Signal Processing for mmWave Communication for 5G and Beyond
Prof. Amit Kumar Dutta
G.S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

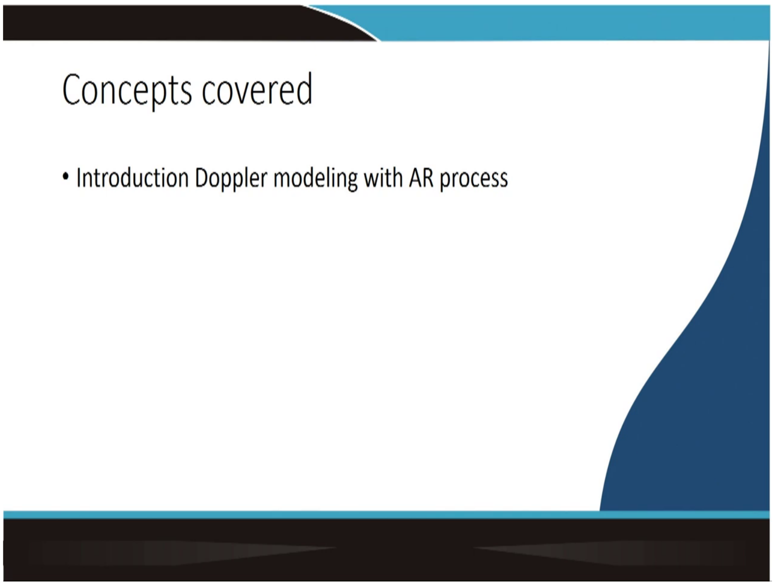
Module - 03
Understanding of various channel related parameter statistics. Narrow band and broadband aspect
Lecture - 17
Doppler with AR process model

Welcome. So, we were talking about the channels model in time varying case, right. So, if there is a Doppler what happens to the channel, and we tried to model it using the time series. And then, we defined 3 time series model, one is a autoregressive, the moving average, and the mixed ARMA. And then, we have defined the spectrum, how to find the spectrum.

Now, today we will give some more example on those time series, and we will also show you instead of going through the spectrum directly through the backward operator, how you can proceed to find the spectrum from the autocorrelation function because that is also important. And it gives you certain information, this autocorrelation function and that why it is very much required for your simulations or for characterizing the channel natures and the time varying natures.

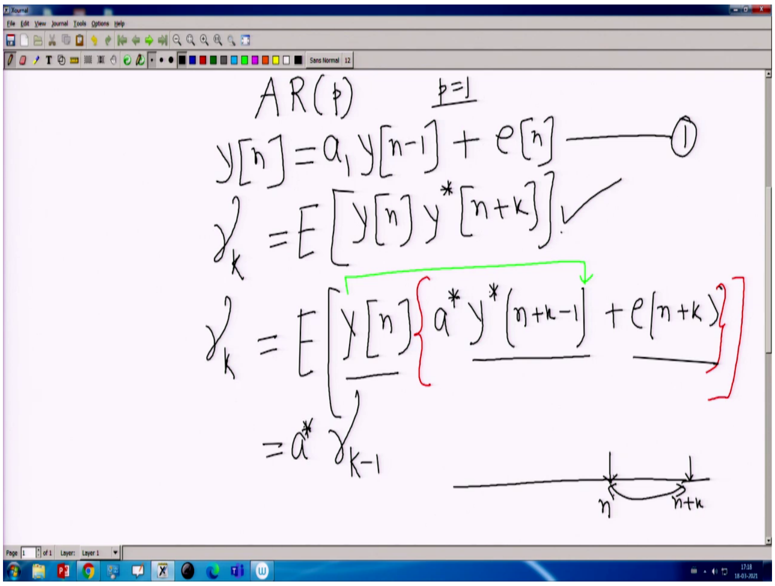
So, we will continue that, a module number 3, which we will be talking about the Doppler with the AR process model, ok. That is the concept which we will be covering today.

(Refer Slide Time: 01:35)



It is a very important concept because we will be talking about how exactly a Doppler can be mapped or modeled with a autoregressive process.

(Refer Slide Time: 01:53)



So, let us say I am taking the autoregressive model. So, autoregressive p model, ok, AR p model. And we will try to find out how you can estimate or how you can determine the exact coefficient and how you exactly estimate the complete autocorrelation function. So, let us say my sequence is y n, let us say its standard y n sequence.

And let us define it say I have a 1 y n minus 1, ok. And for the time being let us say p equal to 1 just for our basic understanding, ok, then we can generalize it and this is our AR 1 model, right. You can think that this is one you know channel model. So, now, if this is the case what is the definition of an autocorrelation function?

If gamma k is the definition of autocorrelation it would be and it is just it is a stationary model. So, the statistics do not change over time. So, that is the key assumption that we are making it here, ok. So, the exact definition of the autocorrelation would be like this, then it would be y n plus k that is the definition, ok.

Let us take an example. So, this is the definition of autocorrelation function. And let us take the first equation where p is equal to 1 for easy understanding, and let us see how it can be formulated. So, my autocorrelation function can be formulated like this. So, expectation of you, just put whatever you get it y n and you just put it here.

So, let us say I give y n here and instead of y star n, I you know I just use a different, I just express this, say I have a star, then you have y star, n plus k minus 1 plus e n plus k. So, that is the typical definition, ok that is the typical definition. As per the definition this is what I got it, ok. Now, what it is? You can just you can just determine what it should be. Now, this is y n, this is y n plus k, and this is e n plus k, ok.

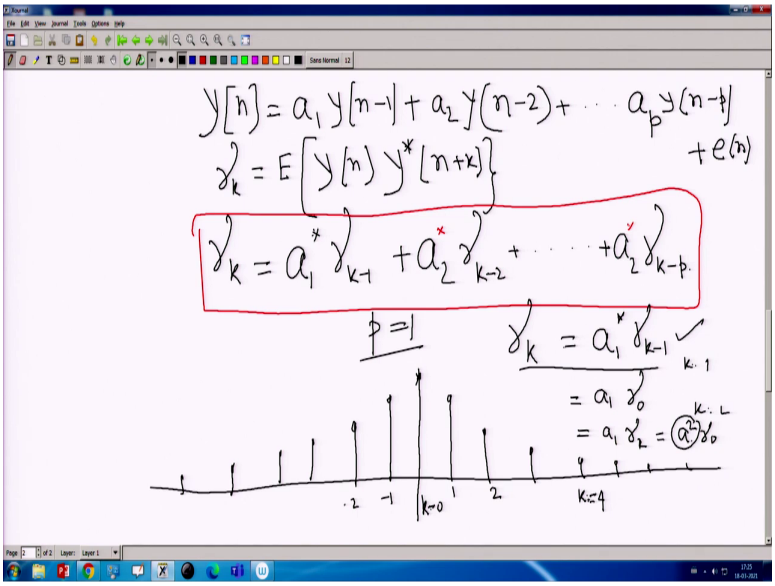
Now, what is that? If you take the whole expectation with respect to y n and with respect to y n and y star n plus k. So, this will be if I take the expectation thereafter this will be nothing but gamma I would say, k minus 1 that is typically it will be done and there is an a here, and there is an a star here, the a part will be put there plus y n multiplied by e n plus k with an expectation that will be 0 because n plus k is ahead in time, right and this is up to n.

So, which means that if I plot it, so y n will be somewhere here this is n and n plus k will be somewhere here, right. This is n plus k. So, naturally n and n plus k will not have any correlation because the at least with respect to e, where e n plus k will not be contained within the y n.

So, the expectation of such elements would be 0. So, I do not consider this whole element here, ok. So, that mean expectation with respect to y n and e n will be 0. So, that is it. This is what I get it a star y k minus 1.

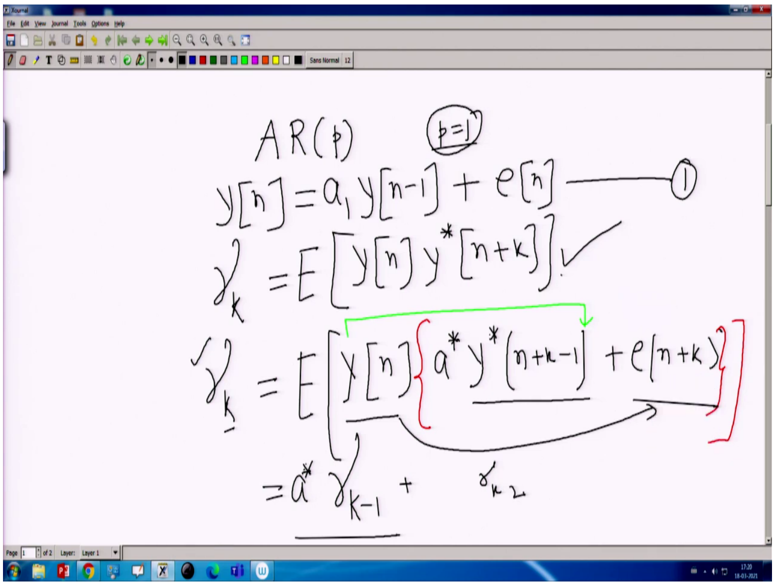
Now, you see that this is actually a recursive equation, right. Because this one gamma k is kind of a autocorrelation function at the kth point, with respect to the kth data, and k minus 1 is the previous data. So, autocorrelation function itself is becoming a recursive kind of things that is coming into picture. Now, if I generalize it what will happen? Suppose, I generalize it with the AR p, p can be any value. So, what can I get it?

(Refer Slide Time: 06:33)



So, it would be, so let us write down the equation first y n, you have a 1 y n minus 1 plus a 2 y n minus 2 plus dot dot dot. What will happen? a p n minus p, this is the definition, plus e n. This is the typical definition, right. Then what is our job? Our job is to calculate the gamma k which is equal to E of y n. So, this is typically I will be doing it; n plus k. That is my job is, right.

(Refer Slide Time: 07:31)

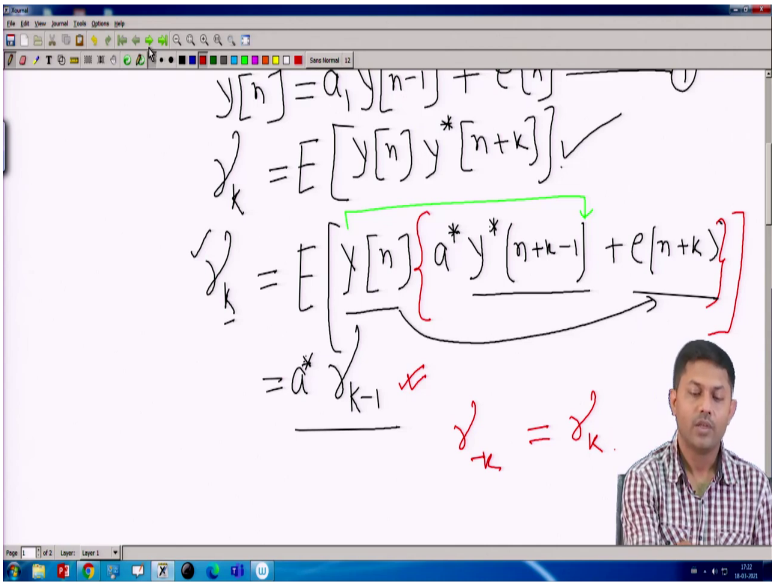


Now, if I take a, if I take; if I take the learning from the previous one, what will I get? I will see that y k, and sorry gamma k and gamma k minus 1 exist, if the if it is; if it is p equal to 1. See, if there are two more element or one more element extra comes. So; that means, there will be gamma k minus 2 will come into picture and so and so forth. So, that is the extension I can think of.

So, which means that here without you know, without much calculation I can just directly take an extension. So, this will be y n star you can put it gamma k minus 1. Now, it can be real also, a 1 can be real as well, need not to be a complex. It should be a 2 gamma k minus 2 plus this will be the just the recursive part. You see this is actually our recursive part. So, this is what my autocorrelation, when I am a AR p.

If I plot it what will happen? Now, let us not plot the p equal to any value, let us just plot the p equal to 1 value. What happens? So, you see the equation first, a star gamma k minus 1 here you go here. So, there may be star here I am not writing it here, because you can assume it to be real. So, that is not a big issue. But what if I plot it, how does it look like? Because it is a recursive, right.

(Refer Slide Time: 09:23)



So, which means that if I plot the first one with k equal to 0, now can k take a positive value or negative value, it can take all it is just an integer, right. Because I can have gamma minus k as well, where k is a negative value or it can be positive value as well, appropriately the definition will be adjusted.

But as it is a stationary process, so in this particular case this will behave gamma minus k equal to gamma k, because it is stationary process. So, assuming that, can I do like that? Can I do a plotting if it is a p equal to 1, for p equal to 1; I am just plotting it. What can I say? I write, rewrite the equation gamma k is equal to a 1 star gamma k minus 1, all right. This is what is coming.

So, I will plot from 0 to infinite or minus infinite to 0, if I plot it, what happens? Let us look at. Something I will get it, right. So, if I plot it k equal to 0, what will happen? At k equal to gamma, k equal to 0. So, that means, a star gamma k minus 1, ok. So, something whatever it is, so that will be multiplied. So, some value will be there.

Now, if I increase k equal to 1, what will happen? If I say k is equal to 1, what will happen? Then, it will be a 1 gamma 0. If it is k equal to 2, what will happen? a 1 gamma 2 or gamma 1 which will be further a 1 square gamma 0, like that. It will be like kind of it is like infinite series can because it is a recursive equation, right. It has no end to it. So, if I plot it, so I will see something like that. It will be gradually decreasing.

Why? Because you see this, as it moves as it moves away it just keeps increasing, provided a is a less than 1 value and we can easily find out that a has to be less than 1. Why? Because if you look at the way we have formulated the b operator you remember, the b operator; what was the b operator case and how the system of linear equation comes into picture. So, this becomes just an IR filter, right. So, obviously, it has a stability point, right.

Now, for it to maintain a stability my a coefficient has to be less than 1 or it should be within my unit circle, right. It cannot go beyond the unit circle. So, keeping that in mind, I am saying my autocorrelation function will be, it is an infinite series because it is a recursive series, but it will go like that.

Similarly, as because it is a symmetric function it will be go, it will be in both side. So, this is k equal to 0. This is for k equal to 0 this. Now, this is for k equal to 1, 2, and so on. This is for minus 1, this is for minus 2, and so on. So, this is an interesting thing. So, what can I infer from this diagram?

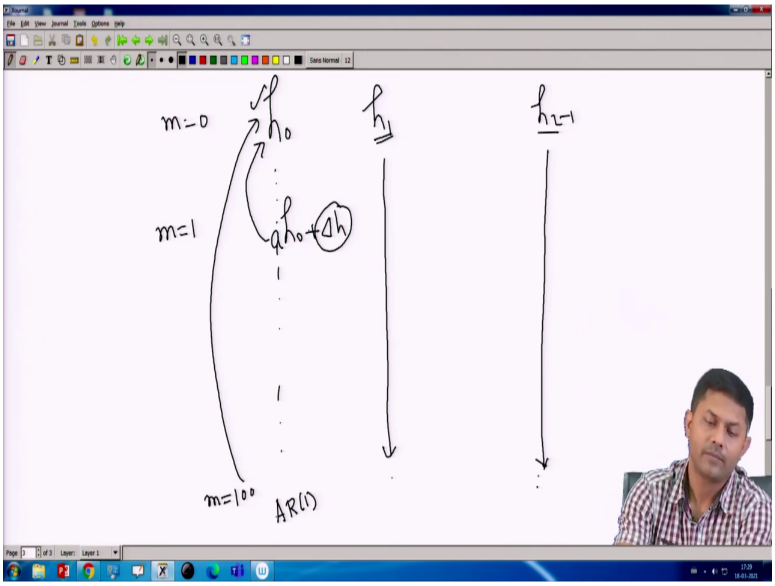
The only thing that I can infer from this diagram is that if you have an autoregressive model the autocorrelation function is actually a infinite plot. That mean, you always have some sort of a effect of any data with respect to any other previous data that is what it means, right.

So, suppose I take k equal to say 0, 1, 2, 3, 4 here. So, what does it mean? That mean with respect to my first data there is always a correlation with respect to the 4th data as well, 5th data as well, ok. So, that is the reason it; and it is very evident also because if you look at the way system of linear equation being formed from the autoregressive model it was like a IR filter, right that is what we have concluded with the b operator.

If it is an IR filter, what does it mean? It means the effect of the input on the output will be for infinite cycles. Like even if we just give one input the effect on the output will be for infinite samples, because that is the definition of infinite impulse response.

Similarly, the similar way I can also conclude that when you have a IR or a rather a autoregressive model for time series the effect of the correlation will be for every samples to every sample. So, the effect will be there. Only thing is that as you move away the effect of the correlation will die down, that is evident from this diagram, if it is a less than 0, if it is sorry a less than 1 then the effect will be always dying down, but there will be some impact. That is what it means, ok.

(Refer Slide Time: 14:07)



This is a could this is a good correlation and we can imagine also why it can happen because if you remember our channel model, so, our channel model was what? This was the h 0, this was h 1, h 2 and so and so forth. Let us l tap channel. And then we saw, if I draw it in the time domain I this is for m equal to say 0, this is for m equal to 1. So, what was the case? h 0 plus some you know delta and so on and so forth, right. Now, this delta is somehow I can model or I can correlate with this e n, ok. Here it is.

Now, there can be some sort of a you know there can be some sort of a coefficient here also just for a stability point of view, but that is ok. But it what I try to say is that this has an impact or this has some correlation with respect to the b plus. Does say m equal to 100 have some correlation with original h 0?

That mean when I sample after 100 cycles, so if my keep on sampling the adc and I look at the sample at the 100th point of my adc, then can I say that 100th point will have some correlation with my original observation at m equal to 0? Yes. Why it is yes? Because this is somehow modeled as a autoregressive model.

An autoregressive says that you have a correlation no matter what your time delay is. So, there will be some correlation left. So, that means, with 100th sample and the first sample, this sample and this sample will have some correlation. However small it is, some correlation will be always there because if I want to model it as a autoregressive one or p model this is how the conclusion would be. So, this is an interesting conclusion that I can make it.

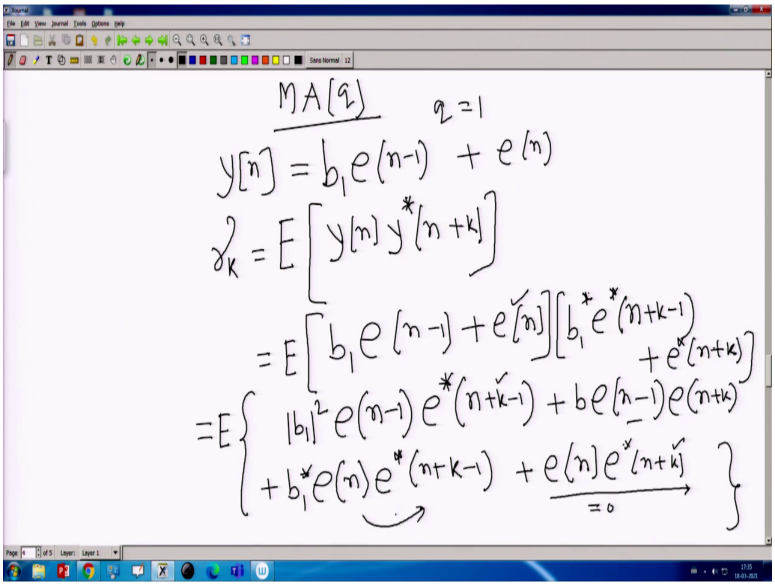
Now, similarly, I can make the similar conclusion for h 1 or h you know l minus 1 taps also because all of them are individual time series, individual time series. This could be some AR, some AR 1 model, this can also be some AR 1 model, this can also be some AR 1 model or AR 2 model or AR 3 model whatever, but some autoregressive model can be thought off for the Doppler modeling of the channel, ok.

Now, next what happens when I go for a moving average modeling of the channel? Now, I am not very sure or rather I am not sure from the logical point of view that how it would be so conclusive to say that channel in the Doppler can be modeled as a moving average, may not be so conclusive or a or it need not to be like that because autoregressive is more convenient way of modeling that.

Because I can say that at particular time my channel tap value was having some relationship with this previous samples because that is how the physical movements going on. Will it be with respect to moving average? Probably that answer I cannot give it, ok.

But let us from a general idea point of view what happens if I can model the channel taps variation with respect to the Doppler in a moving average manner, how the correlation would become, ok. Let us look at that. Because we know how to get the spectrum part, that is clear. But we would like to know how the autocorrelation part comes into picture, ok.

(Refer Slide Time: 17:33)



So, let us take a MA model. Now, I will not go for ARMA model because if I know AR if I know MA it is just combination becomes the ARMA. So, now let us take the MA model and try to understand what would be the autocorrelation function for it. Let us take a model here.

Let us say I have y 1 model, ok. Again for simplicity let us assume q is equal to 1, for simplicity. Let us say this is b 1 e n minus 1 plus e n, ok. Same definition of autocorrelation expectation of y n and y n plus k. There will be a star here, ok. Now, here there will be little confusion here because if I take just n plus k in a general term it may not be working so well, ok.

But still I can take a, equation wise I can take it. Let us break it and see what happens. This is what it is. So, I have just put back what I have got it here and then I say b 1 star, let us not worry about this complex and real path but; so, you just put it here, plus e of n plus k, ok. This is simple.

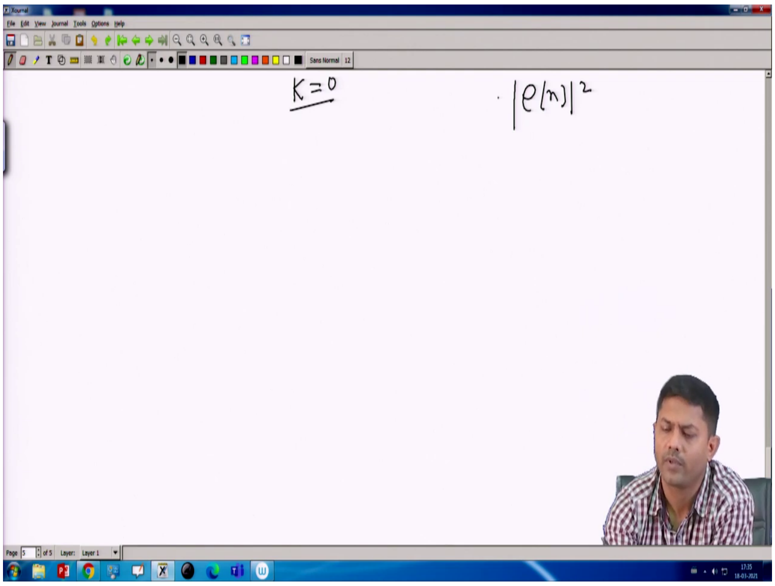
Now, there will be 4 cross terms here and there. So, I can just for the each for the easiness I can b 1 mod square here. I just take that e n minus 1 e star n plus k minus 1 that will be there plus b 1 e n minus 1. I am breaking it, I am just breaking the equation. This one plus and there is an expectation obviously.

So, that part is anyway there. So, let the let me put the expectation part anyway, that part is there, ok. So, I got two here. And then then with respect to that. This plane algebra what will be the case here? This will be e n and you just write down what whatever comes into. Just look at the equation and break it down, simple, ok, n plus k, ok. So, this is what the 4 terms comes with expectation.

Now, here I am little stuck because these I do not know what will be the value. We have already said e n is a IID process, ok. So, it mean e n, e n minus 1 or e n plus 1, they are all independent and they are their cross correlation is 0, that we have already assumed it. But I cannot say for sure that this will be 0. I cannot say that. It all depends on k.

Similarly, the similar situation here. Here I can say that when k is equal to 1, they only exist, otherwise they will not exist. Here if k is equal to 0, this exists. So, that means, for different value of k, this whole equation can be of different type.

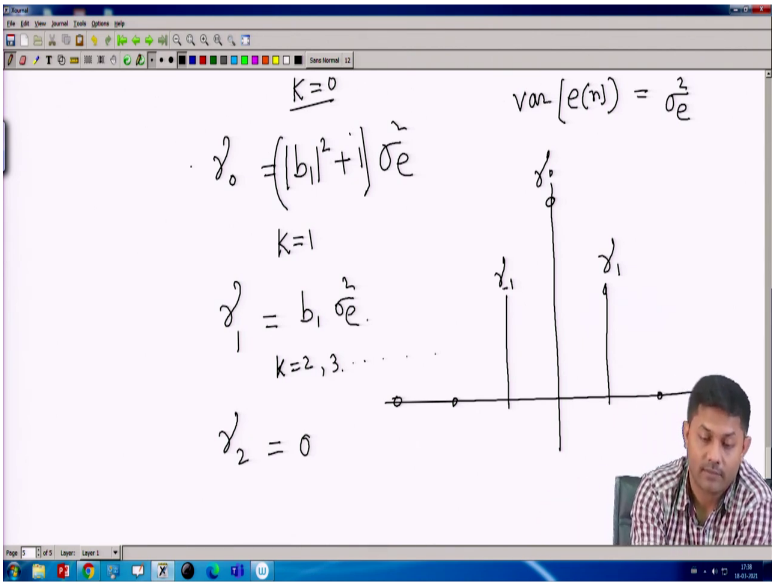
(Refer Slide Time: 21:51)



So, if I consider what happens when k is equal to 0, ok. Then say k equal to 1 or plus 1 minus 1; plus 1, minus 1, I do not have to consider because if it is plus 1 minus 1 will be almost equal it is the same because it is a its a stationary process. So, I need to consider k equal to 0, 1, 2, 3, 4 and so on, right.

So, when k equal to 0, what happens? That is gamma 0. Now, this will exist n minus 1, n minus 1, this will exist; this will exist second one. So, first and 4th will exist. However, the second and third, the second and third will not exist, right because it is a different e n, ok. So, there are two terms exist, whatever it is. So, let us say it is b 1 square and this one will be something.

(Refer Slide Time: 22:40)



Now, let us assume that this e n as you say it is an IID process. So, let us say the variance of this or expectation of this, the 0 mean process that also you can assume it, not a big deal. But safely I can say this is kind of a, let us assume it is a 0 mean process for our simplicity. Variance of e n is you can assume 1 or you can assume anything.

So, you can normalize it that is not a big issue, or 1 or you can also assume sigma square e, no big deal; sigma square e, ok. Let it be a 0 mean process, ok. So, for so, this one will exist something will come into picture. I am not so worried about what the exact value and so on, some value will come because that is not the intention what exact value will come.

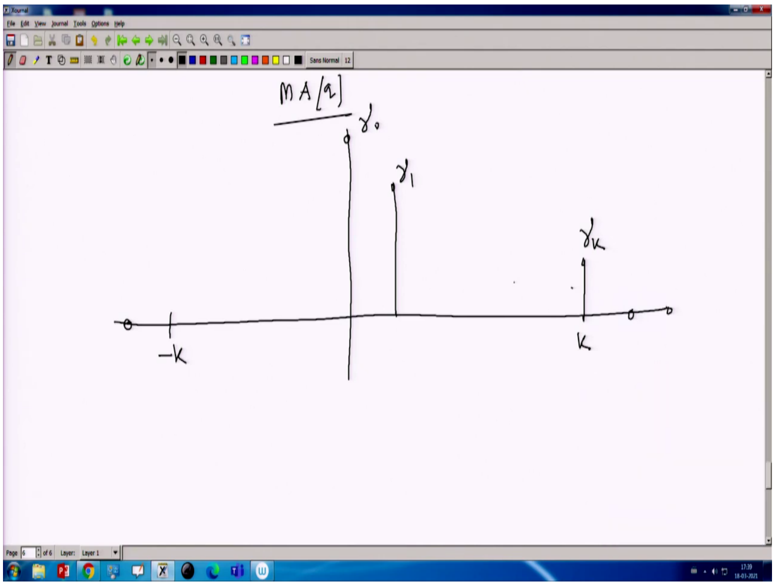
So, what is that? It is a b 1 square, sigma square is plus this one. So, it will be b 1 mod square, b 1 mod square plus 1 multiplied by sigma square e that is what would come here, ok. What happens when k is equal to 1? What will happen? That mean it is gamma equal to k, this is gamma k is equal to gamma 1. So, this would be gamma 1. So, let us see what happens.

Go here. k is equal to 0 mean first term will be 0 because it is e n minus 1 and it is e n, so this term will be 0. This term will also be 0, k equal to 1, third term will also be 0, right. However, this will exist, ok. This will exist. So, how much it will be? b 1, b 1 into this. So, it will be b 1 sigma square e. This is what it is, right.

What happens k is equal to 2? You go back to the equation and you will see that gamma 2 is actually 0, k equal to 2 or anything else 2, 3, 4, 5 whatever greater than, greater or equal to it will be 0. So, the conclusion is that if it is an MA process the autocorrelation is very straight forward. It is not a recursive one.

So, this will be the highest point because sigma square plus 1 is coming, ok. So, whatever it is. This gamma 0, then you will have some value gamma 1. Here also you will have gamma minus 1. But after that you will not see anything, everything else is 0, ok. So, can I generalize it? Obviously.

(Refer Slide Time: 25:27)



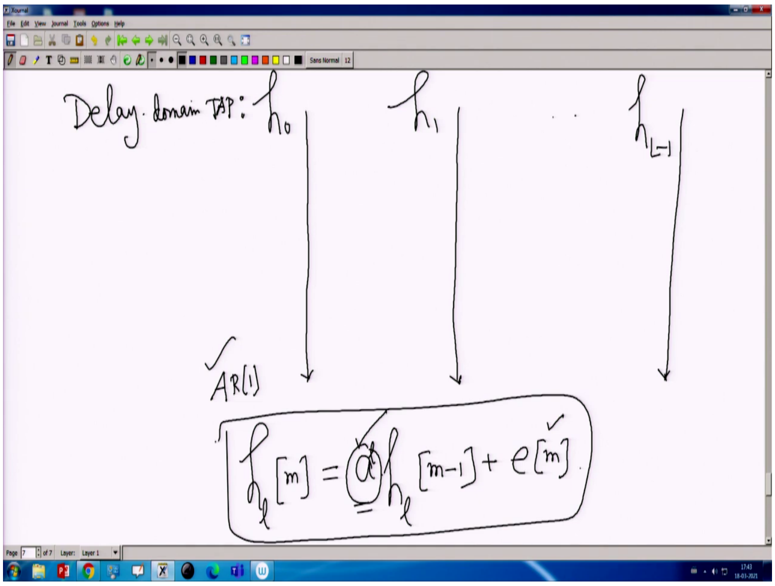
I can generalize it that if it is for k equal to p or k equal to q. So, now if the if the order if it is an MA q order, if the order is q, what is the conclusion? Conclusion is that you will have only q number of gamma k existing, rest will be 0, that is what the conclusion.

So, which means if it is a MA q process, you will have value gamma 0, then some gamma 1 up to gamma k. And similarly, here up to minus k, here up to k, but thereafter it will be 0, thereafter it will be 0, that is the conclusion, ok. So, I can get an MA q process, ok.

Now, what is ARMA? Just a mix. So, obviously, ARMA is an infinite process again, because there is an AR inside it. I am not getting into the exact autocorrelation for the ARMA process because that would be again it is a mix of; if you know how to do it for AR p you know how to do it for MA q process, ok. So, that is precisely how it is evaluated.

Now, from a channel point of view, what is more appropriate for us? Is the AR model is appropriate? Is the MA model is appropriate or ARMA model is appropriate? Well, it depends. Usually, AR 1, AR 2 or autoregressive is more familiar for us and that is more logical for us. So, that can be taken off, ok.

(Refer Slide Time: 27:01)



Now, for the channel thing that we have just concluded or a fraction of times we have just showed you. If it is say h 0, if it is h 1, if it is say h n minus 1, they are all tau domain taps that is what we have seen, right. This is the delay domain taps. So, these are all delay domain tap. These are delay domain tap, right.

Now, this is a time series model. This is an individual time series model, this is an individual time series model. So, every you know every tap becomes a individual time series model. What is that model? Can I model it as an AR 1 or AR 2 model? That depends.

Again, there is no hardened first rule that, so and so channel has to be AR 1 or MA or ARMA model, that again depends on some of the measurements, where people do measurement and try to see the correlation and try to guess what the correlation will be, based on that the modeling starts. So, that is the different ball game altogether. But whatever it is.

Usually we can take safely an AR 1 model very conveniently, because that you know I mean you do not have to go into a very high speed, moderate speed. Moderate speed probably AR 1 is good enough for us. Moderate means 50, 60 or maybe probably 100 kilometer per hour. AR 1 itself is good enough for us. That models tentatively a good. I mean it captures the time varying behavior of the channel pretty well.

So, how do you model it then? It will have a h 0, at a time m. This should be some coefficient h of 0, m minus 1 plus e of m, ok. So, this can be done very easily. Now, the trouble that comes into picture is how do you get the a, because that is the coefficient, because that is the relationship with which you can get the correlation.

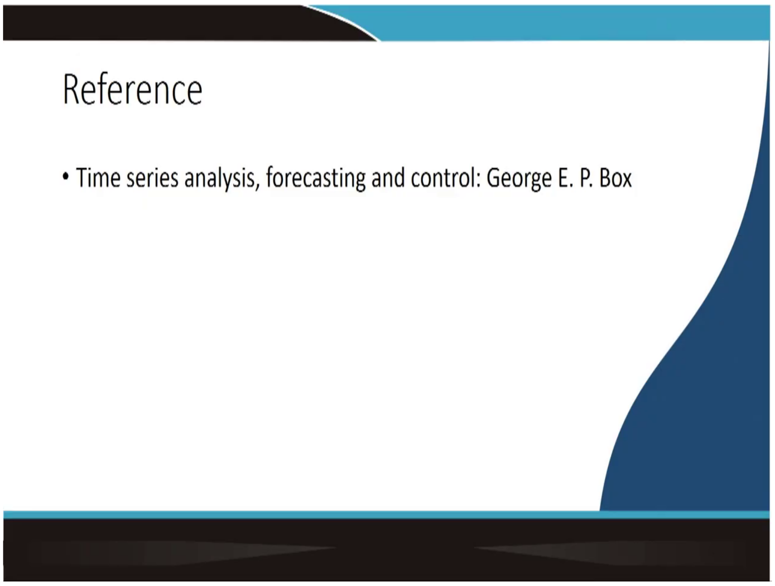
So, in from a from a channel modeling point of view is there a way to get the a, because unless you know the a there is no point [Laughter] and you cannot model it, right, your modeling is stuck. So, you have to know a, ok. You can assume some e m and you can you know you can you can do some something that part. But what about the a? What is the coefficient part? So, how do you estimate the coefficient part? Is there a is there easy way to estimate that? Ok.

So, in the next class, I will tell you what is the tentative framework for estimating these coefficients part because unless you know it, and the channel modeling itself will be incomplete task. Now, this is for h 0. Similarly, I can generalize this model very easily. I can say this is for h l, this is for h l, and so on.

Will the a be the same? Need not be. For lth tap probably my a can be different, can be same also, it depends on what estimate comes into picture, ok. So, probably I can say this is l, l, just for differentiating. So, this is how I can model my channel tap, lth channel tap in a Doppler way. For the Doppler this is how a good modeling for channel taps, individual taps I can model.

Now, if I know the model then what else? I know the autocorrelation, I know the spectrum that mean I know the Doppler spectrum very well from this, ok.

(Refer Slide Time: 30:41)



This is the reference. Though this reference does not talk about the Doppler modeling in the AR process, but this is the standard book which can be followed for the AR modeling purpose. So, that is it for this class.

Thank you.