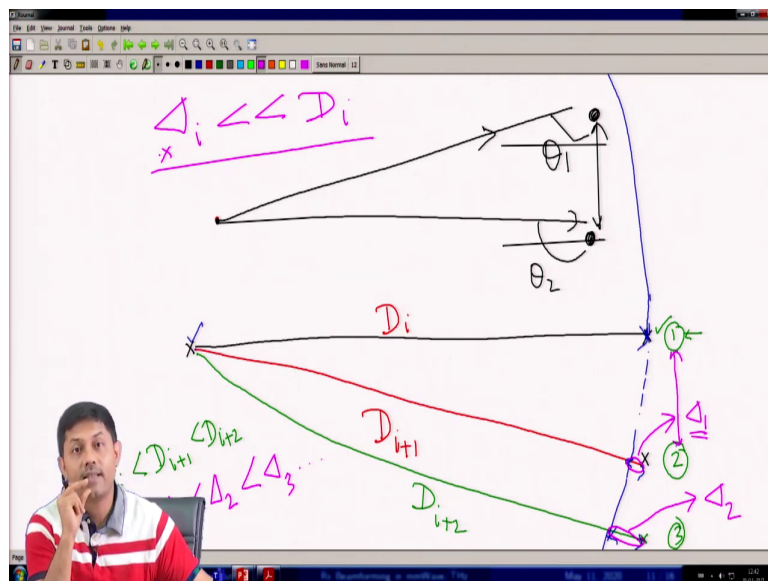
So, you can say that the only steering you can have you can do is in the elevation part that mean only one direction you can shift, but not in the azimuth part. So, let us assume that part and then we will extend it when you go for UPA you will see that you can this angle of arrival will have new angle arrival will have both the component elevation and azimuth.

But here I am putting elevation because that is the only component I can change it azimuth is fixed for us. So, now, what happens to my angle of arrival ok, this is another approximation. So, first approximation distance and we have seen its consequence. What is the second

approximation? Second approximation look at the first diagram here, the first one, first one, first one here so, these are the two antennas received antennas.

So, if they are slightly spaced apart no matter how small; obviously, the ray that will be falling on them will also have two different angle of falling because the source is a point source right. So, if you think of from here.

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So, this is a point source I am assuming right probably this point source. So, the rays are like going all direction because this is a isotropic ray I am just assuming that. So, even if you have one more antenna here and another antenna somewhere here. So, this will never have the same angle of arrival though that is a distance if there is a distance and this will have some distance right. Now this angle of arrival so, this is say this is what our point was theta 1 and this angle of arrival was say theta 2 with respect to horizontal ok..

So, now, go back to our diagram will this  $\theta_1$  and  $\theta_2$  be really different if the distance between  $T_x$  and  $R_x$  is assumed to be large enough compared to the distance between the two antenna ok. So, that is the point. So, that mean if I say I am in a far field what does it mean? My  $T_x$  to  $R_x$  antenna is in the order of say few meters compared to the distance between two antennas which is in the order of millimeter.

If that is what the dimension then it is as if like both the antennas will see that they are receiving the ray in a parallel fashion, just the way we see the sun if I see the sun the way I am seeing it 100 meter away, also another person will see the same way it will it is as if like both of us see the sun in a parallel manner.

I will not see any inclination difference or angle of arrival differences, because the distance between sun and earth is so large that difference so, but if somebody is standing on earth and somebody standing on say Jupiter yes, the angle of arrival will be different because the distance between a Jupiter and earth is also of the same order, but on earth surface just 100 meter away, I will not see that angular of arrival difference. So, this is precisely the another approximation of course, it comes from the same approximation of far field the distances are almost same.

So, here also in the second angle of second approximation of millimeter wave which is the angle of arrival estimation that each and every antenna is receiving the rays in a parallel manner that mean their angle of arrival or angle of you know actual incidence of arrival is same for all instead of  $\theta_1$  and  $\theta_2$  now it becomes  $\theta$  for all of them ok. So, this is another approximation ok.

So, let us see what are the other. So, what is the what are the two approximation that we have learnt, one approximation is that every channel tap has equal gain because the distance is out to be almost same, but the phase is different.

Then each and every antenna will see the parallel rays coming as if it is not a two different antenna I will see two different you know angle of arrival because the distance is as again

assumed to be far large compared to their on to the spacing among the antennas ok. So, angular arrival remains again the same. So, these are the two assumptions so have learnt.

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### Channel Angle $\phi_i$ Determination

- Let us now approximate the channel angle  $\phi_i$  i.e  $h_i = \alpha e^{-j\phi_i}$ .
- Let us assume  $D_i - D_{i+1} = \Delta_i$  for  $i = 1, 2, \dots, N - 1$ .
- In mmWave/THz, this  $\Delta_i$  is very small as  $D_i \approx D_{i+1}$ , but  $D_i \neq D_{i+1}$ .
- This  $\Delta_i$  will not have impact on  $\alpha$ , but it will have impact on  $\phi_i$ .
- We can write any  $D_i = D + \Delta_{i-1}$ , where  $D$  is the common part among  $D_i$ .
- Hence, we can write

$$\begin{aligned} \phi_i &= 2\pi \frac{D + \Delta_{i-1}}{\lambda} \\ &= \phi + \Delta_{\phi_i}, \end{aligned} \quad (3)$$

where  $\phi$  is the common phase part of channel TAPs and  
 $\Delta_{\phi_i} = 2\pi \frac{\Delta_{i-1}}{\lambda}$ .

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Let us see, what is the consequences that it is various phi and all these things will have it ok. So, let us see that. So, what does it mean, distance is same phase is different, now if the phase is different what should I do, we should now say somewhere in our equation this difference of phase should be evaluated right, do you know phase will be different so, can I evaluate it, well.

So, for the evaluation purpose I bring the second point here, why the phase is different because  $D_i$  and  $D_{i+1}$  if the distance the  $i$ th path and  $i+1$  path distance are not same they are slightly different and that slightly difference itself is good enough for our phase. So, when I do a phase estimation that mean if I want to estimate how much my phase would be at

that time I would say these two distances are not equal so; that means, this is the second point comes into picture.

So, let us assume that the difference between two consecutive you know incoming rays that is the  $i$ th path and  $i + 1$  path is  $\Delta_i$ . So, this is what, this part the second part. So, let us assume that this  $D_i - D_{i+1}$  is  $\Delta_i$ ,  $i$  is equal to 1 to  $N - 1$ . So, it is the two consecutive path has a small difference of  $\Delta_i$  ok.

Now this is consecutive difference now let us go back to our again some of the discussion. So, what does it mean it says that I have a this is my antenna here, this is my another antenna here I have another antenna.

I let me just magnify the distances and spacing for our own discussion. So, let us assume this part is this path. So, let us say this is my  $D_i$ , this is  $D_{i+1}$  consecutive mind it and there will be one more antenna let us assume there is one more antenna here. So, this should be my  $D_{i+2}$  ok. Let us assume this is the first antenna, this is the second antenna and this is the third antenna and they are just linearly placed and this is my frame of reference; that means, I am standing there and from there everything is looked at.

Let us assume that  $D_i$  is the smallest one among all of them because that is may be the smallest one it is not a big deal to assume it for simplicity.

So, let us assume this  $D_i$  is the smaller compared to everybody the first antenna among these three let us assume this this is easy I mean without hampering the generality aspect this you can safely assume the first one you can assume this is the smallest one and as you go down to the antenna the distance is it what does it mean, it means that if I draw a circle from this point to this point take that as a you know radius and I draw a circle.

So, what will I get? I create a circle right. So, it is like as if like I am drawing a circle from this point with radius of  $D_i$  I will cover I will cut like that right if all of them are greater. So, that mean as I move along my antenna positions I will see this distances increases. So, this is the first point, then there is a second point and third point and this is what so; that means, can

I make an small point here, till here or till here till here the phase will be equal why because the distances is a exactly same for all of them.

But from here to here the phase difference is caused because of the small tiny difference in the distance because of these small tiny difference in the phase in the distance. So, that small tiny creates the problem for the phase one ok. Now what we have assumed here in this particular context that this one let us assume this one is the  $\delta_i$  let us assume this is the first one is the second one or let us say this is  $\delta_1$ , this can be assumed to be  $\delta_2$  and so and so forth right.

And you can predominantly see that for this configuration  $\delta_1$  must be greater than  $\delta_2$  must be greater than  $\delta_3$  and so and so forth. Now this is how the configuration is and these deltas are responsible for the phase differences, this what our assumption here. Now will do some sort of a geometry and some trigonometry and try to see how again one more level of approximation can lead to another interesting result ok.

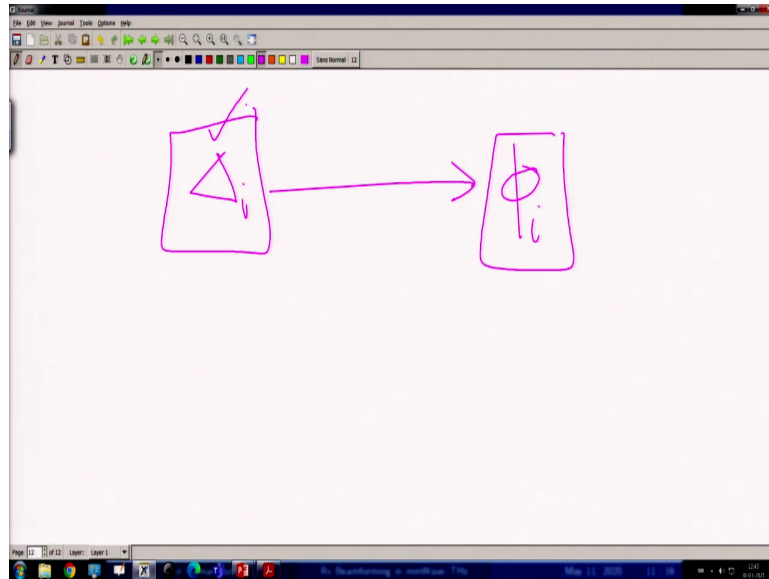
Now, because now this  $\delta_1$  and  $\delta_2$  are not known to us, why it is not known to us because I do not know the distance between them between each and every antenna to the Tx that is not known to us. The only thing we know that they are whatever the distance is it is almost equal that is the only difference we know and this  $\delta_1$  this  $D_i$  is so large even if I draw a circle this small  $\delta_1$ ; that means, this small  $\delta_i$  is very very less than any of the  $D$  you can take it very very less and that is; obviously, right.

It cannot be large enough because this distance itself is very small that is a small tiny lambda like a 2 millimeter 3 millimeter. So, how much this difference, they are also in the order of millimeter only they would not be a very large because this distance this  $D_i$  itself is a very large distance may be in the order of say 20 meter compared to say millimeter ok. So, this  $\delta_i$  is like a small millimeter dimension compared to  $D_i$ . So, I can safely assume, now what does it translate to our phase, does it give us an advantage ok that we need to understand.



Secondly, I still do not know what is delta I this I do not know, can I make some approximation ok and that is precisely our next level of approximation that how exactly the phi.

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Because this delta i determines what this delta i drives my phi finally, that drives my phi that difference because of this delta i get a difference in my phi very straight forward ok. Now the question is that I do not know. So, can I do more one more level of approximation by which I can guess what my delta i should be ok?

Now, with this I stop the class today. So, in the next subsequent class we will be talking of the generation of this phi, how I can make that phi determined. So, what are the three assumptions that we have made so far one is a distance to be equal, then channel that makes the channel gain to be equal, then the phase we have not made it equal because that is key

point and third one what is the third approximation AOA angle of arrival we have said angle of arrival is also equal for all of them if it comes from the same source ok.

That mean  $T \times$  no reflector our scatterer when it reaches to the all antennas their angle of arrival is same. So, this three approximation we have the next level of approximation is  $\Delta I$ , the difference between the distances ok, conclude our session today. So, we have learnt some more approximations predominantly AOA and the distance things again and in the next class will be  $\phi$  or the angle differences how exactly they prop up in our discussion that is it.

Thank you.