Signal Processing for mmWave Communication for 5G and Beyond
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Module - 03
Understanding of various channel related parameter statistics. Narrow band and broadband aspect
Lecture - 16
AR, ARMA, MA Process

Welcome. Now we will be talking about the time series of different models. So, far we have just looked at some of the; some of the ARP models. We will do some of the examples later, but let us define the problems first AR, ARMA and MA process which were integral part of the stationary time series which we introduced in the last class.

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The concept that we will be covering today are the AR process then MA process and then ARMA process. So, AR is basically Auto Regressive process MA is a Moving Average and ARMA is Auto Regressive Moving Average process.

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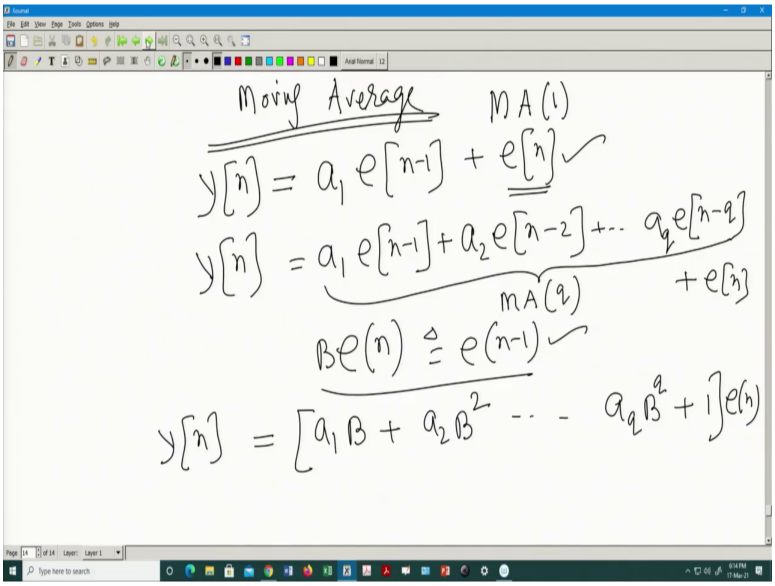

Now, today we will be explaining the moving average part moving average ok. So, how to define a moving average? Moving average let us take the first one. So, it is called MA. So, let us define MA 1 simple to start.

So, you have y n ok then you have a e a 1 e n minus 1 plus e of n right that is the only thing that will be coming into picture right. So, this part is there anyway that is the randomness, but now Y n does not depends on previous value of y n, but its previous value of my randomness ok.

Now, in our channel how much it is effective it is doubtful, but it just as per the our definition. So, the whether we really need it for our channel model that is a secondary question, but as per the time series this can be useful. Now similarly this is MA 1 model and I can generalize it say let us say I want to have a MA q model. So, you have a 1 n minus 1 plus you know e 2 a 2 e n minus 2 plus dot dot a p or a q plus e n.

So, this is the MA q model. So, here you can see it depends on q number of previous disturbance or previous you know innovative innovations or the random values. So, that is the moving average its a MA q model. So, this is your MA q model ok. Now given an MA q model how do I you know how do I get some sort of a spectrum analysis because that is the our ultimate goal right.

Again we can use the backward operator because that is easy to you know easy to deal with right instead of a auto correlation. Auto correlation also you can use it, but let us use the backward operator that is the I mean that is what we are comfortable with here in this case. Let us the definition remains the same if B e n comes it is nothing, but e n minus 1. So, that definition is anywhere there.

So, if I express the whole thing in terms of you know backward operator here backward operator here. So, how can I express it? So, this will be y n ok y n is equal to a 1 e a 1 B plus can I say a 2 B square because just 2 operator a q B q plus e n sorry there is small mistake here plus 1 into e n this is what it is right correct.

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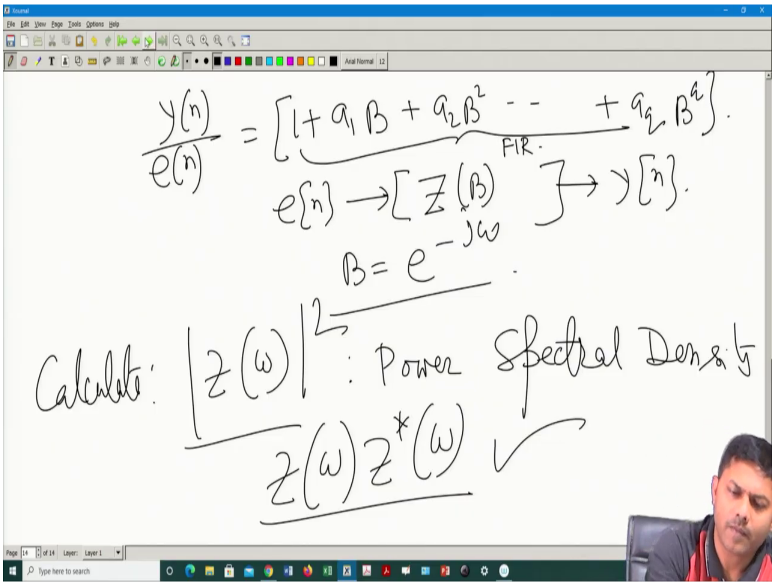

So, what is my ultimately; again I come back y n by e n what it would be it is like 1 plus a 1 B a 2 B square a q B q. Same thing same linear system of linear equations you have an e n you give this Z of B you get y n ok and you can see this is an FIR filter just like an FIR filter. So, how do you get a spectrum of FIR filter? Replace B by e to the power minus j omega right.

So, you get Z omega and calculate this one. So, calculate Z omega square. So, what does it give? It gives you the power spectral density I am done if somebody gives me power spectral density I am really done because that is the part I am looking for ok. So, I have to just replace this instead of B I have to write e to the minus j omega and then take the conjugate of it and take the just multiplied it that mean Z of omega and Z star of omega you get it you get the power spectral density of it.

And if you know power spectral density you know how much this bandwidth would be you can exactly know how much the bandwidth would be right. So, this part we want to make it this means standard process.

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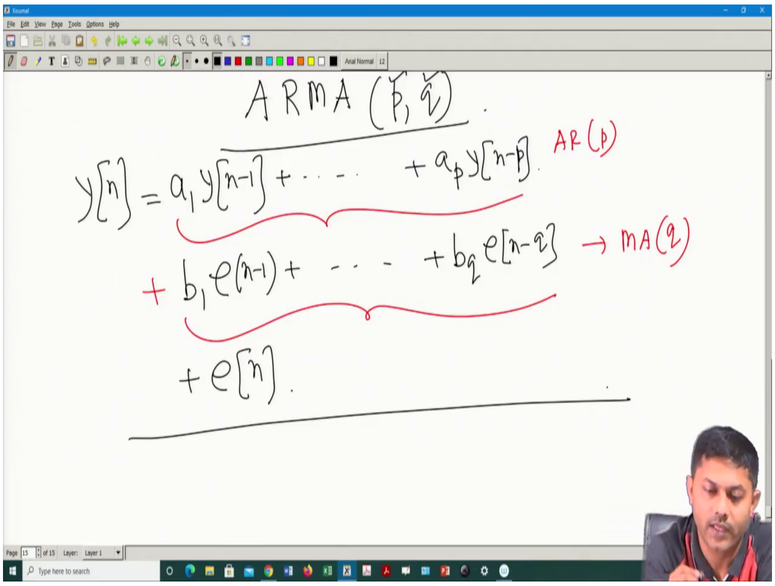

Similarly, next go to the ARMA process is the ARMA process. So, what is the ARMA process? So, it is a mix of auto regressive process and moving average process ok. So, it has a auto regressive part. So, if y n is my data of observation. So, it will have a auto regressive with respect to p order and it has a moving average with respect to q order ok. So, it would be say a 1 y n minus 1 plus a p y n minus 1 sorry a p y n minus p. So, this is the this part is your AR part plus plus now there will be a MA part.

So, let us put a different notation e n minus 1 plus dot dot dot b q e n minus q ok now this part is your MA part ok apart from that; obviously, you will have a noise the noise of course, noise meaning the current noise this is the one. So, this is your ARMA pq model ok. Now if you go through the you know if you want to get the auto correlation function of such ARMA process its very complicated its not easy to get it right.

So, best way to get the power spectral density you put a b operator replace b equal to the power minus j omega and then you know take the conjugate and multiply you get the spectrum at least you get the power spectral density part and that is it you are done. So, now, the same thing I will I want to express it in terms of the b operator here let us do that.

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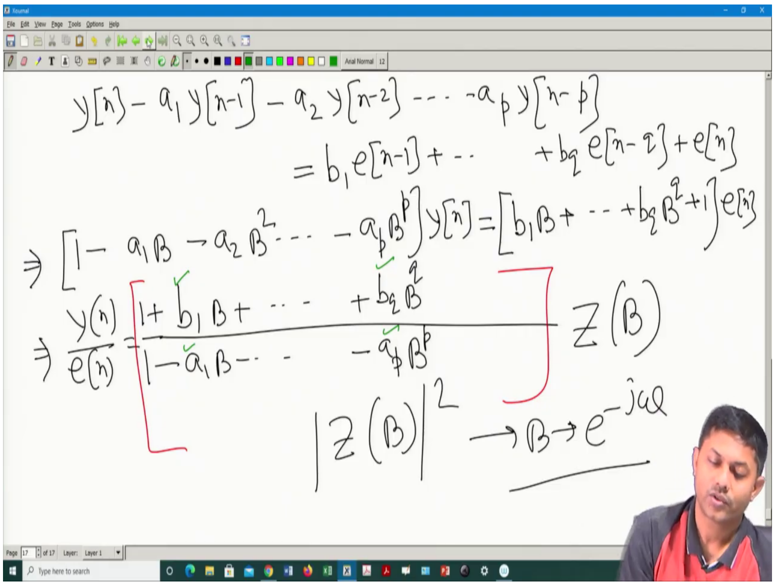

So, take the y part in the left side. So, this would be a 1 y n minus 1 I am taking all y pi left y side y y component left side and e component in the right side minus a 2 Y n minus 1 1 minus 2 a p y n minus p one sided equal to left side right side we will have all the e e component.

So, what are the b? So, this would be b one e of you know n minus 1 plus dot dot dot b of q e of n minus q plus e n this is what I am getting it I am just rearranging it now use a b operator use b operator on that. So, how do I express it? So, this will be 1 minus a 1 into B minus a 2 into B square minus a of p B to the power p correct I am just into y one just use a b operator.

Similarly, I will have the similar kind of things b 1 B plus dot dot dot b q B to the power q plus 1 e n this is what is happening. Again you create some sort of a input output relationship what will I get? Say y n by e n what will I get? I will get some sort of a you know polynomial by polynomial. So, it is as if like some sort of a IR filter kind of things.

I will get something like that or probably I will put I will beautify this one 1 plus this one similarly I will get this other one a 1 B dot dot dot minus a p B to the power p and so and so forth. So, this is my filter you can think of it as a filter ok. So, this is my filter this is my complete input output. So, this is the filter with complete input output relationship ok.

Now whether it is stable that you have to see you have to do a pole zero analysis and see whether it is stable or not, but that is a secondary question because if it is not a stable of course, there is problem. So, and that drives us how exactly you know how exactly the value of b 1 to b q or a 1 to a p should be there ok. So, that really drives us whether its really a stable or its not a stable part ok.

But whatever it is let us assume it is a stable and let us assume it is also a causal system if that is the case then I can think of a spectrum very easily right. So, what is the how do you get a power spectral density of such system? So, this call it this you this whole you know this the whole quantity within that red box I call it Z of B ok that is your you know that is your filter part. So, how do you get a power spectral density? Replace B by e to the power minus j omega you are done.

So, its very easy to get the power spectral density of such system and you can easily know how much your you know Doppler shift, but the crucial point that I am trying to convey here is that the b 1 b q a 1 a p these coefficients how do you really get it they are not easy to get it they have to come from lot of measurements they have to come from you know lot of other things. So, they are not something which are very straight forward, but there are ways to do that.

So, you can create a model and you can try to do some sort of an estimation that is ok, but there is a way to you know estimate this coefficient. So, given the fact that you can estimate these coefficient well I can very well create a time series model ok. Now let us go back to our channel part because this is where we started with.

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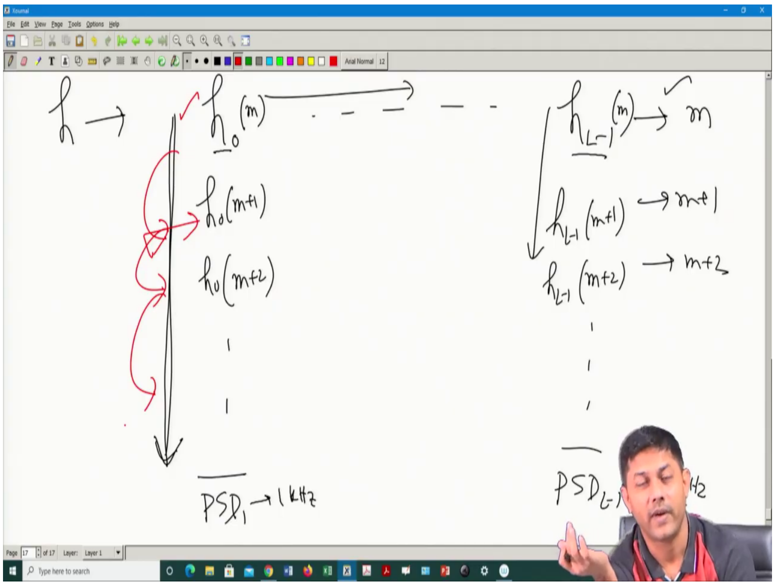

So, what we started with? We started with our channels which are say h 0 to say h of L minus 1. So, let us say I have a channel which has L number of coefficient frequency selective channel L number of coefficients I have. Now, this is only at a particular time m.

So, when I change it to m plus say 1 m plus 2 I can think of the whole thing as a time series modeling like for example, now the time series modeling in which direction. So, there is one direction here and there is one direction here these also a times if you look at the right side this is also a time series right, but in a tau domain in the delay Doppler in a delay spread domain.

But when I look at the vertically it is basically your Doppler domain. So, both are actually time series. Now this one the right one or the right and left side which is coming in the below direction that gives us a flavor of the Doppler spectrum. So, this gives us a flavor of the original coherence bandwidth spectrum. Now there are another issues point here is that you have one time series here and like that you have L number of time series right.

Because there are one coefficient and each and every coefficient I can think of it as a time series in the Doppler's case right so; that means, h 0 will vary over time, h 1 will also vary over time and so, and so, forth till h L minus 1. So, h L minus 1 will also vary over time and each and every time each and every you know each and every taps we have so, far tentatively assume they are kind of uncorrelated independent.

So, that mean I can say there are L number of time series parallelly going on I can think of that way. So, that mean this h 0 then this. So, I can say this is h 0 m this is h 0 m plus 1, this is h 0 m plus 2 and so, and so, forth. I can also think this is h L m h L minus 1 m, h of L minus 1 m plus 1 and so, and so, forth h L minus 1 m plus 2 and so, forth. So, there are L number of such time series parallelly going on ok.

Now, the question would be the effective bandwidth that we are seeing it right is due to what? Is it all individually or is it the summation of all or is it kind of a different way ok? So, what I am trying to say that, if this is a time series if this is a time series this will give you a power spectral density it will have some power spectral density one. If this is a time series this will also give you another power spectral density L minus 1.

The question that arises are all these L taps give you the same power spectral density or not we do not know that right. So, the effective change in my channel that we would see is neither due to PSD one or neither due to PSD L minus 1 it is a composite right its basically each and individual tap is changing that. So, the its basically a composite effect that I am seeing ok.

So, it may happen that this changes say up to 10 kilo Hertz whereas, this changes only up to 1 kilo Hertz well the effect will be more prominent somewhere here right because that changes. So, I will say bandwidth is getting shifted more towards 10 kilo Hertz, but that is in between 1 kilo Hertz changes also happen.

So, you cannot really say exactly which one will take the dominance whether it is 1 kilo Hertz or 10 kilo Hertz tentatively 10 kilo Hertz will have something to say more because it shifts more there right.

Now, what if all this h 0 h L minus 1 have the similar kind of statistics all of the similar statistics that may happen that precisely we will be discussing in the millimeter wave. Because in millimeter wave you cannot simply say that just because that h it is sitting at h L minus 1 it will have a very low power you cannot really guarantee that because everything is in even indoor environment ok. This is all depends on how the power absorption at the far off places happen. So, its not a very outdoor environment.

Outdoor environment yes this L h L minus 1 you can see the taps really going down, but when it is a millimetre wave you cannot guarantee, you cannot guarantee that the powers will be you know going down very drastically.

So, there may be even significant power at some instance of time not just at h 0. So, there if you have that kind of scenario, it is this more or less you can say more or less they can have a almost the similar statistics. So, the effect will be same for all of them, but when you have not a when you have a different scenario.

In case one shifts only 1 kilo Hertz another shifts 10 kilo Hertz. So, what is the effect? Is it 10 plus 1 kilo Hertz or it is 10 minus 1 kilo Hertz? No the effect will be the one which shifts maximum right because that is the one which shifts maximum the others will be just hiding behind it, but if you have almost equal probably that is the almost equal impact will be there ok.

So; that means, if you have multiple taps in a system and if something has shifted say 20 kilo Hertz you cannot guarantee that a that one particular tap has shifted 20 kilo Hertz, it may be one of them can be worst case and that has been impacted more, but usually what happens you know because all the taps will be equally impacted by the Doppler right its obvious right because even if it is far off tap Doppler can impact all of them suppose my receiver is moving.

So, whether its a nearby reflector or a far off reflectors the impact of the Doppler shift will be more or less the same ok. So, in the in a sense that all the taps in that case will give you the equal amount of power spectral density not equal may be a similar amount of power spectral density. So, that is the kind of a very rough conclusion, but may not be very correct conclusions in this case ok.

So, this is a very good way of modeling my Doppler part. Now how do I do? I mean what is the impact I mean is it like that the spectrum is of only interest to us is not there anything else that we I would like to have from this particular mathematical scenarios? Yes, there are many things that we can do it here. One thing that we can do it here is that what about the channel estimation and tracking.

For example, I know if there is a Doppler there is a spectrum set, but what does it buying to us? Does it buy anything to us? What will I; what will I do after knowing a spectrum shift? Does it really make a big deal for us? The big deal for us is the channel estimation because that is the coefficient I need to know for my data detection.

So, which means that even if I have h 0, h 0 m plus 1, h 0 m plus 2 and I created nice time series and do all this thing my final goal is not just the power spectral density of my changes, but rather I will be more interested to know what is the changed time what is the changed you know channel coefficient at a particular time that I am interested to know.

So, which means that channel is changing suppose I have done a channel estimation, but it is suddenly changed at m plus 1, I may have to do a channel re channel estimation channel again changes at m plus 2.

I have to again do are estimation of my channel because it has changed again channel changes. So, that mean at every time do I have to do a channel estimation that is the question we like to answer is it required that I at every point I need to do a re channel estimation re estimation of my channel probably it is not required what I need to know is that I need to know what value has become right.

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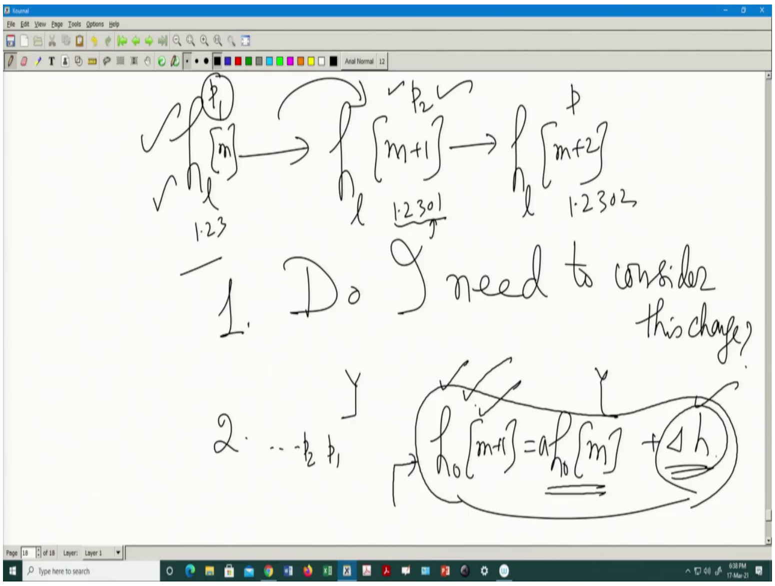

So, what I am trying to say here is that, the channel say my lth tab lth tab at m at m time which is m time it has become or it has changed slightly because as we know its a correlated changes right its not a very random change its a slight change correlated change at l m plus 1 or m plus 2 the channel also change slightly ok.

Now if it changes slightly do I need to first of all do I really need to know whether channel has changed or not? So, first question I am asking here do I need to consider that change ok? This is very very valid questions suppose I am sampling at say 100 mega Hertz at my ADC or my effective sampling rate is 100 mega Hertz then 0.01 micro second. So, at every 0.01 micro second do I really need to know how much the channel has changed ok 1st question.

2nd question is that, if I think that that changed value is of important to us; that means, if I think that yes whatever changed has happened I would like to know how much changes happen ok. That mean the channel has changed whatever value suppose it was 1.23 value and this has become 1.2301 and say this has become 1.2302.

The 1st question is, is this 1.2301 important to me or is it significant to me that is what I mean to say important to me do I need to consider the change. Second aspect is that even if it changes to 1.2301 do I need to know this whole value? Let us say I need to know that point is that do I need to re estimate this whole value just the way I have done a channel estimation at mth time.

What it means that at every time do I need to re estimate my channel again and again and again? All these you know bulky algorithms that I may have deployed for you know channel estimation do I really need to do that? The point is that if the answer is yes, then who will give you pilots?

Because pilots because how do you estimate a data? I mean without a pilot estimating the channel is very difficult right because that is the basic fundamental way of estimating you can do you can do a joint estimation of channel and data as a blind estimation, but performance will be very poor.

So, if I really want to get a very good estimation, I would always prefer a non blind estimation which means that every time I need to have a pilot to estimate my channel that mean I have a pilot here I have a pilot here I have a pilot here to do estimate my channel then why do I then when will I send my data because every time I have to send a pilot only right say I have to send one pilot or the data here. Suppose, my channel is just a single you know single antenna to single antenna and every time I have to only send pilot only right.

There is no other way because at p 1 we using p 1 I estimate this channel by next time using another pilot I need to estimate because there is a separate pilot I have to at that time whatever pilot comes I cannot use my older data and estimate my h l minus m plus 1. Yeah so; that means, there is no data I have to only say estimate pilots. So, it is not possible it is not feasible.

So, which means that I cannot have the luxury to estimate my channel re estimate my channel completely using the fresh pilot again and again and again that is not something that I can do its not virtually possible. So, the question that remains is that, can I have just one pilot here which means I only observe data here and estimate my channel next without having a pilot there ok.

If it is possible how and what is the philosophy behind it? Yes, it is possible. So, this kind of concept is very much well known as a channel tracking. So, basically you estimate the channel and then you do not re estimate fully rather you only know how much the channel has changed only the delta part you keep on you know estimating the delta part and readjust your channel just a matter of readjustment why?

We do I need to re estimate the whole channel not required why it is not required? Because I know at m plus 1 my channel is a small delta from my previous data that is precisely what I have shown it in the last classes right because it is h 0 at m plus 1 is nothing but whatever is h 0 m plus some extra delta h that is precisely happens here there may be some factor here also.

There may be a factor here also, but this is how the actually the channel is moving at m plus 1 it is whatever was earlier plus some extra you know that random due to the Doppler that comes into picture. So, which means that essentially it forces me to think or model in a time series manner since in this particular case I have done an AR 1 model, but I could have done different ARP or whatever model, but let us say I am just thinking in this very simple simplistic model.

So, this is one interesting part that at that m plus 1 I just create a simple AR 1 model and say that hey the only thing I need to do is that I just have created a model and now using this model can I guess h m because that is my job. My job is that. I know this I may know the statistics of it can I guess this that is the simple as simple as that ok.

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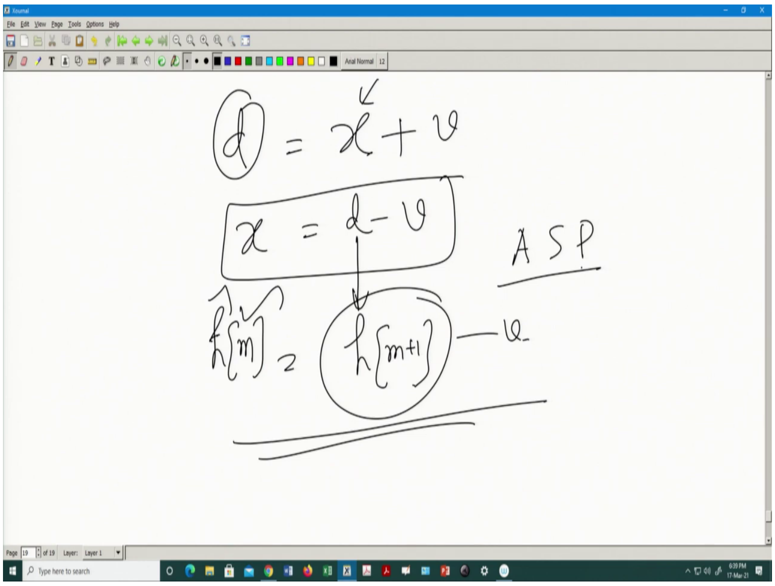

So, it is as if like as if like you have a data called say d or observation called d or this is your input data or maybe you know that and this is your noise data and this is what your new data which you want to you know estimate it. Then I say x is equal to some sort of a d minus v this is what my new data model, where d is your new data say let us say that is your channel at m plus 1, this is my channel maybe at m which I have estimated it now this is my noise.

So, my job is to re estimate this h m plus 1 from this observation because observation do not change its very simple data model. So, this sort of you know you can you use this kind of technique or you know you can use normal estimation. So, this kind of you know way of re I should not say it is a re estimating rather its a readjustment of your channel.

So, there is a new field not a new field there is a new dimension of thought that is your adaptive signal processing ASP. So, using adaptive signal processing you can you know you can actually re I should not say re estimate rather you can readjust your new channel at m plus 1 ok. So, this kind of thing you can call it a prediction. So, you have channel at m, I want to predict what it would be at m plus 1 using my old data.

So, there is a different I am not getting into that because that is completely a different direction of thought process. So, this time series is very important for us either from a modeling point of view to know the spectrum or to you know or to estimate my channel without getting any pilot just to you know readjust my timings. So, this time series is very valuable in that aspect ok yeah. So, we end it here.

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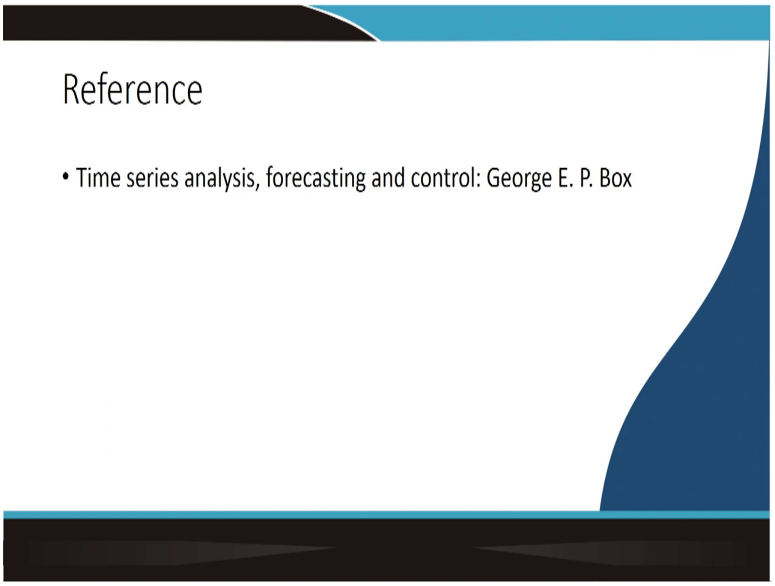

So, this is the reference book which contains the stationary time series.

Thank you.