Signal Processing for mmWave Communication for 5G and Beyond
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Module - 04
Detection, Estimation and mmWave channel
Lecture - 20
Channel estimation and Equalizer

Welcome to Signal Processing for millimeter Wave Communication 5 gen G. So, far, we have covered the channel model extensively and some of the time varying nature and how we have solved the time varying as well as the static one.

Static one is straight-forward, the channel is modeled as a FIR filter right. And the time varying part was more of a time series modeling so, like auto regressive model moving average or ARMA and predominantly, we have said that it can be the auto regressive model that takes the precedence.

So, today we will be covering the channel estimation part for the 6 gigahertz and equalizer. So, we are still not in the millimeter wave to be very frank, but this is required as a fundamental because the same concept will be propagated to the millimeter wave as well.

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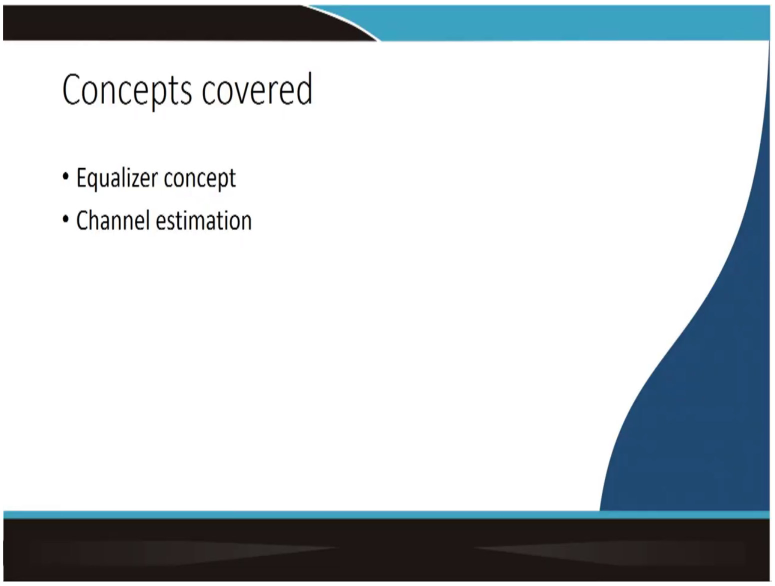

So, the concept to be covered here are the equalizer concept, channel estimation concept, but this is a very general concept nothing to do with millimetre wave or any 6 gigahertz. Now, let us do some signal processing application using such kind of channel model. So, we know what is channel model at the end of the day, right.

Now, what will I do with that? So, what should be my application or what should be my signal processing activities that will be involved in the transmitter side or in the receiver side that needs to be known, because that is the core of the problem here right. So, let us proceed here.

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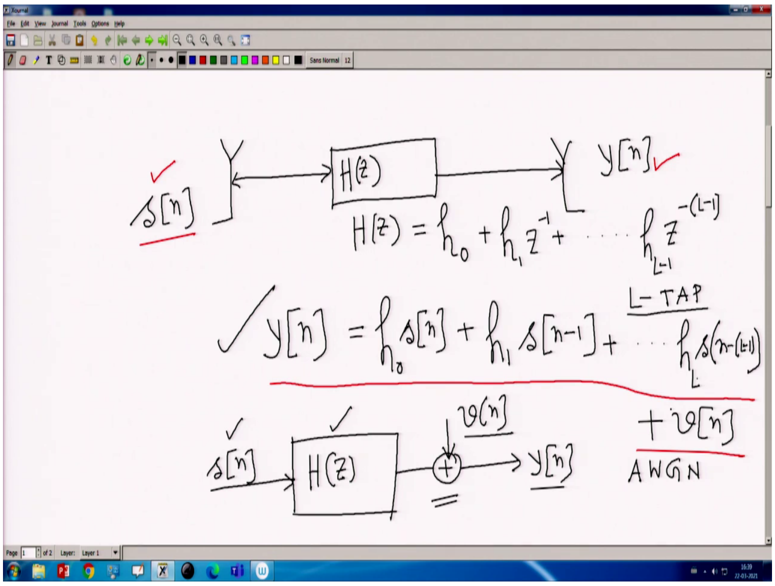

So, which means that I started with a single antenna case. So, I have a single antenna and then finally, we will move to the multiple antenna and we will see what needs to be done and then, we will slowly move to the millimeter wave context, ok.

Now, this whole thing whatever I will be talking about are equally applicable for normal sub 6 gigahertz or any millimeter wave things. So, this is independent of that, now this is few of the applications that we have; that we are going to highlight here. And once this is done, we will be moving to millimeter wave channel models. We have already discussed little bit, but now we will be going deep into that later on. So, some of the applications here.

So now, we have understood that the channel between the antenna to antenna is actually an FIR filter. So, if it is an FIR filter, I can always have some sort of a transfer function. In fact, I should not say this FIR filter is a time invariant in fact, it can be time variant because if there is a doppler, then it really becomes time variant.

Now, let us not worry about the doppler part again that we can handle it separately. So, let us assume it is a static environment; so, the channel is a pure filter which is an LTI filter, linear time invariant filter. Let us say the transfer function is something like H of z; H of z. Let us take for H of z, we can have let us I have L number of taps in this case, these are all digital taps mind it, this is not analog, I think we have explained in details.

What is digital tap? It means that when I sample the actual you know array for analog some analog taps, this is what I am getting it at the output of the ADC as per the sampling. So, if I change my sampling rate at ADC, I will say different number of taps right. So, this tap is the output of the ADC tap. So, I am getting L number of taps. So, L tap channel, this is an L-TAP channel right.

Now, when I say L tap channel, I think you have understood well understood that all this channel do not simply take into account the antenna to antenna behaviour, it start from DAC ends at a DC. So, when I say antenna to antenna, it inherently the inherently it also takes into account the effect of all the DAC, you know filters plus modulator and demodulators, impairments, everything all the effects whatever is present into the whole chain; apart from the digital side, everything is your H z ok. It is not just antenna to antenna, but I may draw it like that just for a simplification.

Now, here as it I am dealing with H z, obviously, I will always take into account the digital data, right; I am not going into any analog. So, everything is digital for me because this is where the signal processing is so, everything is digital for me ok. So, all the you know analog, RF, everything is now everything is now inside this h 0 to h L so, everything is just within it.

So, now what will be my data model? So, it is as if like I am giving x s n to a filter, it is a filter right, it is an FIR filter, and the output is a y n. So, what is the output? Just a convolution so right. So, I can say my y n will be h 0 into s n, it is a convolution say LTI system it is a convolution plus h 1 plus dot dot dot h L minus 1 and s of you know n minus L minus 1, right. So it is a, it is like a delay lines right like a FIR filter. So, this is what my system of; this is this is my system of linear equations kind of I can think of. So, one equation that I can think of.

Now, this is not a signal processing part, this is just the output representation. So, what are the signal processing that I need to do, because that is the intention of this particular talk; that what are the signal processing that I can do based on the channel model right.

Now, let us put effort on that. Now, look at this equation y n is equal to so and so; now, what is the intention of this equations? What is the intention of this trans receiver? The intention is that I will be sending s n, I will be receiving y n now from y n I want to recover what I transmitted this s n; from which equation? From this equation ok. Now, this is a very typical signal processing application. Now, if it is a normal digital signal processing, this is done, but for us the task is different. For us the task is that at the receiver, you do not know what has been sent right.

Say for example, in fact, there is a slight more things will be there so, I will be having a noise also ok. Let us assume that this noise is independent of my data system and that is kind of a you can assume it to be a additive white Gaussian noise so, you can model it as an additive white Gaussian noise why? Because this noise is more of a thermal noise right.

If it is a thermal noise, usually thermal noise are mostly like a Gaussian noise and it should be white in the sense that it can be present everywhere in the spectrum. So, it is a white and this is natural, it is an additive in this case and it is independent also, most of the time it can be independent like independent of with what; independent with s n, independent with channel those kind of things. But there may be some correlations depending on how the systems dynamics and complexity happens.

But in our case, in our simple example that we are taking, we have to say assuming that this noise is a completely additive, white and it is a Gaussian noise for us. And this noise component is very important for us, because this noise makes the whole y n completely the way we extract the data, the way we view the whole system completely different, this noise is so important for us. Now, this is what my data model, complete data model, whatever you are seeing, this is my data model along with the noise, of course the noise has to be there.

Now, what is my job? So, in a simple sense, I can think of this is some sort of a equivalent model of like this. So, you have an H z here ok, you are giving an s n which is unknown to you currently, this can be you know this I am just trying to you know model that system of linear equation so, as if like I am getting this; so, the same thing I am just modeling in a different way. So, you have a filter called H z where one side is your s n and then, later on there is a additive component of the noise added and then you get a data.

Now, what is the job? Job is to get back your s n from the y n and there are two more things that is hidden here, one is that this v n is not known to you ok, because you never know what the noise is right. And this is a white noise so, that mean at every time to time, your noise samples can be different. So, even if you estimate the noise, next time the noise can be different. So, there is no point in even estimating the noise. So, noise is unknown to you.

So, that mean it is like a; it is like a; it is like a garbage, it is like a garbage bin or a kind of a dust bin where dust is basically your v n and within it some gold part called s n sitting there and your job is to extract that gold from that dust bin. So, that is all the job is right. So, this is the signal processing part from the communication aspect so, this extra part, this noise makes the whole thing different.

Now, here two points that come into picture first of all, when I try to try to know what the s n is what about your channel, is it known to me ok number one point. If not known to me, how do I know the channel because what are you doing; you are sitting at the receiver, if you sit at the receiver, you really do not know what is the front-end system right, it is virtually impossible.

So, you do not know s n definitely, at the same time, you really do not know your channel is right, you do not know what, how many reflectors, scatterer at the same time, you do not even know from the transmitter side what are the power amplifier characteristic, what are the DAC characteristic; nothing you will know it right, no impairment no information available to you.

So, the question is that the channel itself is not known to us ok so, when you are at the receiver, you do not know the channel. So, now, if you do not know the channel, there are too many unknowns right, you do not know channel, you do not know v n, now you are saying you do not know s n now, out of all these unknown I want to extrication is it possible? Well, that is a tougher job. So, we solve one by one, the way the communication system works is that it is solved one by one.

So, first is that there is no point in knowing v n, because at every point your v n is different, but channel something that we can do right. Because we have already learned the channel has a coherence time so, what does it mean; it means the channel do not change so significantly for a long period of time depends on Doppler and we have well defined it, what can be the Doppler. It is in the order of 1 by 4 ds right. So, if the ds is small or ds is almost negligible, your channel is like a, for a long time it can be constant.

So, the time of observations like the compared to my ADC speed I am talking; at the receiver so if I sample at say 100 megahertz which is 0.01 microsecond, if that kind of system I am talking of or even a 1-megahertz bandwidth system where my ADC sampling rate is 1 microsecond, channel hardly changes.

So, it is ok to assume channel is constant for wide number of cycles or quite long cycles, but the point here that comes into picture is that even if it remain constant, you have to know you do not know, it remains constant fine, but how do you know what the value is.

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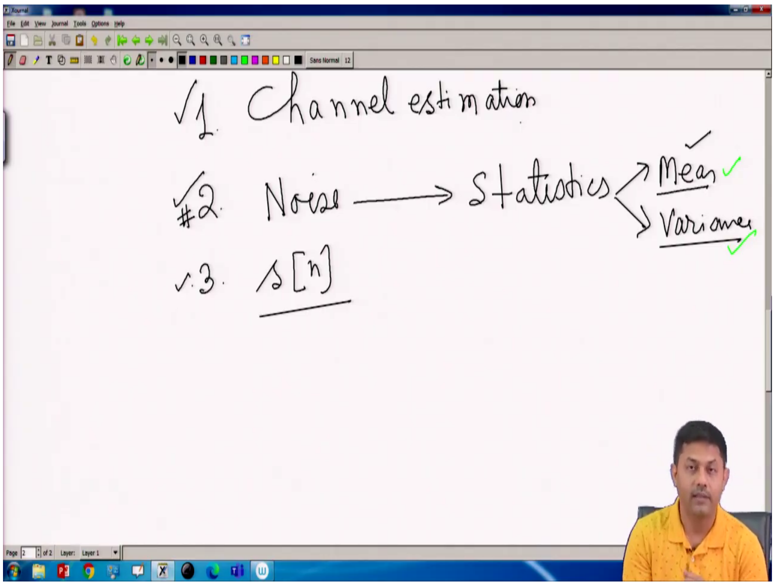

So, the first task that every system encounters is the channel estimation, because you have to first know the channel and that technique will be known as a channel estimation. Though this topics is a detection estimation topics, I will not get into that, but I may show you some of the; some of the data model by which you can do that, the exact techniques can be just; can be just mentioned there. Some of the cost function probably I will explain it in this concept, because the same thing will be used in the millimetre wave as well. When you go to the millimetre wave, we will talk about the same language like what is channel estimation, what is signal detection all these things should be talked about there as well.

So, before I get into the channel estimation so, there are two tasks here. So, one task is that I have to get into the channel estimation, but before that I have to know one more thing noise. Noise as I said, you do not know noise right because at every sample, your noise can be different not like your channel; channel act sample to sample, they do not vary provided the

Doppler is manageable. Even if the Doppler is high at least for how high it could be, I mean you know what is the maximum speed that that on earth you can travel, right.

Even if you talk about say bullet train, it would not like a speed of light right, it will never be. So, when it is not like that kind of range so, you can assume that the bandwidth that you are dealing with, it can well handle that channel coherence time. That means, the coherence time is much much longer than your ADC sampling rate. So, that is the our key assumption here ok.

So, before I do anything channel estimation anything like that, I will do first know what is the noise part, but the unfortunately noise changes at every sample to sample, it is a white noise, right. White noise meaning at every frequency it is present and at every time that channel can sorry; the noise value can change it.

So, it is virtually impossible to estimate unlike your channel, it is virtually impossible to estimate a channel sorry a noise in that case, the remedy would be to estimate the statistical parameter of it. Because if you look at the kind of equation that we deal with, we always deal with statistical phenomena's right, because in our case, h 0 to h L minus 1 under tap just statistical quantity. Then, data s n, s n minus 1 can also be thought of a statistical quantity for us at the receiver level because I do not know what has been transmitted, right.

See if I do not know something, what is transmitted I can assume it to be a statistical quantity as well. Similarly the noise, and we are already assuming this is a Gaussian noise, what have been its distribution itself is like a statistical quantity. So, for us everything is statistical. So, under that scenario, we have to think in a statistical language right.

So, that mean I do not know the noise well, no problem, may I know its statistics because that is important for my activities moving forward. So, instead of estimating the noise, I want to know its statistics; I want to know its statistics. What is the meaning of statistics? PDF can be a statistics. If I know PDF, I am done.

Now, we have already assumed that noise is a Gaussian for most of the communication system. The kind of communication system we are dealing with, we safely can assume noise thermal noise is just Gaussian noise.

So, which means that for a Gaussian noise what I need? I need the mean and variance that is the only two quantity I need to define a Gaussian PDF well. So, which means that for this noise if it is a Gaussian, I just mean it is mean, I just need the mean and its variance; that is all, this two quantity I need it right. Question comes that how do I know the mean and variance right, ok.

Now, 3rd point. Once I know my noise statistics, once I know my channel values, then only I can think of detecting my data, right. So, this will be my third priority for us, but first, I have to know the first 1 and 2, after that 3 can be done.

Now, the question comes how do I know the mean and variance, is it such a big task? The usually for noise statistics to know, this is one of the easiest task in a communication system right. And there are many estimation techniques for noise, I am not getting into that but trust me that is one of the easiest technique.

For example, say I am not doing any transmission, say I am just transmitting a 0; so, what does it mean? The only thing that I will be receiving is the noise right, in this equation, look at this equation that red marked equations here. See if I do not transmit anything that is kind of a null time right that mean I am not transmitting anything.

For example, suppose you are speaking on a mobile, it appears that you do not speak all the time right, the whole 24 hours it would not be speaking, you may be speaking for a while after that you know your mobile can be just sitting idle. There may be some data transaction going on, but usually it can be idle right.

So, when that kind of scenario happens, when you know in null time, I would say that there is no data transmission. So, what the receiver can do at that time maximum? It can just if it wants, it can keep on sampling the data at the receiver; what will it receive? It will receive only noise nothing else it will receive right because there is no data, there is no channel so, it will receive only noise. So, it takes a large amount of noise sample ok.

So, if you need; if it takes a thousand noise sample, pure noise sample it will get in that case right, if there is no data, then what it can do? It can extract all the statistics from the large data. What it can do? It can estimate the mean. How to estimate a mean? For a large data you can just take an average simple method, or you can do some sort of a squaring and then, take an average. So, you can get a variance, you can get. So, these are the different techniques of estimating the spectrum of the noise, I am not getting into that.

So, my point here is that these two term; these two term are easiest quantities to be available to the system ok. So that means, these things can be given. Second thing that is important for us is that given the second number 1; number given this point number 2; how do I get into the channel estimation part?

Now, again, this is an estimation topic I am not getting into that, but I want to create a framework where you can think of a channel estimation, you can think of a data detection both possible in the same framework. So, let us think of that framework first ok.

So, to create that framework as it is a single antenna system I am dealing with currently and it is a channel with a multiple taps. So, let us assume just for the time being that I know the channel, somehow ok I know the channel. Once I create the framework, then you can you know create some mathematical manipulation and do some channel estimation framework. So, because this number 1 and number 3, the framework remains the same.

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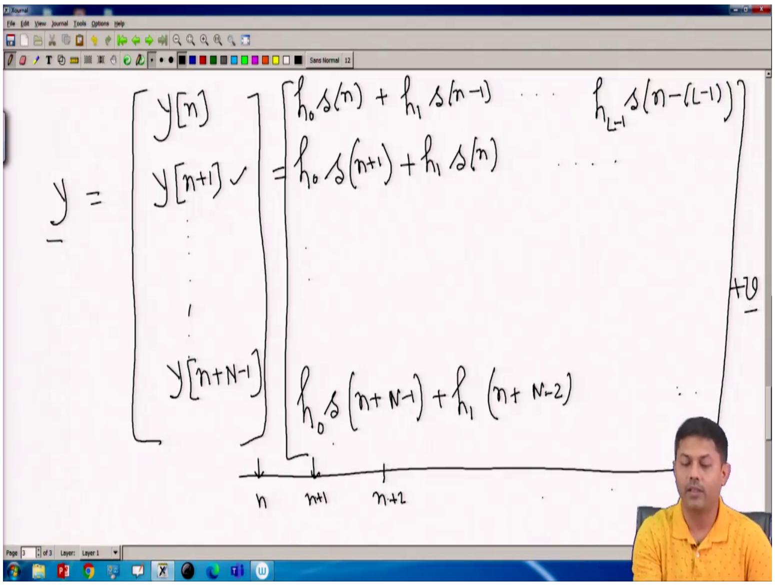

So, let us assume that I know the channel, just for the time being you assume it ok. If that is the case, I can create some sort of a data model. Now, we know what is y n right. So, let us say I know y n, may I know y n plus 1? Of course, if I know y that is an observation right, I know y n that means, n-th point observation, this is my n plus 1 point observation so like that. I can take lot of such observation.

So, let us say I take, let us say I take N number of observation, straight-forward. So, you have the ADC, this is my n-th sample, this is my n plus 1 sample, this is my n sample, this is n plus 2 sample and so on so forth. I just take the observation point and I keep it in my memory.

So now, let us say I stack it. What is mean by stack? Stack meaning I just put them together ok, I just put it in one, it is kind of you can think of a row, mathematically I can say this is kind of a column not a row, column in a matrix form so, I stack it. Let us call that as a vector, this is a DSP, this is what the signal processing in the communication.

So, what can I think of? What can I think of here? Go back here, what is the data model here; h 0 s n, h 1 s n minus 1 and so and so forth, this is for the data model. Now, for your simplicity ok, let us go into here. So, for your simplicity, let me write the equation, then it might be easier for you to understand here.

So, the first one, the y 1, what was the y 1; y 1 was h 0 s n plus h 1 s n minus 1 and h L minus 1, this is the case correct. For the second one, what is that? You just put you put as per whatever observing here right. So, this is h 0, this will be n plus 1 right plus this will be s n will be there right and accordingly, it will be adjusted right. So, whatever here, it will be plus 1 here, ok.

So, what about the last one? Last one will be s of n plus N minus 1 right plus h 1 of one level will be down plus n minus 2 and so and so forth right. This is how, and there will be up to that you can adjust it what with the indexing. Plus I will assume this is a vector noise because the noise is like v 1, v 2 and so on, I just stack it together and make it like a vector. So, what will be the.

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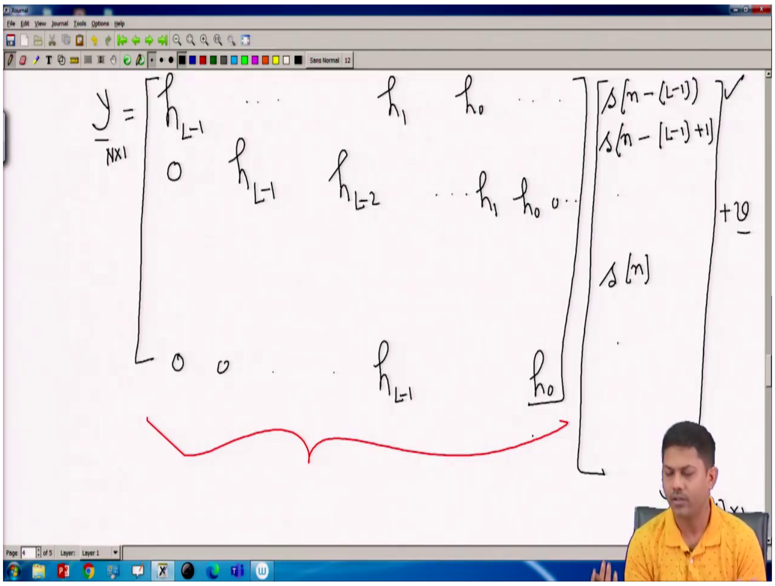

So, if this is the equation, how can I represent it? So, I can say this will be some sort of a y bar, what will be the case here? I can think of some sort of a Toeplitz matrix in this case. So, can I build some sort of a Toeplitz matrix in this particular case ok? We can do that, definitely we can create a Toeplitz matrix in this equation.

So, I can just think of it, h L minus 1, then it will be going like a h 1, h 0 lot of 0 will be there, and from here it starts from s n minus of L minus 1 and then, it will be s n minus of L minus 1 plus 1 and so on, index will be increasing here right. At some point of time, you will find s n and so on so forth ok. So, this will be the data plus it will be some vector will be there.

What with the channel matrix then? This will be the case h L minus is just plain simple application of linear algebra whatever you can write it here, whatever is shown here, the same thing you can see it here ok, then it will be h L minus 2, h 1, h 0, 0, 0 with that, it is like a shifted like a Toeplitz matrix that is what we are saying it right. The next will be again double 0 and finally, at the end, you will be having at some point of time h L minus 1 and this all will be 0's; this something like that right.

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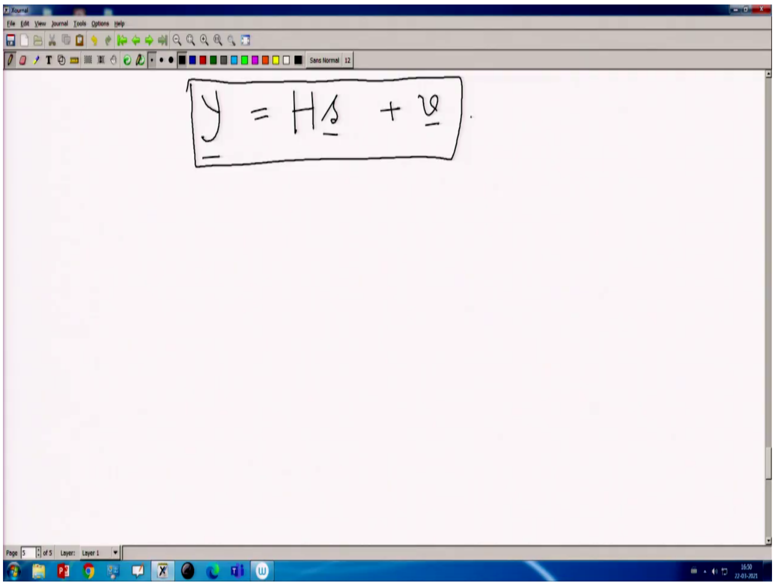

Now, this kind of form is very popular for all our communication system, this is system of linear equation, just think about it. What am I getting? Left side, I have a observed vector, right side I have bunch of matrix and vector so, can I write this whole thing as y bar is equal to some equivalent matrix called H. What is that H? In this particular case this whole; you know this whole matrix and it is a Toeplitz matrix right, as you can see it is a right shifted. You can form one row and then it is getting right shifted ok.

So, now, can I think of it some sort of a system of Linear? Yes, this is what is coming s bar plus v, this is the system of linear equation. What is s bar? This s vector that is your data so, that is nothing but this vector. So, it start from here and ends here. So, what is the dimension of it? n plus L. So, you have L data extra and then, you have n x. So, this will be n plus L cross 1 that many number of data is present ok. And accordingly, see if this you have N here so, this will be N cross 1 dimension and this will be adjusted accordingly N cross N plus l and so on so forth. So, that adjust dimension will happen at adjusted accordingly.

So, this is the key part. So, that means, this is the start point for our signal processing, this should be very clear. So, this is how I can create and this signal processing you know this equation is so important, it gives rise to most of our analysis, most of our you know detection estimation algorithm everything comes from this system of linear equation ok.

So, this equation can be somehow formed and this is you can think of, though it is a matrix vector involved, but whole thing I am just making it using single antenna. So, here what is what my key point here? I am observing multiple data at my time.

So, I can do something in the time, I can do something in the space, I can do something in the frequency, but this is a simple observation where my; where I am observing the data at different instances of ADC and trying to create some sort of a you know some sort of a system of linear equations. Now, once I am given this kind of equation, I create wonder out of it ok.

So, next class, I will talk about how a data can be extracted, how exactly a channel can be estimated and after that we will move to millimeter wave. So, this is the fundamental equation that or a signal processing first part of my signal processing in a communication system ok.

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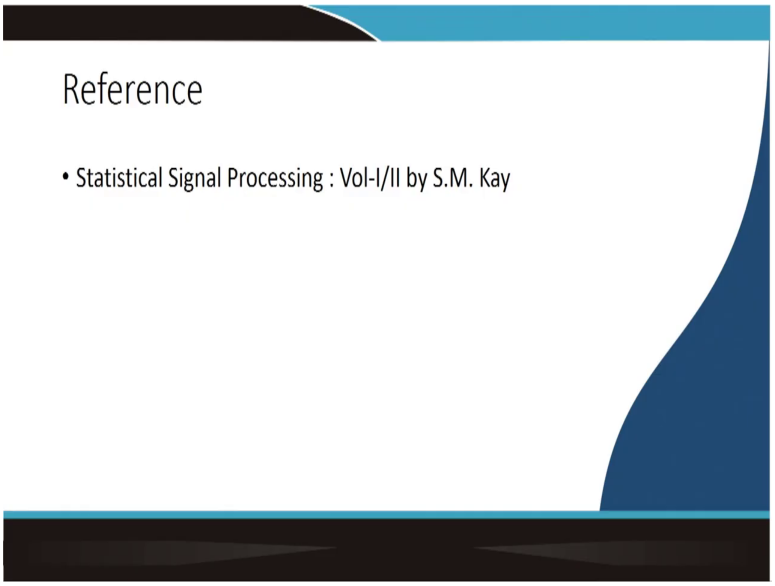

So, this is the reference.

Thank you.