Signal Processing for mmWave Communication for 5G and Beyond
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Module - 03
Understanding of various channel related parameter statistics. Narrow band and broadband aspect
Lecture - 18
Coherence time and parameter summery

Let us continue the discussion on the channel modeling part from the doppler point of view that means, there is a time varying channel let us discuss that the lecture number 18 on Coherence time and parameter summary.

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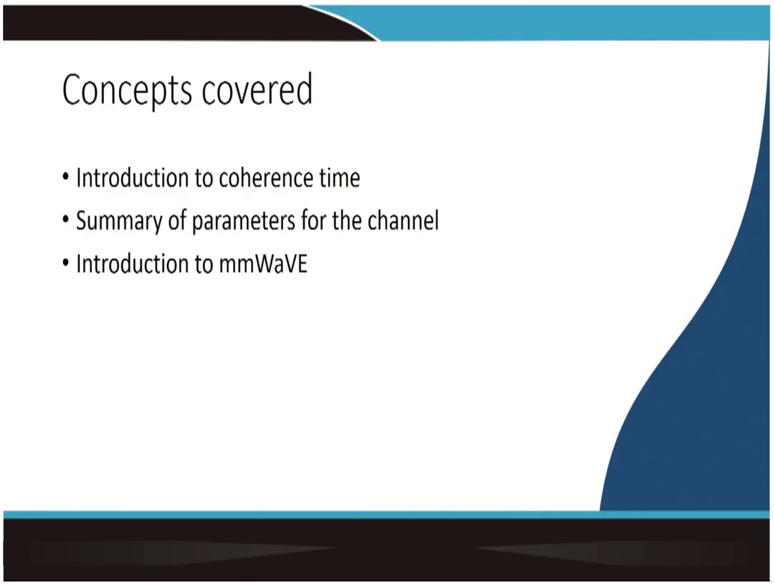

So, these are the concept which will be covered today introduction to the coherence time and then, we will be summarizing all the parameters of the channel that we have learnt so far and then, the basics or the introduction to the millimeter wave.

So, we have learnt that a channel tap can be modeled in a auto-regressive manner. You can model it using MA, you can model it also using ARMA process, but it all depends on the measurement. But the problem that we thrown, we have thrown in the last class was that how do you determine the coefficient that is the central point of the whole problem because unless you know the coefficient, you cannot really determine the correlation ok. So, is there a way to do that? Yes.

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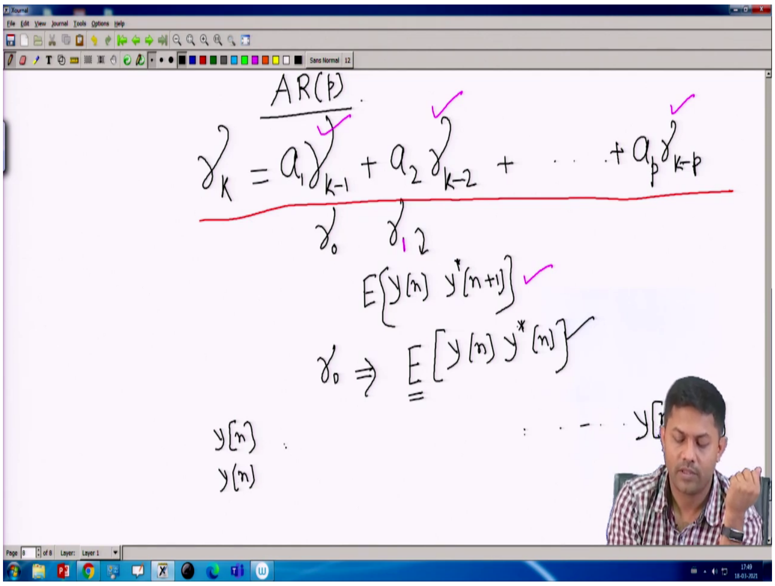

So, let us take a very simple example, let us take AR process for our explanation. Let us say I have an AR process, AR p process ok. So, we have already learnt how you have already got the auto correlation function. So, what was the auto correlation function definition? Gamma k if you look at it is already written here.

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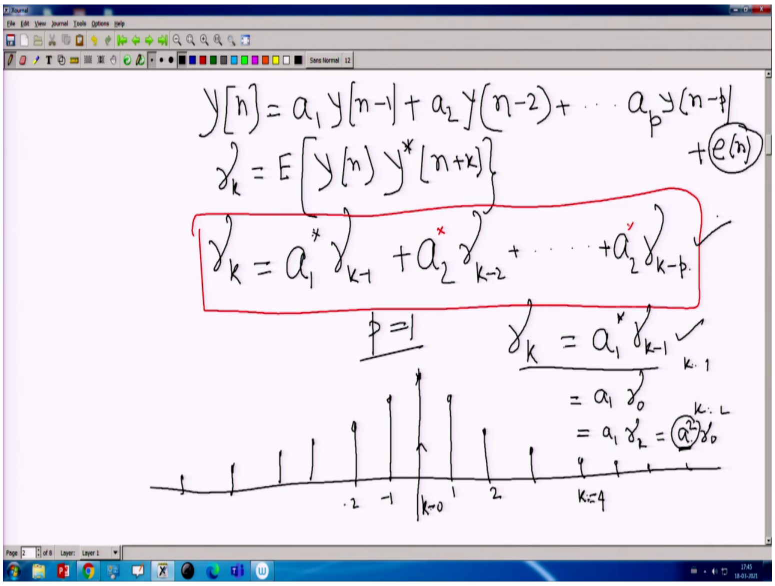

Yeah so, this is the case right this equation, you look at that equation that was the standard AR p model for AR p model that is the auto correlation function ok. So, I take the same thing, I just copy the same thing to the to this class, I just take that. So, let me just write it down. So, this is gamma k a 1 gamma k minus 1 ok yeah. So, this is a 1 let us not worry about the real or imaginary complex all this thing, let us assume for the time being this coefficients are real which is a; which is a fair assumption.

Unit can be I mean y n can be a complex, but coefficient need not be complex, it can be a real also let us assume for our simplicity it is a real number. So, this is gamma k minus 1, this is a 2 gamma k minus 2 ok how much it goes? Say this is a p gamma k minus p. So, this is the original equation ok, this is the original equation we know it, but we do not know what we do not know is the coefficient ok fine.

Now, let us a form some sort of some set of equations and try to see is there a way we can do or we can obtain some of the points or some of the data from measurement ok. Now, what is gamma k? Gamma k is a correlation right. What is gamma 0? Gamma 0 is the variant which is the 0th coefficient that means, that is nothing, but the variance of the sequence.

What is gamma 1? Present with respect to the one step ahead or one step back that coefficient that mean, y n correlation with y n minus 1 or y plus 1. If it is stationary, both are equal. So, that is y 1 sorry gamma 1 that is that represents the gamma an expectation of this quantity will be representing the gamma 1 ok.

So, now, when I look at this equation, when I look at this equation, what is my focus? My focus is to get back my coefficient, but along with the coefficient, what I am missing here is that what about these points, can I get, or can I obtain the correlation components not the correlation the auto correlation component by some sort of a experimental basis ok and the answer is yes.

Thus, what is gamma k; what is gamma 1? The gamma 1 is nothing, but this quantity right if it is a 0 mean process that is it that is the problem. What is gamma 0? Gamma 0 is expectation of y n y n and so on and so forth. So, if you have a sequence of y n to say y 100, I have a sequence from here I have a sequence of y n plus say 1000 many such sequences I have it ok so, it is a long sequence I have it from that sequence, can’t I get this?

Yes, if somebody gives you a sequence, cant I get the expectation of y and y star n? Yes of course, I can get it because you have the data, you multiply it, add them up and just take an average, you cannot get an expectation where expectation meaning you need a; you need a statistical tool to determine that.

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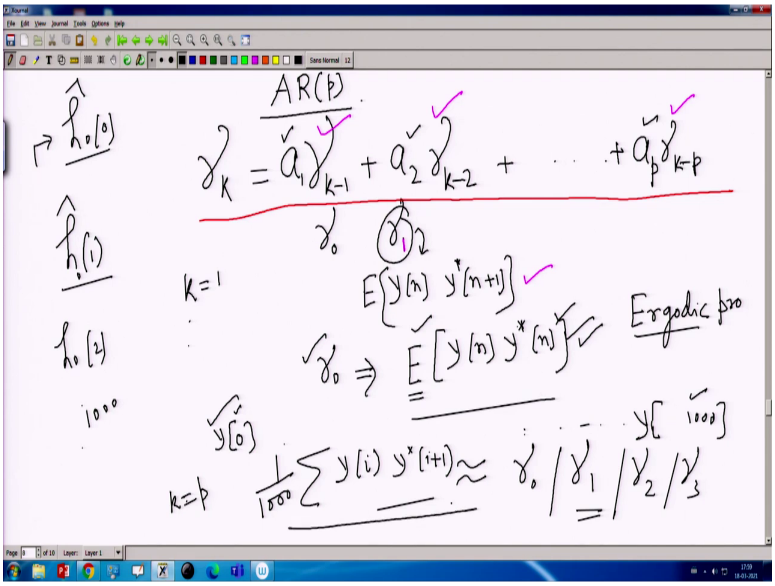

But from an experiment point of view, what can I do? I take a sample, I just multiply with its own sample that mean if I have say I have for simplicity, let us not worry about this n I mean here say this is 0 and say this is 1000, I have 1001 data can't I get the variance of the sequence if it is a 0 mean process? Of course, I can get it.

So, what do I do? I just take each an individual sequence value, take summation, divide by as many as numbers I have it 1000 or 1001 whatever, I tentatively get what is that? Gamma 0, this is a very very crude way of getting your expectation.

But probably there is one underlying fact is that this should be an ergodic process, this should be an ergodic process otherwise it would not hold true so, where the statistical average is same as your ensemble average or time average. So, I assume this is an ergodic process.

So, I can given a sequence, I can get the gamma 0 because just this just follow this equation, you get a number 0. If you take large number of data, asymptotically this will be almost equal to the variance provided this is an ergodic process.

Can I get this then gamma 1? Definitely, I can get it. Why? Because I know the sequence. So, if I have the sequence instead of i just make it i plus 1 here, do the same process what will you get? You will get gamma 1 right instead of here, you just let me make it slightly clear this should be the case, do this operation, you have the sequence, do this operation you get gamma 1.

Similarly, you get gamma 2, similarly you get gamma 3, now how accurate would be that depends on how many data you have taken so, you get that. So, what my pointer is that you can get the estimated value of your correlation point that auto-correlation values for different value of k you get it provided you have a huge collection of your estimated value.

Now, in this case, if it is a channel, it is not a big issue to estimate the channel in a normal way right. If the channel is something like this h 0, then h 0 suddenly becomes say this is at time 0, then this is at time 1, then this is at time 2 and so and so forth if the channel is moving like that, I can do a normal estimation; normal estimation I do not have to do any tracking, I just do a channel estimation here, I do a channel estimation here meaning I use say 100, 200 pilots and I each and every time I re-estimate my channel freshly.

So, I get my channel data. So, like that. So, I get my y. So, y is the channel in this case so, I can use a 1000 pilots or 100 pilots right. So, I if it is just a say simple data, I instead of a normal unknown data, I serially send say 1000 you know 1000 pilots data, it is a single antenna to single antenna, what will I get for each and every pilot?

I can estimate my channel, I can first pilot comes I can estimate my channel, second pilot come estimate my channel like that 1000s say pilot or maybe 100 pilot, you do not have much time say 100 pilots I give you, 100 channel estimation I can do. Similarly for h 0, h 1 all the you know taps I can estimate it. So, that is not a big issue. So, you can estimate the channel using pilots. So, that can be done one time ok.

So, using those data's so, that mean you have the estimation of your data and using that, you estimate re-estimate the correlation. Now, you can always ask what if there is a estimation error in the channel? Well, we are not determining that part. So, it is a chicken and egg problem kind of. So, you estimate the channel, using the estimated channel, you are further estimating its correlation so, that is a error over errors ok.

So, that we are not dealing with. So, we are assuming that system has a good SNR that mean whenever you send the pilot, probably you just increase the power so that there is a good SNR under that you do a very good fair job of channel estimation.

So, if there is a fair job of channel estimation, there will be a fair job of correlation, this correlation factor this gamma k estimation also be done fairly. So, that mean my point is this is known to us kind of relatively a better position we have.

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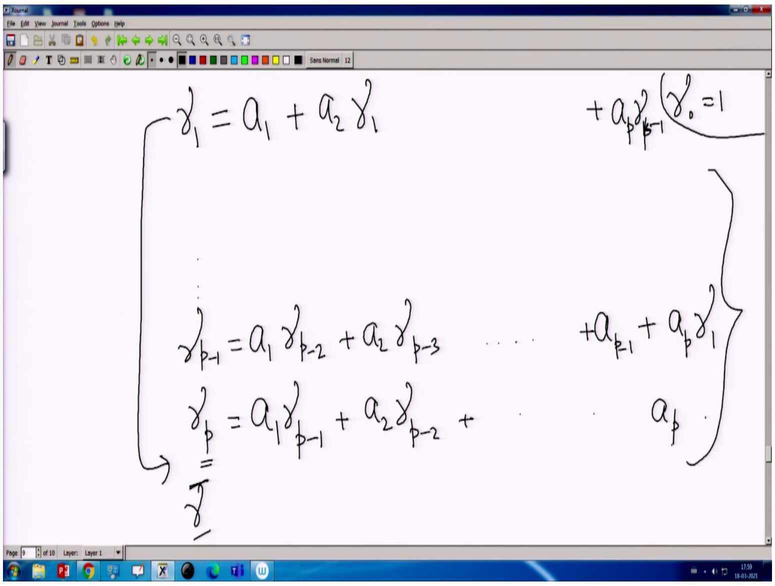

Now, my job is to what? My job is to create or my job is to estimate this fellows. So, what I do now? I just put this equation. So, in this equation, you just put k is equal to 1 to k is equal to p keep putting that term ok. So, what will I get? Suppose I put k equal; gamma equal to p so, this is my gamma p, what will I get, just I put k equal to p so, what will I get?

A 1 gamma p minus 1; a 1 gamma p minus 1 plus a 2 gamma p minus 2 plus dot dot how much it can go? Gamma 0 well, gamma 0 we can just assume it to be something gamma 0. If you say this is an autocorrelation and it is a normalized auto-correlation, you can assume gamma 0 is equal to 1 safely, no big deal ok. So, you can say this is just 1, just 1, gamma 0 you can assume 1 that is ok because if it is just a normalized auto correlation function, this is what is coming fine.

Similarly, I can have a gamma p minus 1, what it would be? You just replace the value here ok. So, it would be a 1 gamma p minus 2 plus a 2 gamma p minus 3 it will come plus a p minus 1 nothing will be there, but at a p, you will have gamma minus 1 which is equal to gamma 1 so, this will be gamma 1, this is what it is right. Similarly, you go up to gamma 1.

So, what I am trying to; what I am trying to build up here? I am trying to build up the correlation up to my known autoregressive model. So, we will let us assume that I have assumed the channel has a AR p model so, I am just trying to build the gamma p like this equation, it is not a impossible to build it why?

Because I know all the gamma so, that means, from gamma 1 to gamma p, these are all known quantity to me just the way we have explained it, you can if you have a large data, you can fairly do a good estimation with that.

Now, what about gamma 1 then? Gamma 1 would be you just put a value 1 here, what would be a 1 gamma 0 here so, it will be a 1 will be coming into picture plus a 2 gamma 1 will be there and so and so forth finally, it will be go here a p gamma k minus 1. So, this will be a p and gamma k minus 1. This is some sort of a equation that I have built up.

Can it be a system of linear equation? Look at that yes, this is a system of linear equation. I have k number of equations sorry p number of equation, if my order of AR is p and I have a corresponding equations for them. So, can I do some sort of a matrix operation on that.

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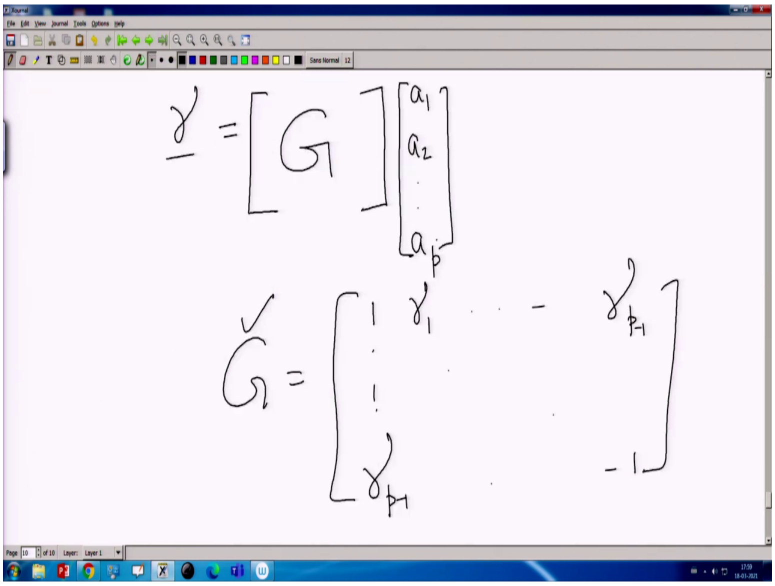

Let us say I express this gamma this whole thing as a vector, this whole thing; this whole thing as a gamma vector. Now, notice that everywhere I have a 1 to a p everywhere, you see every equation has a 1 to a p. So, I can express this whole thing as a matrix multiplication that is precisely, I will do. So, this one I will express it as some matrix G, but a 1, a 2, a p.

What is this G matrix? G matrix will contain all these coefficients so, it will be something like if I just look at these equations 1 gamma 1 to gamma this should be p minus 1, 1 gamma 1 dot dot gamma p minus 1 and then, it will be last will be gamma p minus 1 1 in between other elements will be there.

We just look at these equations; look at these equations and you can build up your G. So, G is known. Now, you see G is a known matrix to you why? Because you know every gamma because you have estimated how you have estimated? Using this concept, using this is an experimental way of estimation these ones, experimental way of estimation ok.

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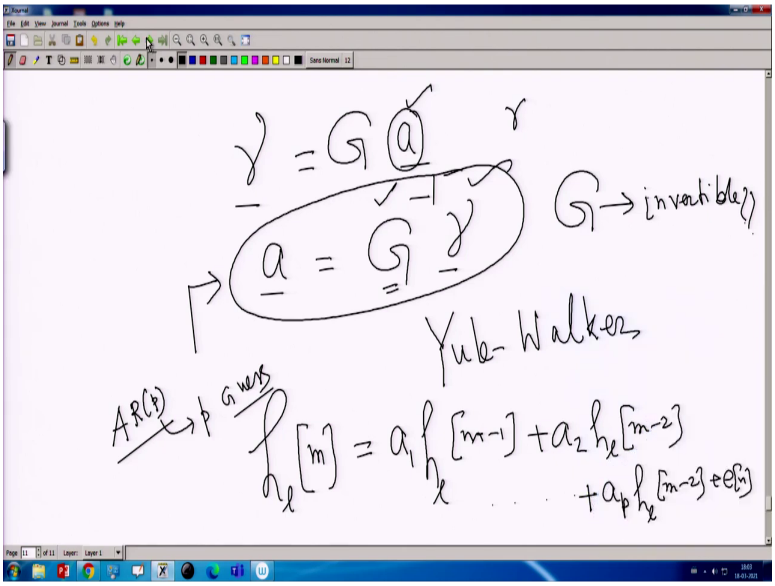

Now, I got it so, can I write it like this? So, can I say this whole gamma which is equal to G into some a vector, system of linear equation ok. So, what is my job is to find out a that is all that means, this is known matrix, this is a known matrix, I now know what is my coefficient, what my coefficient would be, I know my coefficients.

All these coefficients can be known easily provided I do the exercise of estimating the correlation so, that part has to be done first and that is also not a very tough job because you can experimentally do that ok.

So, this is a typical method, and this is also known as Yule-Walker equations, these equations are called Yule-Walker equations basically it is a recursive equations. So, when you have this kind of recursive equation, you can easily create system of linear equations, but there is one again a catch, how do you know your G is invertible?

How do you guarantee that your G is invertible? There is no guarantee right, it all depends on how good you have done an estimation probably, you may have done a good estimation, but you do not know whether finally, it will come as a invertible or not. So, that is a of course, an open problem.

If it is not invertible, then you have to you cannot just take a G inverse in this case, then you have to do some sort of a pseudo inverse or at least you may not get all the coefficient properly because it is a system of linear equations right. So, it means that this does not have a full rank so, in that case what will happen?

In that case, you if the rank of this matrix becomes r, then you can estimate or you can probably get the values of a only for r elements rest of the elements will be like a dependent element, independently you cannot estimate them. So, it is a; its a system of linear, whatever system of linear equation guide us, you take it, but the main focus is that you can easily get the coefficient.

So, if you know the coefficient, you are done, then you can you know you can for each and every channel; for each and every channel you know all these coefficients say you know the coefficients so, now, you can just happily put all values that we have got it here, we can get. So, this is AR p model of your channel. So, you assume a value of p and you can estimate it.

Now, another question come from any questions that can appear, how do you guess the p guess? How do you guess the p? Because you do not know p right, unless you know the p, it will be difficult so, this is called a model order estimation that is again another thread of discussion, I am not getting into that, but we assume in this particular case that I know p ok.

So, this is the modeling part of my channel so, that mean, if I have a doppler, I know how to model it from an auto-regressive point or from a time series, any time series model that I want to do it ok.

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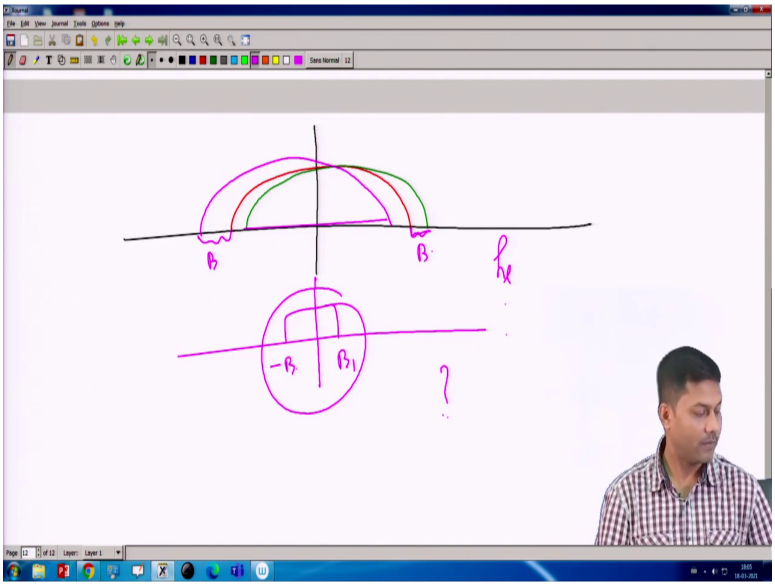

So, from that I know how to get an autocorrelation function, how to get a doppler spread or doppler frequency that I want to get it. So, that mean if I go back to my original discussion ultimately if I see my channel spectrum, if this is a static spectrum, this will be some a fixed spectrum. Moment it becomes a doppler, what will happen? It gets slightly shifted either in this right side or it can be shifted into the left side.

So, the amount of shift in the right or left is actually your doppler spectrum ok. So, that means, if this is shifted say B 1 here, if this is shifted by B 1, your doppler shift I can think of it is B 1, this is minus B 1 so, that is the doppler thing that mean, if I take h l and create a time series model, the spectrum of it would tentatively give me this kind of spectrum not this kind of spectrum ok. So, that is the idea here. Now, this is from the modeling point of view.

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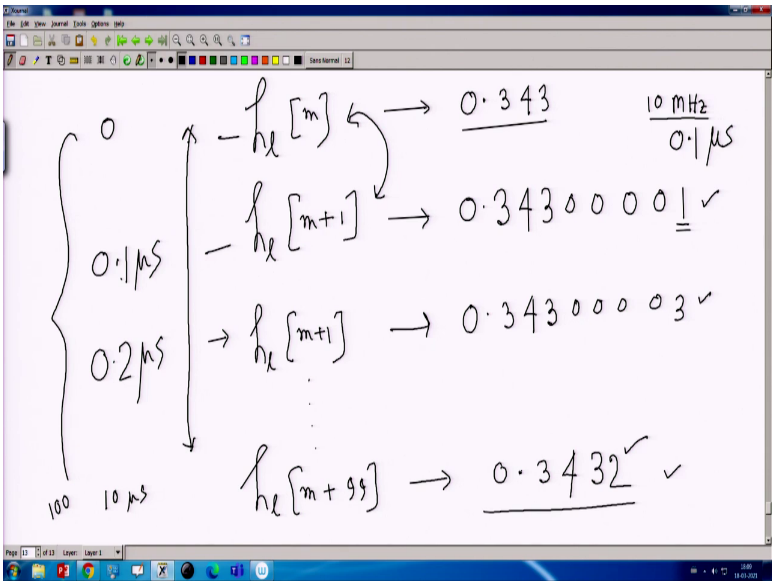

Now, the question is can I guess this value? Because from an engineering point of view, it is always; it is always useful to guess, can I guess it, can I guess how much this should be ok? That is one question or there is one more question that I asked some time back some maybe 3-4 classes back is that how the changes of the channel over time important to me ok?

What I am trying to say is that the channel is changing say I am taking the lth tab at time m next ADC sample is a h l m plus 1 and it has changed because that is a doppler. Now, it may happen that this change may not be significant for you ok.

Let us say; let us take an let us take a simple assumption let us say this is 1 point the channel tab value is say 0.343, I am observing that lth tab has that value. Let us say in the second case, second sample and let us say my sampling distance or ADC sampling at say a 10 megahertz, at 10 megahertz at 10 megahertz my ADC sampling so, that means, 0.1 microsecond is the distance between this sample and this sample on a times real time. If I see the real time, when h l come after 0.1 microsecond, the second sample comes ok.

Now, let us say my doppler is not so significant or I do not know what is the doppler, but I see the value is like this, it changes, but like this order it changes, its changes I mean you can well model it using auto-regressive or any other time series model, but the point is that the changes is so small that you may tend to ignore it ok.

Let us say I have one more say this is my 0 microsecond, this is at 0.1 microsecond, this is at 0.2 microsecond and so on. Let us say this one is also not so significant change so 3 ok and so on, but I see that if I go on for say probably 100 samples ok that mean 10 microsecond, around 10 microsecond now, I see that the h l value has some significance changes. So, probably this becomes I do not know, but let me just probably this has come after say 100 samples ok. Now, this is the reality, this kind of changes.

Now, it may happen that this change value, this much change value create some disturbance in your performance because you may be assuming that ok 0.343, but you actually see that whenever it makes this kind of changes, it may not impact your performance or if you even plot it, you would not even see it I mean the digits are so small, I mean it is kind of coming at a you know how much? 7th decimal hardly it has any impact on the performance or you cannot even represent it by you know the same number of bits that you already have it.

Suppose this is represented by say a 6 number of bits, this probably you cannot even represent it, but channel has changed this much. So, the quantization the illusion that you have already taken, this will just miss it because it may happen that you would not even, you do not get in I mean it would not impact any of your performance such a small number, but however, here, there is a significant changes that has happened and that can impact my performance.

The question that comes into picture how long I should consider the channel does not impact my performance that is the parameter that I need to do design parameter that is coming right because it is a very critical decision, why it is a critical decision? Because if the channel do not change significantly, why should I do a channel estimation?

Suppose channel is remain constant, it remains constant for say 1 hour, do I need to do any channel estimation for 1 hour? No, because its constant. So, I need not to do any channel estimation or if it changes like this kind of changes 0.00001, do I need to do any channel estimation for that? Probably not ok.

So, even if you have a small you know walking, the channel would change however, small it is ok, but point that we are trying to make here is that is this change significant or do I really need to say that I ignore that change and how long I should ignore it ok so, can I quantify it, can I quantify that change or can I say that ok beyond this point, if the channel I mean I would say channel changes significantly now, I cannot ignore it anymore like this kind of things. So, probably after 100 cycles, I probably may not be able to ignore the data.

So, now, let us give a formal definition of such kind of points ok. Now, this this definition that mean the time throughout which the channel do not change significantly. I never say the word channel do not change, channel will change however, small it is. The moment you have a doppler, moment you have a you know however, small velocity you have, even if you move at 1 millimeter per hour, channel will change after 1 microsecond theoretically.

Now, whether that is significant or not that is a secondary question, but it will change. So, I cannot say channel would not change. If there is a doppler, channel will change. The question is it impactful? So, that is what I bring in a definition that a time throughout which my channel do not change significantly ok. So, that is a new definition for the channel parameters.

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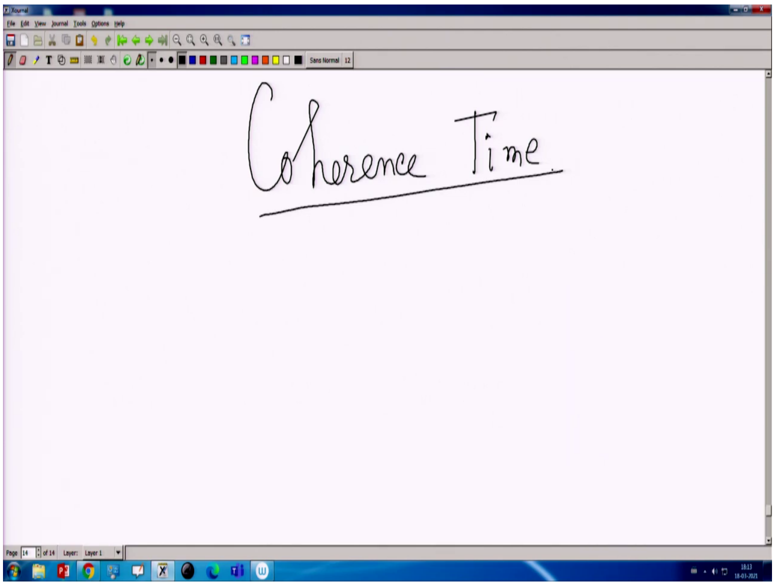

And this is called coherence time, it is a coherence time because it is the time over which channel do not change significantly. Now, what is that significant change? Is it 1 percent change? Is it 2 percent change? Well, that is; that is a different issue, but it do not change significantly and also, what about the application?

Suppose I am just doing a plain low bandwidth wise versus a case where I am having a say 4k video streaming, both needs a different data rate, one is a very small maybe 2k-3k good enough another is will be in the order of Mbps data rate, but let us say in the both the cases, I have the same amount of doppler.

Now, in the first case, even if the channel changes significantly, probably I may not care, but in the second case, I have to care.

So, when I say coherence time so, it has two-dimension of definition, one is that what is your application, is the application demanding a channel to be saved so, one percent change meaning that application will fail is it like that or this is one point or is it really making my changes significant such that you know channel its parameter completely changes, completely different and so that I cannot track anything.

So, this coherence time depends on both the aspect, changes as well as the application because application demands how much change you can tolerate it ok. So, we can in the next class, we will give some formal definition on the coherence time, but I am not bringing the application aspect right now just a formal definition. With this, I conclude the session here.

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So, this is the book same Fundamental of wireless communication book by David Tse and P. Viswanathan. I will be talking about the formal definition of coherence time and the complete concluding remarks for the channel models whatever we have done so far except the millimeter problem after that we will start the millimeter channel.

Thank you.