Signal Processing for mmWave Communication for 5G and Beyond
Prof. Amit Kumar Dutta
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Module - 04
Detection, Estimation and mmWave channel
Lecture - 21
Precoder and MIMO

Welcome back. Welcome to the Signal Processing part of millimetre Wave Communication for 6G. So, last class we have stopped exactly at this point where we are stopping at the system of linear equation and this is what this is what we built it from the single antenna I s; I mean single antenna multitap channels ok.

So, today we will be talking about the lecture number 21 where it will be discussing the Precoder and the MIMO part. Again this is more generic irrespective of your millimetre wave or 6 gigahertz because the same concept will be again used in the millimetre wave. So, conceptually there is no difference.

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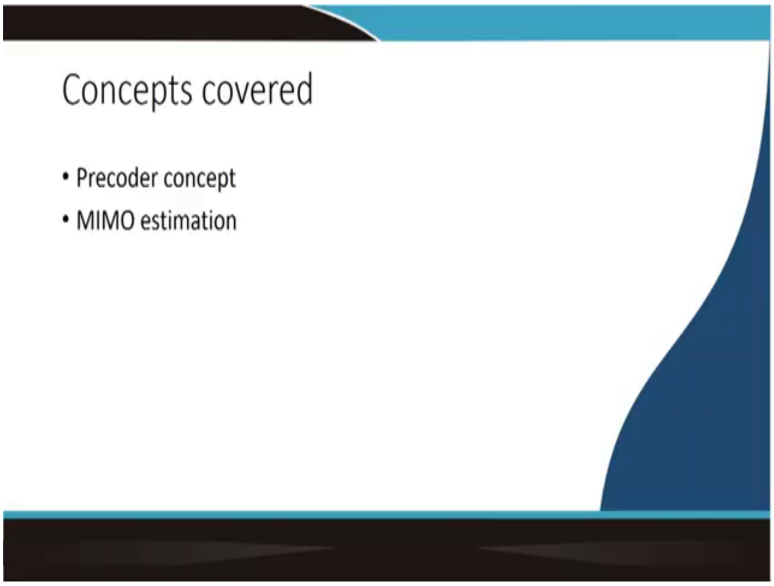

So, we will be introducing the concept of precoder, few criteria will be covered there like SVD and all and then we will be talking about the basic MIMO part. Now what is the job of this equation and I got the equation, but what needs to be done here?

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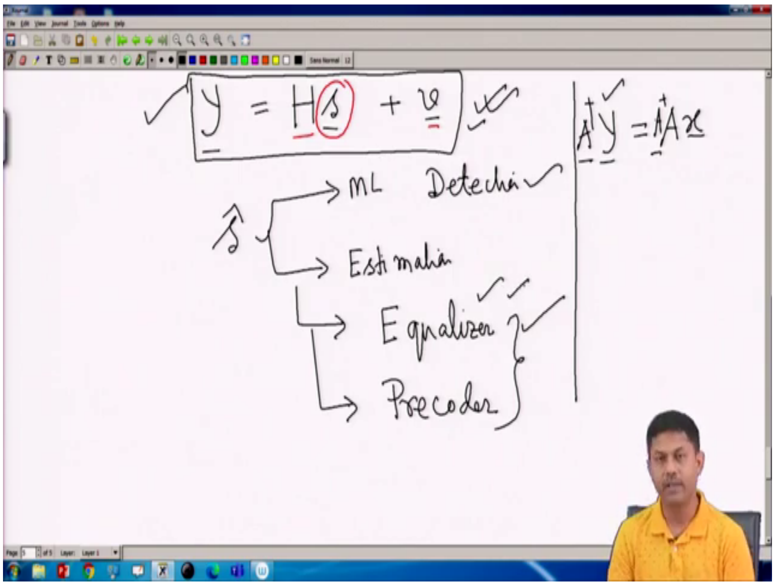

What needs to be done here is that, my intention is to extract s that is the final intention. From what, from why; what is given, of course H is given to me, v is given to v is not given, but it statistics probably may or may not be given ok. Sometime there may be estimation technique where you may know, there may be estimation technique where you do not even know that ok.

And I am not getting into the really a detection estimation part, because this is not the this is not part of this course. I assume that at least you have some idea of detection estimation. But, so from that point of view you know that you may or may not have the statistics of v ok.

So, now, once this is done I can create wonder out of it. So, I can create I can estimate my s, I can do a detection of s, many things I can do. So, if s is a data for us, suppose for example, when I send my communication signal what is s for me; s could be pure constellation point, s could be some complex number at the end of the day right, if that is the case how do I do a detection for it, how do I do an estimation for it all things can be done.

So, from here I can create ML detection, from here I can create estimation. When detection is performed? Detection is performed when s is kind of a discrete constellation point so, there I need a detection. If it is like a continuous data say for example, channel yeah it is not a; it is a discrete, but it is kind of a range. So, within the range it can take any infinite points.

So, when this kind of thing come I have to do some sort of an estimation. So, it is your choice what type of data what you want to do, but this data model is important. So now, from here I would like to have one or two point discussion because those points will be very very important in the millimetre wave context.

So, one is called equalizer design and second point is called precoder design. These two points will be very important in the context of millimetre wave and obviously, it is equally important in the context of other communication part. So, this point I would like to highlight what exactly they are, because they are you know kind of bread and butter I would say in the millimetre wave context as well.

So, now let us think about the first part here equalizer part here ok. So, what is equalizer here, let us understand that and then we will see; what is precoder here. And these two concept can be you know extended for even a MIMO system suppose a MIMO system. So, in this particular case I have not used MIMO system it is a single antenna system, but my observations are multiple in different times.

So, when it is a MIMO I will show you that the observations are across the space or across the antennas. But data model, this part data model somehow you know remains more of an linear system of linear equations. So, that is the reason I really do not you know; I really do not stress upon where exactly the data model comes from, at the end of the day it is a system of linear equation and for me I will just try to solve what is the variable here.

So now, there are two points that will be kind of bread and butter for us are the equalizer and precoder. So, this concept needs to be clear here. So, what is equalizer here? So, let us look at the system of linear equation here. So, y equal to H s plus v right. Now when I say equalizer what does it mean, it means that I am kind of nullifying some of the effects. So, it is like you have y equal to H s H of s plus v so, this equation. So, let us say you keep the v part aside ok.

So, y equal to H s v is there inbuilt you do not worry about the v part. So now, what is the job? Job is to extract s. So, H is not your job H is of your interest, but H is not a ultimate interest. What is your interest? Interest is your s, because s is the data that you have transmitted from transmitter. So, that is what you want to that is one that is what you want to get back at the receiver not the channel. Channel is just an intermediate component which will come and you have to remove it; that is the only job.

So, you say I am removing the effect of channel what exactly you are doing. So, if you say I have a equation like that, it is a general equation nothing to do with channel A of x this is a general equation ok. I say that y is my observation, A is kind of some sort of a matrix which is known to us and x is not known to us and I know this model and this is an observation.

So, what if I say how do I; how do I know what the x is the first thing that any, any person who knows system of linear equation or linear algebra kind of thing is that; hey why do not you just do an invert of a and multiply both side. Well, whether it works or not that is a secondary question it is immediately we will just make some sort of a inversion here and multiply it here and get try to get back your x here right.

That is the first I mean that is the first approach you may have it. Now invert inversion may not exist for all the matrix, it may be a singular matrix or the matrix itself is not a square matrix. Then he said ok, what did I do instead of inverse why do not I do some sort of a pseudo inverse.

Well, it nullifies the effect of A. If the pseudo inverse also you do not like pseudo inverse can I do something else. So, that can I do something else by which I can take out the effect of channel is what your equalizer would be ok. Now it could be linear, it could be non-linear really does not matter.

Now obviously, when you deal with a very gigantic system say for example, in this particular case it is a single antenna, but my observation says 100 or maybe 50 observation. So that means, the dimension of such kind of equation is also very gigantics, right.

Now, when you deal with this kind of gigantic system you tend to have always a linear system that mean the equalizer is also kind of a linear for you I mean that is a natural choice always right. So, when it is a linear equalizer this kind of thing that comes into the picture; that mean you want to deal with matrix, vector and so on and so forth, you do not want to get into some non-linear equations, right.

So, here whenever we want to do the processing I mean later on also every times we want to deal with linear system. That means, my linear equalizer, linear precoder, linear system of equation because that gives me my a complex that keeps the complexity low in at every stage, right.

So, let us concentrate on the equalizer front which basically tries to you know remove the effect of channel in a linear fashion. What does it mean? It means that I need to deal with some or I need to create some sort of a matrix or a vector whatever by which I can you know take out the effect of channel and I will get back my data. So, what exactly my linear equalizer is?

So, let us take a very simple example.

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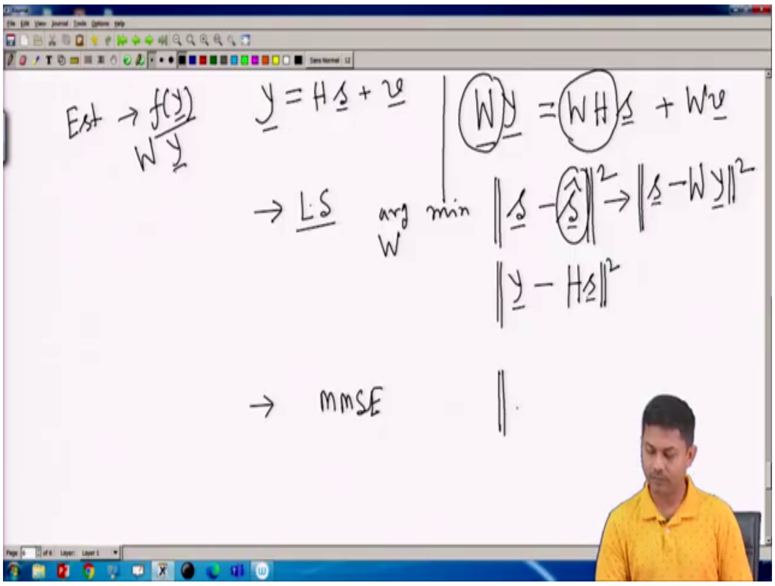

Let us say again I get back my equation here. This is my original equation I want to get back I want to remove the effect of channel, what I need to do is that if I see it is an equalizer what does it mean I need to multiply with some sort of a matrix ok.

So, something like well I want to no, I should not put it here I want to you know multiply with some matrix which is to be known. And if I multiply it the effect will be; the effect will be as if like it does not exist, it is like a identity matrix effect ok. Whether it is really making it or not that is a secondary question, but effect will be very close to identity matrix.

So, now there are many techniques by which I can design such kind of W and this literally comes from optimization equation. So, moment is optimization equation or I want to develop a W; obviously, you have to think of a cost function. Because in communication when you want to design a parameter that parameter has to do some job, right. I mean either it can you know maximize some of the effect or minimize some of the effect those kind of things it has to do.

Then only I would say I have designed that parameter in a good sense otherwise if I just take some ad-hoc manner I do not know whether it is performance will be good or bad. So, that is the precise part of every communication system. So now, mean how can I design this W ok. There are many techniques again I am not getting into the details of it. One such technique is that can I design the W in a least square manner. So, that mean there should be some cost function which will minimize something.

So, let us say can I design a W such that my actual s which has already been transmitted, I do not know what has been transmitted. But what I come out to be what my decision would be, suppose s is something what has been transmitted and with all this equalizer I somehow says that well this is what that s that has been you know transmitted. So, let us say I call it s cap because, what I say what has been transmitted and what has already been transmitted really may not match right. So, that is what I just keep that difference s cap I call it.

So, when I say it is a least square probably this could be one of the one of the you know one of the cost function that I would like to have it or this can also be one of the cost function in all in the; all in the umbrella of you know least square ok. Now this can be really parameterized.

So, this is an estimated value this is what you your decision is and you know every estimator is a function of your observation right because that is precisely what you do that is the only thing I have it right. So, I have the observation. So, my estimator has to be a function of observation. So, that mean even if I have some sort of a estimator it has to be some function of my observation that is fundamental I mean that comes from detection estimation theory.

So, now whether it is a linear whether it is a non-linear that depends. So now, in this particular case we are assuming everything is linear to make our life simpler. So, this f y is some sort of a linear assumption what does it mean, it means that I have a vector, but I want to multiply with some matrix. So, it is kind of a linear things. So, here this will be the case.

So, these are the cost function different type of cost function; then what you want to do? You have to minimize this cost function over the argument W. So, it is a standard optimization problem. I can also have cost function which is MMSE cost function; Minimum Mean Square Error cost function. The same thing probably this is not useful here. So, let me not keep it here.

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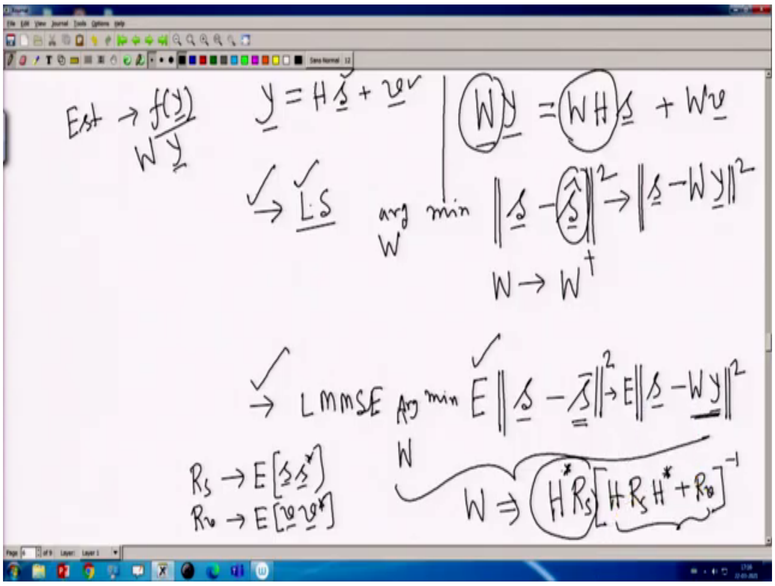

When it is a mean square error the only difference should be there is an expectation here. So, here also there will be an expectation. Expectation with respect to what; expectation with respect to whatever variable present into the system of this linear equation. Who are they, could be v is a random variable, this could be a random variable, this at it is s itself is a random variable, whatever is present with respect to the that expectation ok.

And then what I need to do I need to do I need to minimize this right I need to minimize this quantity over the W. Again you see this is some sort of an optimization equation ok.

Now, again I am not getting into the exact way of solving the problems and there are standard methods that has already been formulated for such kind of you know equations. So, these are the two popular method by which you can use some equalizer, but sky is the limit ok. So, you come up with your own cost function define a W, you can create an you know you can create some sort of a optimization framework and solve it.

For example, what if I say my W should be such that it you know it maximize the capacity of the system whether these two can maximize the capacity that needs to be seen; that is a different story. But I would like to directly get a W which maximize my capacity. Well, in that case what should be my cost function? It should be the capacity equation right involving W and then you maximize it.

Sometime you may say ok I would like to maximize the SINR of the system or SNR of the system in that case what will be the answer, develop the cost function accordingly maximize it ok. So, like that so, sky is the limit. So, you create your own cost function based on that you can get your W there.

Now these two method least square and MMSE, I would say this should be LMMSE because it is a linear LMMSE are two most popular method by which you can do the estimation thing. Now if it is least square usually the solution is that it is the pseudo inverse ok.

If it is LMMSE the usually the equalizer solution is something like that; I will explain what all the. Let me put it in a different format HR of S H star plus R of v whole inverse. What is R S and R v? R S and R v are the covariance matrix of your s vector and v vector. So, it will be expectation of your s vector. Similarly, R of v is the covariance matrix of your noise, but you can create that and there are standard equation.

In fact, the last one has a very interesting format also.

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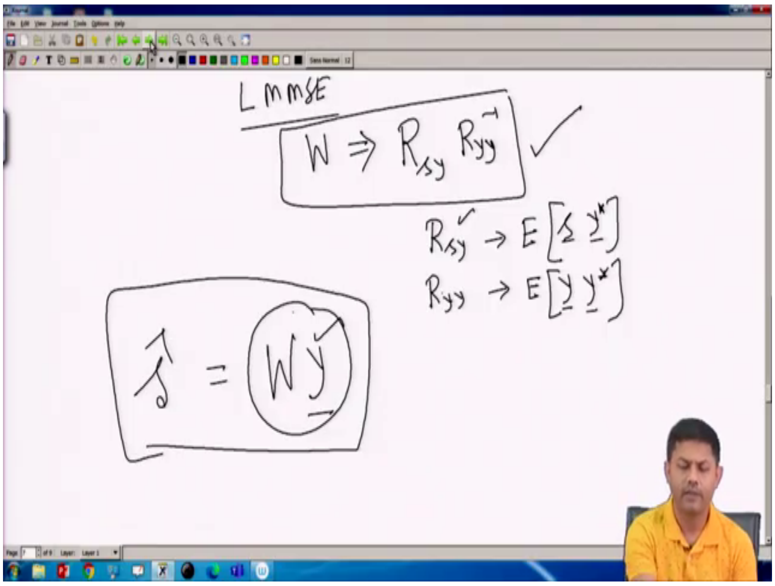

So, if it is an LMMSE; if it is an LMMSE in fact, the W you can think of it like this, this is such an important you know equation. So, what is R s, R sy, R sy is nothing but expectation of your s vector and y vector. And R yy so that is the covariance matrix, so auto covariance matrix. So, it will be y and y star this is what it is right.

So, if it is an LMMSE this is a very standard equation R sy R yy or inverse and if you know what is R sy, what is R sy here in this case; you can calculate this is nothing, but that and what is R yy, R yy is this is R yy ok. So, now, these are standard results available ok.

So now, what is my job, my job is to I go back to my earlier points here.

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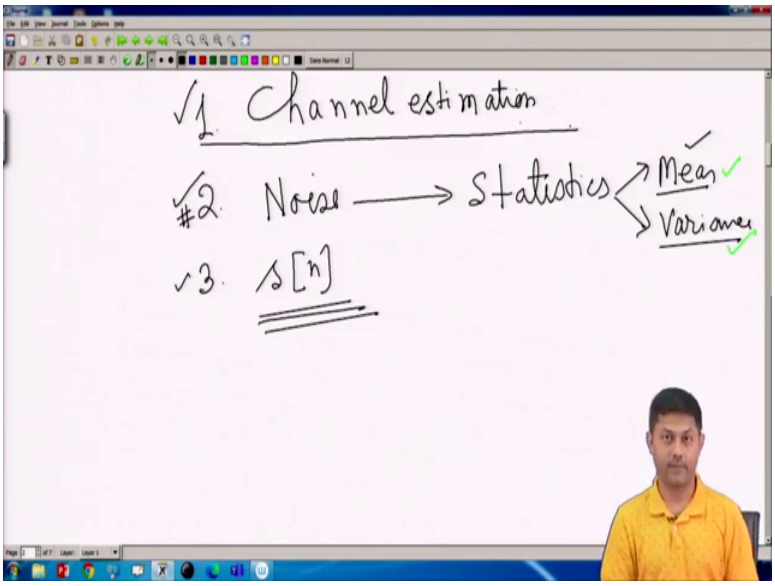

My job is to create data detection that is my final goal here right. But I still have not come to that point, because I have to first know what the channel is ok. Now from this equation; from this equation whatever we have done so far the equalizer build up this does not tell me how to estimate the channel, because it actually estimate the data or rather detect the data ok.

So, now in this case what I will do; that I make my modification into the system of linear equation such that instead of data I will first estimate my channel. So, what needs to be done here? So, look at the equation back again.

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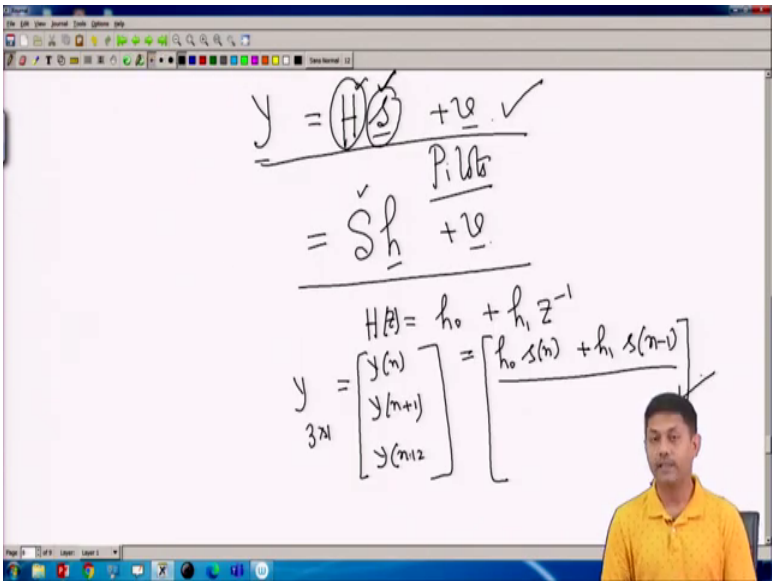

What is my system of linear equation; y is equal to H of s vector plus v vector right this is the system of linear equation. Now, here s is unknown and channel is known to you just make the assumption reverse that mean you first transmit unknown data ok.

Now this is true for anything, whether you know s or do not know s this equation holds all the time. But now I am asking you to make one extra step modification that whatever you are transmitting assume that that whole s vector right, that whole vector is known to you, known to you meaning known to the receiver.

So, what does it mean, what is that nature of that s; I would say this is a pilot that is precisely what is called a pilot, right. What is pilot? Pilots are basically the known data that you are sending. So, that s vector whatever s n minus l minus 1 to whatever n plus you know l length vector that whole thing assume that you know it at the receiver, so that means, it is a non sequence.

So, then how do I manipulate this system of linear equation? So, that mean in this case when s is known or s is a pilot this whole s vector is known to you, but H is not known to you ok. So, can I do some sort of a mathematical manipulation such that this whole thing can be written like that; what does it mean?

It is like instead of small s I construct equivalent channel matrix called capital S taking all the value from s only. But whatever component are present inside the s I put them in a stack yeah; I create a vector out of it I can do that. So, let us take a simple example let us say my channel is a simple channel where it is nothing but h 0 plus h 1 z inverse that is the simple model I am taking out. ok.

So, this is my h z ok. Now in a normal case what would have happened? Let us say I take just for simplicity let us take I take only 3 observations ok. So, it is a 3 cos 1 vector. So, it will be like a y n y n plus 1 and say n plus 2 right. What was the first equation? First equation was h 0 s n plus h 1 s of n minus 1 that is it right that is the first equation right. Now, here instead of taking h 0 h 1 and s 1 in the s n s n minus 1 in the right side I pick an opposite assumptions, right.

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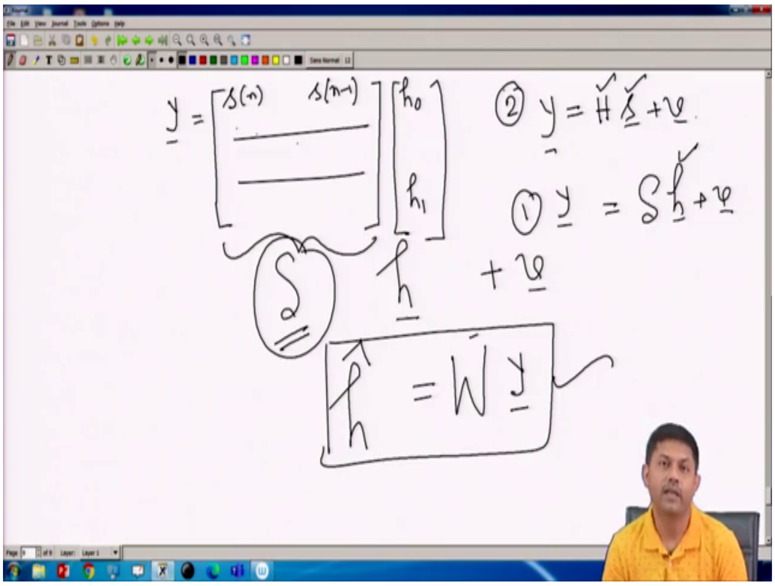

So, can I write it like that instead of this can I write it this y vector I put it like say I stack from here say h 0 to h 1, right. What can I write it here; s n s n minus 1 right.

Similarly, I have the second equation and so and so forth. So that means, this whole thing this s matrix I can form it for rest of the equations, I am not making it you can try it at your own. Write the second equation create the second row here, write the third equation create the third row here. So, it will be like a some components or some matrix created out of this s n s n minus 1 and so on and so forth, in the right side you will only have h 0 and h 1 that is it. So, it is as if like a I am creating S h plus v.

So, what is that form y equal to h S plus v right that same system of linear equation. Then now what is the job, I can create my equalizer to nullify the effect. Now this S is now known to me because it is the pilot every component is known to me it is known to me. So, I can create an equalizer out of it right, I can get least square equalizer, I can get LMSSE equalizer then what will I do, I get my channel estimation as follows develop the equalizer multiply with the y vector.

Similarly, here also we have done the same thing here. Once you get the W; what will you do with the W? So, you have to multiply this W with y vector ok that is the thing that I wanted to know. Now what does this quantity give me, this quantity give me a value which is the estimate of my s that is the ultimate intention right, because this is what the cost function was.

I should have a estimation of s cap which is nothing but W into y such that it is closer to s. So, it is basically giving you an estimation of your s. So, which means this will always be true. So that means, if I have an equalizer multiply with my observation I will get back my estimate value of my transmitted data or the parameter.

The similar way if it is a channel; if it is a channel I create an equalizer, but this equalizer may not be the same equalizer which I have used it in the data case because this W will be a function of s only. Now whatever method you use least square LMSSE or some other method some linear equalizer you build it ok. And then you multiply with the observation you get back your channel.

So, that is the easiest way to do a channel estimation in this case. And once you know the channel estimation go to the third step, you create your data detection. So, that mean your original signal was y equal to this one y is equal to H of s plus v.

So, first you transmit a known sequence s then get y S equal to h cap plus v this is your first time you do once you get your h you reconstruct your the capital H which is your equivalent channel matrix and then you transmit your unknown data and get the data. So, the process would be you first send a pilot set of pilot and make any number of observation.

And then you create your data, then you create your you know channel estimation, once you do the channel estimation you use those estimated value of your channel into the second equation where data will be not a pilot; data will not be a pilot and then you can you know you can do a data detection.

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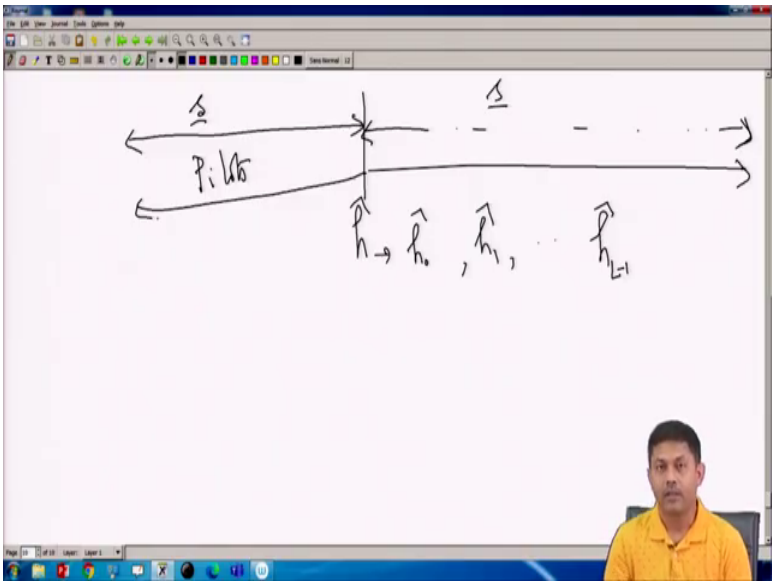

So, the steps can be if you do a signal processing steps the steps can be for a sufficient number of data they will all be pilots right. So, all your s in initially how many such pilots you want, well that depends on your system that depends upon your confidence, how many really data you need to estimate the channel. So, again it depends on the system.

Once you know, once you know your channel, so that mean you first you send your pilots you estimate your channel h cap what is the h cap, h cap basically the edge it will estimate h 0 cap, it will estimate the h 1 cap, it will also estimate all your h l minus 1 cap.

Now, you know your channel, then you use it use those value of your channel for next data transmission. So, here you may not have any channel. Again if the channel will changes what will happen? Suppose during this time your channel changes due to Doppler. So, naturally you have to again re estimate your channel, but how do you know your channel has changed; again that depends on your coherence time.

So, on an average you find out what the coherence time is see if it is a 1 millisecond. So, make sure that this time never, this time never crosses I would say this whole time never crosses 1 millisecond otherwise channel would have changed ok. So, that is the typical way of the fast level of signal processing in a single antenna case.

Now we will move to the MIMO case in the next class ok.

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These are the reference; Statistical Signal Processing, Fundamental of Wireless Communication by Viswanathan.

Thank you.