Signal Processing for mmWave Communication for 5G and Beyond
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Module - 04
Detection, Estimation and mmWave channel
Lecture – 22
Precoder and MIMO (cont'd)

Welcome back. Now, we will be talking about the Precoder as well as the MIMO system, how exactly we want to do the signal processing there where it will be discussing the precoder and the MIMO part. Again this is more generic irrespective of your millimetre wave or 6 gigahertz because the same concept will be again used in the millimetre wave. So, conceptually there is no difference. So, we will be introducing the concept of precoder.

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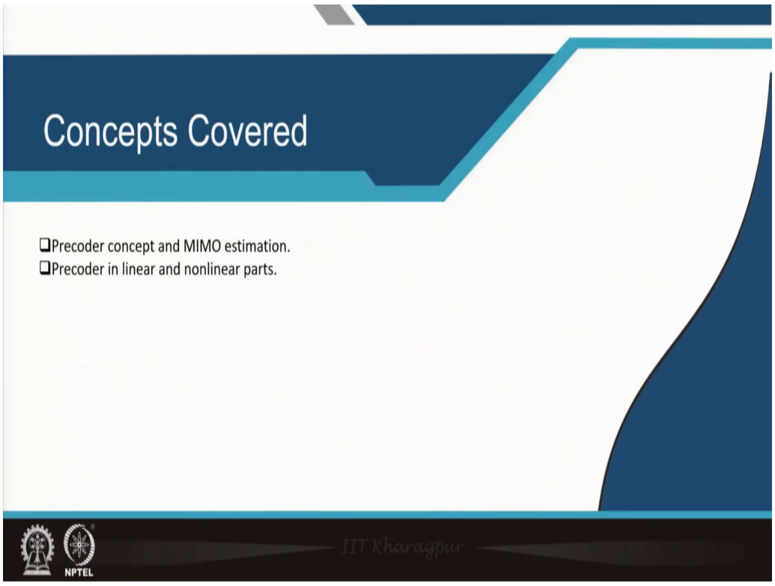

Few criteria will be covered there like SVD and all and then we will be talking about the basic MIMO part.

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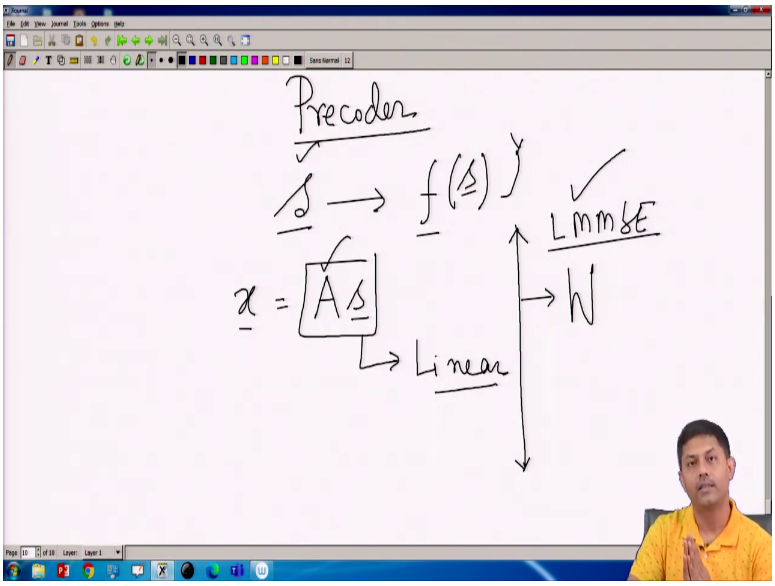

So, now, if it is a MIMO before that I need to talk about the precoder part. What is the precoder part? Now, precoder usually it is again it can be linear it can be non-linear it is like you are its kind of a coder as you as the name suggest. So, what does it mean? So, you have some data. You are modifying the data with something. So, who can modify it?

May be coding system can modify it or some amplification or some you know phase shift or something some complex number multiplication division can make the data changes. So, precoder is something like you have a data indented data, but before that you are modifying the data for certain purpose ok, for certain purpose and then you transmit it.

So, it is as if like you have the original intended data s, but before that you modify the data through some function f s and this one you transmit through the antenna ok. Now, this f again come to the same point. Can it be any linear, can it be any non-linear, it depends on you.

If you can afford to have a function which is any function like a non-linear function, well if your system can if your hardware can support it well and good, go ahead. If we cannot you can always have a linear precoder in that case. So, in that case what is the meaning of it, linear meaning? It will be some matrix multiplication or some vector multiplication ok.

Now, as this itself is a vector. So, we will be essentially talking of a matrix multiplication. So, now, when I say this s vector the original transmitted vector is really going through a precoder, it simply means that it is actually getting multiplied by some matrix A with the s bar. So, this is what we will be transmitting instead of x s.

So, this could be our transmitted vector ok. Now, what is the purpose of A? So, we will come to that point. So, purpose of A is that many is not 1, the similar way of your equalizer. What was the purpose of equalizer? Equalizer was there in the earlier context to nullify the effect of your channel with some cost function in mind.

Here also the precoder also does a similar job that it can also it can also aid the equalizers job much better. So, equalizer can be for you know for maximizing your SNR or it can be boosting up the power or optimally utilizing the channel power or it can also be boosting for boosting up the capacity, it can also you know it can also make the decoding simpler whatever.

There are many ways that you can have your own precoder just like your equalizer that sky is the limit. What type of equalizer you wanted earlier similar type of precoder also you can have it earlier. So, it was one more level of boosting kind of thing I would say. So, I can have this kind of matrix design if it is a linear.

So, essentially we are talking of linear precoder in this case ok. So, if it is a linear precoder I can have laundry list of methods by which I can design a precoder. So, in this course I am not really getting into the all techniques how to design a precoder because as we have seen from the equalizer design that this is also a optimization problem because it finally, has to maximize or minimize some goal or some quality of service.

So, what are the quality of service? It could be capacity of a system it could be SNR, it could be you know BR can also be one of the point, it could be how ease of your decoding anything I mean there is no limit to it. So, let us design one such precoder which are usually used it here ok.

So, now, let us formulate some sort of a problem statement because that is what exactly my goal hit here. Let us say I want to design a precoder based on a LMMSE method. What does it mean? It means the my precoder should be such that at the receiver after decoding the data using some equalizer it gives me the minimum MSE. Let us say that is my cost function.

So, how can I design the cost function and what is the optimization method? So, why I am talking all these things because millimetre wave communication and terahertz communication we will see that these are very fundamental. This precoder, equalizer are very very you know bread and butter of the complete communication chain.

So, let us really come up with the architectural aspect let us really come up with the optimization problem. Sometime you know sometime the problem may not be closed form. Just like in the earlier case I have designed an equalizer which was closed form and that is a very standard result.

If you open any book or if you search any I triple explore papers you will see that. These are very standard results done long back. So, I am I did not really get into the exact design methodology, but many times such equalizer or this precoder design may not be having a closed form solution. It could be even an iterative solution ok.

It can you may encounter depend on the problem depends on the cost function. Now, if it is LMMSE there is no guarantee that it will be you know it will be kind of a closed form there is no guarantee. For equalizer alone it was, but when it is a precoder it may not be ok. So, moving forward many times what we will be doing is that we will be forming the optimization problem because that is important.

How you solve it is not important that is that is your own choice of solving, but what should be the optimization framework that is required to get the data is more important for us because techniques optimization techniques are standard techniques ok, but formulating the problem in the right spirit is what is what is important for us.

So, in this case precoder also we will be formulating the optimization problem as a whole. Now, let us say I am giving only one example. Let us say I have LMMSE as the cost function and using LMMSE I want to design a precoder. Now, naturally if I really want to design a precoder I would also have the equalizer as well.

So, can I have some sort of a you know joint optimization that can give me this LMMSE? So, in this case probably A is one of the quantity and W equalizer is another quantity which will be used as equalizer. So, in this particular problem statement it is as if like I want to develop a precoder, I want to develop equalizer together ok. You can separate them out also. It is all depends how you would like to achieve the things.

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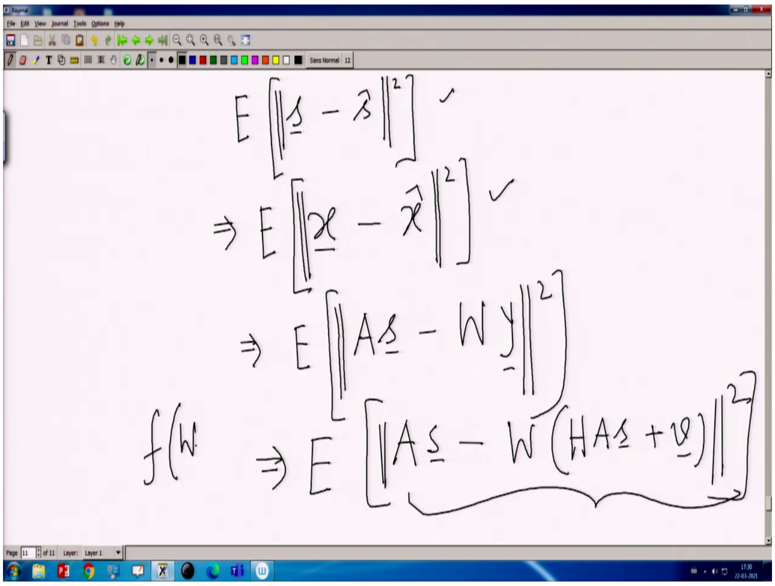

So, let us say I want to build up such kind of cost function. So, what should be my cost function? This is what is the ultimate goal s cap minus this should be minimized right. This is my ultimate goal. My equalizer should be such that this quantity is actually minimized ok.

Now, in this case what are the other inherent quantity that is coming into picture? Now, s is not the actual quantity because s is what I am not transmitting. I am transmitting what? I am actually transmitting x finally. So, I just make my slight modification in this case because I am doing a precoder design.

So, can I make my modification that I want to design equalizer which would ultimately minimize this. This was the earlier case. This is now our new goal right because this is what I am transmitting it. Now, I just replace it. Now, you replace it here s cap here. And what is this? This one will be y ok. Do some more mathematical problem, I just put it back whatever I got it.

So, what is this? This is some function of W and A is some function of W and A. So, this is my cost function.

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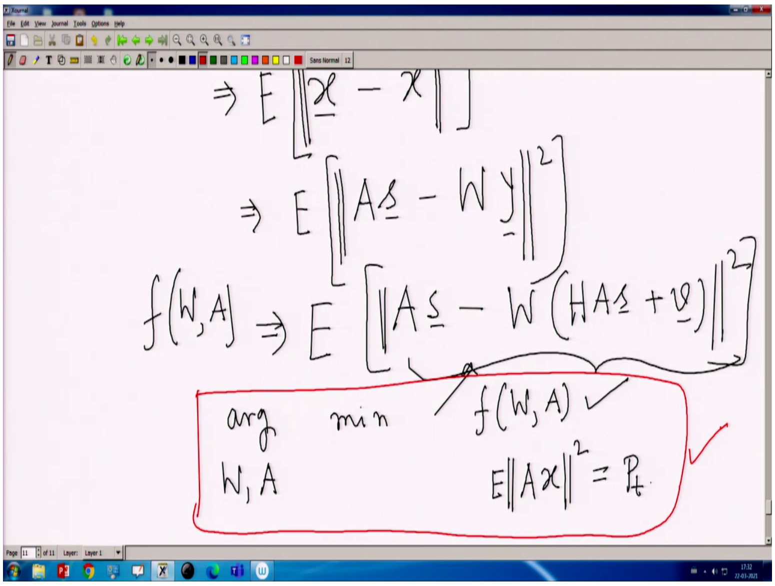

So, what is my job now? Job is to minimize this quantity or rather f of this W into A quantity over W, A. So, this is one typical you know framework for optimization. So, optimize this minimize this quantity such that it can maximize. But you can put some sort of a restriction as well. You can the restriction could be that the power because if I want to do that what will happen?

I can always put a very high power in the transmitter side and you can always minimize it. So, I need to put some sort of a power goal. This should not cross your transmit power right. So, given this transmit power constraint optimize this step. So, this could be a very nice optimization framework.

What is the method of solving it? There are plenty. Let me not get into the method of solution. Probably when you go into the millimetre wave communication, same problem will be encountered. I will explain one or two optimization techniques ok.

So, here more important is the creation of the framework. So, framework creation is more important than solving the optimization problem using standard methods. So, and during that time I will be explaining at least few examples of how I solve the optimization problem in this case, but this framework creation is more important for us ok.

Once you do that your job is half done. I would say. In fact, I job may be 60 to 70 percent because you have created the framework. Now, it can be convex way of solving it non convex way of solving it there are many many many techniques are there. I will explain some of them when I really deal with the millimetre wave part ok.

So, this is the point here. So that means, I can have a precoder I can have equalizer. There is a there is one more interesting precoder available where you really do not have to go through any optimization techniques ok. You can straight away get it and that gives you some benefits in terms of maximizing of you know maximizing of SNR or ease of decoding some benefit it can give you. As I said sky is the limit on how exactly the cost function should be.

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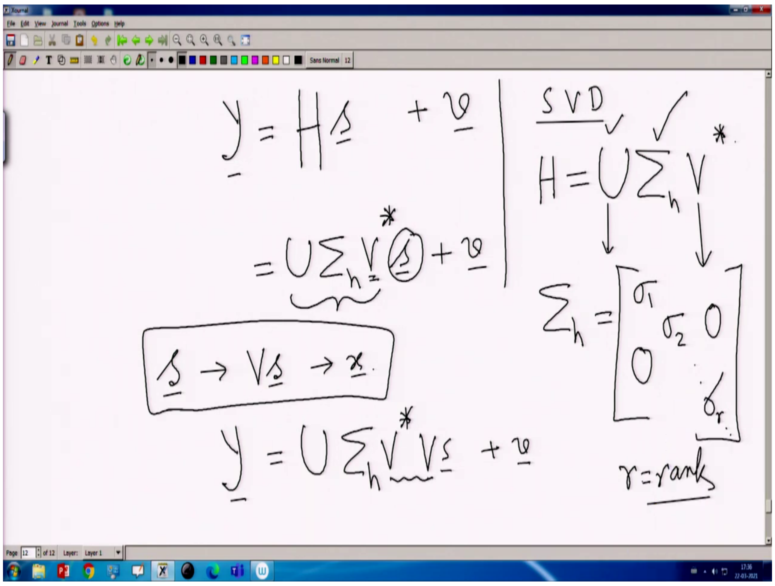

One such you know one such interesting equalizer design as well as the as well as the precoder design is there suppose your channel matrix is this whatever channel you got it. So, let us write down the system of linear equation again ok. Now, let us say I get the SVD, singular value decomposition. What is singular value decomposition?

That any matrix can be whatever dimension is not a restriction there. So, any matrix can be broken into three matrixes ok. So, any matrix. So, let us say H itself is that matrix. What is the dimension of H? We have seen. It is not it is not even a square matrix, it can be a it can be the rectangular matrix.

So, I can always break it like this. Any matrix can be broken into three such pieces. This U and this V are called unitary matrix they are actually unitary matrix and this sigma h is the is a diagonal matrix having singular values. So, this sigma h can be broken into a diagonal matrix and this diagonal elements are all positive elements ok. Sigma 1 to sigma r, I am not drawn the dimension. This r can be basically r is nothing, but the rank of that h ok.

So, if the rank is whatever that many number of positive quantity we will have in the diagonal rest will be 0 and rest of the elements will be all anyway 0. So, this is a diagonal matrix and U and V are two unitary matrixes. So, any matrix can be broken into that. So, assume that I know H. So, if I know H, I can always break it like that because this is the advantage of my singular value decomposition.

So, I will show you another very interesting precoder as well as you know as well as equalizer development ok. This is one of the interesting points. So, now, think about it. So, what is my intention? I would say I want to do some sort of a precoder which maximize something. Let us not worry about the what it optimizes right now or what is the cost function that optimizes.

But can I think of, so, this is my channel. Can I think of a precoder where the precoder A matrix whatever we have drawn it is nothing, but this V. So, what I am what am I saying that s vector first you precode it using this V matrix which is nothing but this V matrix, this is the hermitian part V matrix. So, what I transmit? I transmit basically V into s which is my x is what I may transmit.

So, if I do that what will be the what will be the advantage? If I transmit that mean if I choose a precoder which is nothing, but this V matrix then advantage is that at the receiver I will get U sigma h which is V star and this s is no longer s. Instead of s I am actually transmitting V into s right. This is what I am transmitting. Now, this is unitary matrix. So, that will vanish.

So, this whole thing will vanish. So, that mean moment I put this equalizer this whole thing vanishes. So, it will be like a you this will come back here ok.

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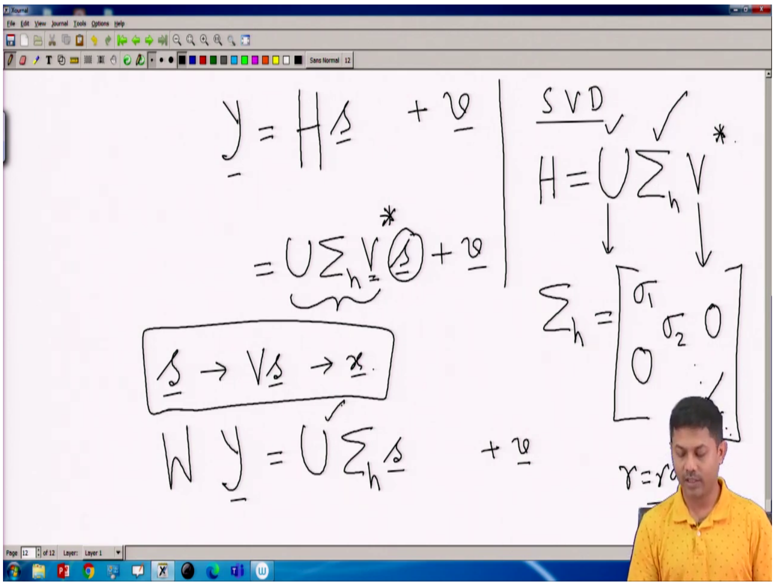

This is an interesting precoder ok without doing any optimization. Whether it will optimize something else that is not part of this course, but this is also interesting precoder. Now, can I do an equalizer design? Well, instead of thinking of all the complicated precoder can I think of a very simple precoder that makes my decoding absolutely simple.

Yes, instead of that W which comes from all this least square LMMSE you know blah blah blah or other some other cost function. You just think of an equalizer which is the inverse or Hermitian part of U.

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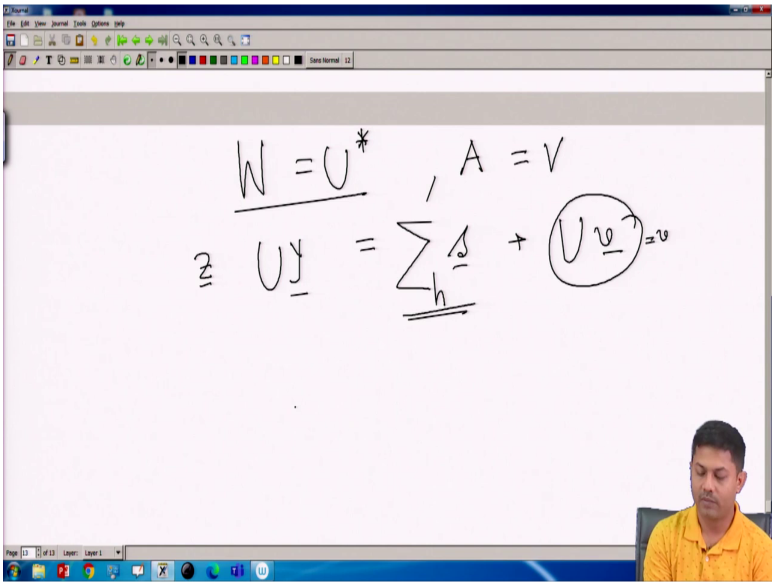

So, that mean now I am saying that what if I take a equalizer W is equal to U star in this particular case. So, that mean the equalizer is W, your precoder matrix is nothing but V. So, what will happen? You multiply this U with your received data what will happen? This U will be again vanishing, this U will be again vanish. So, what will I get? It will be sigma h s bar plus U v.

So, I am not worried about that part. It will be some v dash is equal to v dash something will be there. This is does not change the statistics of my noise. Why? Because this noise is a circular symmetric noise, right. So, if I multiply it with a unitary matrix it that does not change my statistics of my noise. So, this I keep it like a v itself because anyway I am not interested particular value only statistic that remain as it is. Now, look at this. Now, let us call it Z vector.

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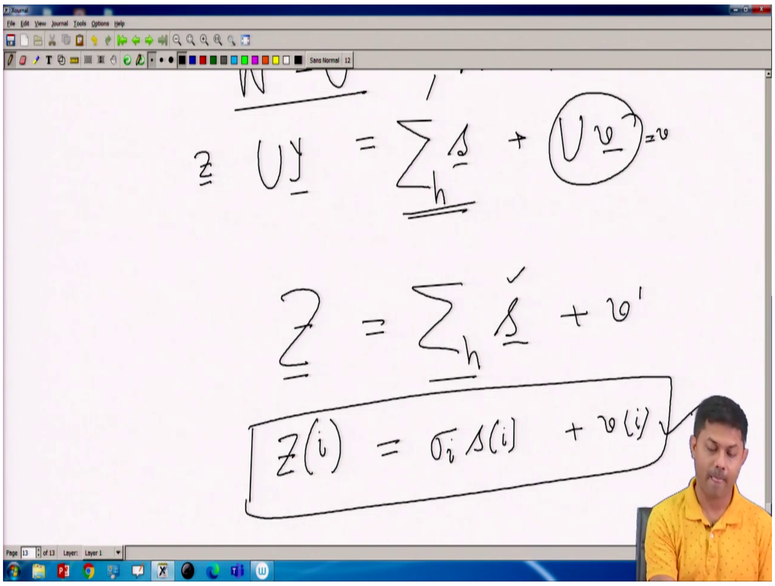

So, that mean my Z vector will be some sigma h into s vector plus some noise. This is what I am getting finally. Look at the decoding complexity, absolutely simple. Why? Because this is the diagonal matrix and each and every row is like an independent row.

So, it means that instead of this vector taking the whole vector as a you know as matter of vector and then try to decode this s, I can just consider the ith point of it, ith point of Z and that is nothing but the ith row of sigma i. So, let us say sigma whatever the ith component into this s vector whatever the ith component plus the noise.

Look at that, whole vector decoding is now come back to me as a scalar decoding point. So, I can decode such kind of things very easily. So, this is also another equalizer and another precoder design where my job of decoding and job of you know decoding complexity is much more simplified ok.

Of course it will maximize something, but this is also another type of decoder. So, there are many such precoder and equalizer available and you can have your own equalizer and you know precoder based on different cost function of your choice ok.

Now, given this fact now I will move to the last signal processing aspect of the single antenna case ok. So, now we have already discussed equalizer and precoder. Can I now think of multiple antennas case because this is the multiple antenna issues that is coming into picture.

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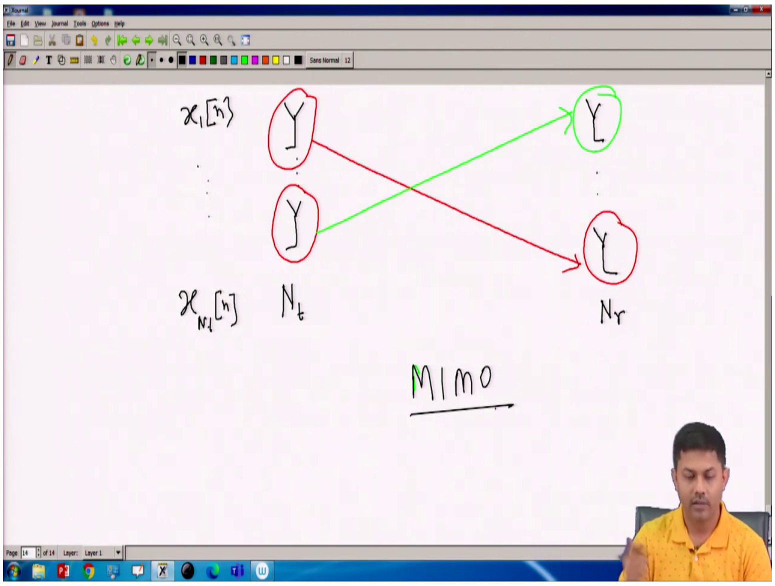

So, instead of one antenna, can I now think of multiple antenna? Ok. Now, it is a multiple antenna. The only you know the only channel that will be coming into picture is that the whole thing is now is like a multiple point to point communication you can think of.

So, you have say N t number of transmit antenna, you have N r number of receive antenna. So, any antenna transmit antenna to any receive antenna is like a point to point communication right. So, you can transmit multiple data here. So, you can transmit x 1 data here say x 1 n and you can transmit x N t n. There are empty number of data stream independent data stream you are transmitting it right.

So, each and every antenna transmitting antenna will be acting like a will be acting like a point to point communication for each and every receiver. So, it means that if I say take the first one versus the last one is as if like a it is a point to point communication.

If I take the antenna here this will also be like a point to point communication here provided they are all independent. They do not you know mingle with each other. So, that fundamental assumptions are given. Even if they are not independent only thing is that your data in the data model things can be correlated, but that is ok.

But when I say it is kind of a multiple antenna system or rather MIMO system MIMO system each and every antenna I can think of it like a point to point communication. So, I can just extend my single antenna channel model here as well. See if it is a point to point communication what does it mean?

It means that one antenna to another antenna the channel that I will be viewing it is a what, FIR filter ok. If that is what the if that is the notion of my channel that is the notion of my channel then what can I write it here?

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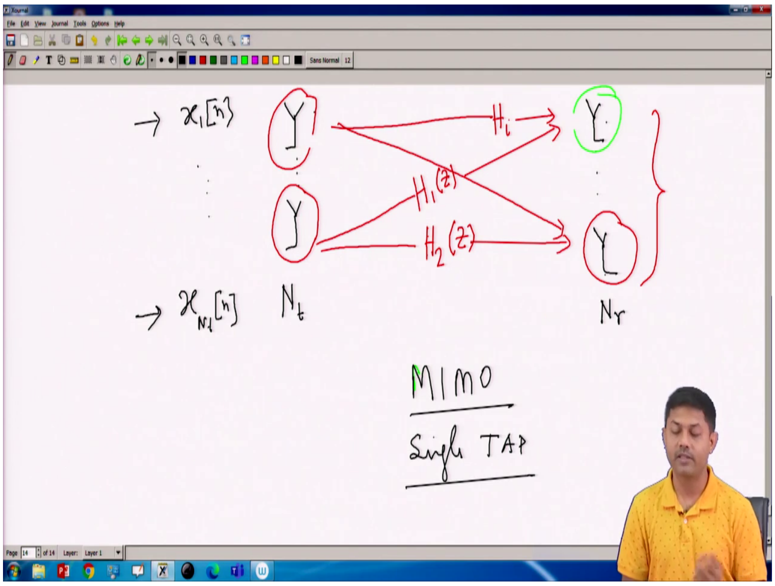

I can simply write it that from this channel from this antenna to this antenna I would see some FIR filter ok. From here to here I can see another FIR filter. So, let us give some name H 1 Z. So, let us say let us assume there are many antennas there. Here also main antennas here.

So, here I can also have some H 2 Z and so and so forth right. I have to only written H 1 and H 2 there may be multiple because there are N r number of data is here. So, here also this point will be some you know some H i. Here also something like that. So, every point to point it is as if like one FIR filter.

So, how many such FIR filter we will be receiving it? N t into n r number of FIR filters that I will be dealing with it ok. Now, how do I create a data model for such kind of things? Well, if it is an FIR filter getting a data model is not so easy. You have to bring into the tensor product and so and so forth.

But I am not getting into that because to tackle it usually we have the MIMO, OFDM concept. So, where you really do not have to go through this complicated tensor product kind of things. You can easily model it as a like a single tap antenna. So, it is very important to know what if my channel is just a single tap.

So, what I am trying to say is that instead of assuming to be a multi tap system can I assume to be a single tap system single tap system. What is the advantage of it? Advantage here is that later on when you go for say MIMO OFDM or because that is what the waveforms that we always follow here, single I mean MIMO OFDM system.

The actual model will come back as a single tap model after all your OFDM thing. So, that is the reason why it is important to know a single tap MIMO system good enough. If it is multi tap system data model can be complicated, but if I use a MIMO OFDM that whole thing again gets nullified.

Now, the concept of MIMO OFDM I will be introducing after the millimetre wave channel model is over because of course, millimetre wave also uses the MIMO MIMO OFDM system. So, at that time I will bring in the MIMO OFDM part with MIMO. So, for the time being let us assume that I am using a MIMO system any multiple input and multiple output meaning multiple data stream that will be you know getting transmitted.

But the channel from tap to tap is just like a one tap ok. Now, when I say MIMO does it mean that tap antenna to antenna it is the channel length will be same? There is no such guarantee. For example, first antenna to the last antenna there may be ten channel taps that you can see it in the digital domain.

And the same first and turn to the second antenna probably another antenna I may see a 9 taps, some other antenna to some other antenna I may see just a 1 tap. So, it is not guaranteed that the number of channel taps will be exactly equal. So, there is no guarantee.

Second thing that, what about the coherence time? Will the coherence time be also different for antenna to antenna? Usually well, usually coherence time may not be so much different from antenna to antenna because these are all individual you know FIR filter. So, if it is an individual FIR filter obviously, every characteristic may be different right may be different. Its coherence time can be different, a Doppler spread can also be different everything logically they are all different.

But usually when you have a physical system if you think of a cellular network or a Wi-Fi network these antennas will not be placed very wide apart right. So, if you think of your mobile phone the antennas are within the mobile phone right, they are not very wide apart. See if it is not wide apart the channel characteristic will not be very wide apart as well.

Similarly, whosoever is transmitting say base station the antennas are not kept very widely like one antenna is somewhere here and then another 100 meter is the second antenna, it will not be like that. Within few centimeter or the second antenna or within a within probably you know half inch or maybe depends on what your arp is that within just 1 feet you have many number of antennas.

So, they are not widely spaced they are not wide apart. So, if that is the case the characteristic of my channel will not be so much different. It is like a more or less everybody is seeing the same kind of you know physical geometry. So, the coherence time, the Doppler spread will be more or less same across the antenna in this particular case ok.

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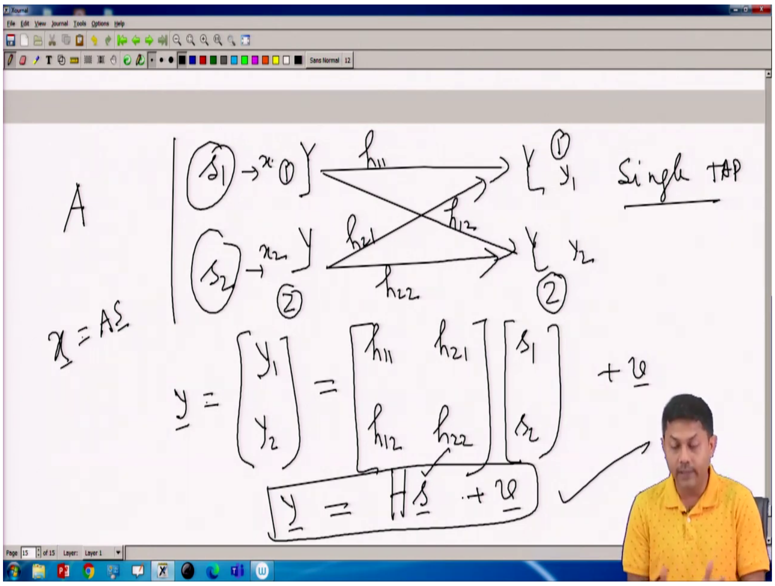

How do I model? See if it is a single antenna to single antenna the modeling aspect remains the same. See if I just take say two antenna here and I just take a two antenna here and I prefer single tap channel because multi tap channel is not at all an issue for us. Why? Because moment I use a MIMO OFDM, I will show you later that it is nothing but equivalent to a multiple such single tap systems ok.

So, it is good to know single tap system. So that means, I am assuming that there are two such antenna and every antenna to every antenna these are all like a these are all the channel part and just one tap. So, this is my first antenna, second transmit antenna, this is my first receive antenna, second receive antenna ok.

Let us assume I am transmitting s 1 symbol here s 2 symbol here. Let us assume this is my y 1 data y 2 data. Now, my observations are across the antenna which is spatial antenna spatial observation not in different time, unlike your you know single antenna case where you had the multiple taps here.

So, let us assume this is 11, this is say h 12. So, this is say h 21, this is say h 22. This is what my channel taps just the single tap. So, what will be your data model? So, what is y 1, what is y 2? I just take it again stack ok, this is a y bar. What will be my data model?

So, this I just created like this s 1 and s twice put it here. So, what should be my s 1? So, at y 1 for which is coming h 11 and h 21. So, it will be h 11 and h 21 that is coming here look at that ok and at y 2 it should be h 12 h 22 plus a noise vector.

So, what is again my equivalent data model? It is like y bar is equal to some channel matrix h which is your 2 in this case it is 2 cross 2, some data vector, some noise vector and this is what my data. Again I am back to my system of linear equation. What is my next job? Next job is to detect s. How do I do? Design equalizer whatever choice.

Can I create a precoder? Of course. So, instead of s 1 and s 2 this whole vector you pass it through some equalizer A first then and give x 1 and x 2 here. So that means, x vector is equal to your A into S vector and create your own equalizer you get your own precoder there. So, you see that moment I have this y equal to you know h s plus v I can create linear equalizer I can create my linear precoder and I can detect my data I can do a channel estimation whatever I want to do ok.

So, this is a very basic signal processing using the channel model that we have already discussed it. Now, our next job precisely is to move to the millimetre web channel model and see how all these scenarios change ok. So, with this I conclude this particular session.

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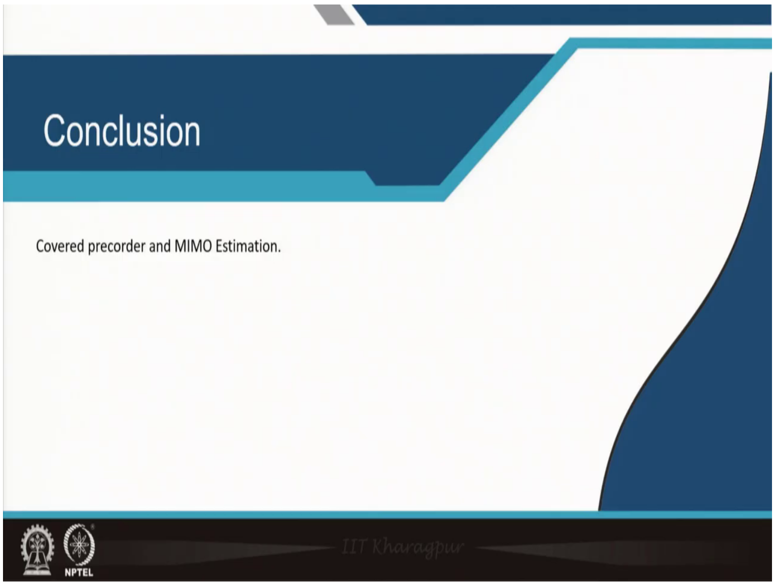

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These are the reference Statistical Signal processing, Fundamental of wireless communication by Viswanathan.

Thank you.