Signal Processing for mmWave Communication for 5G and Beyond
Prof. Amit Kumar Dutta
G.S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Module - 03
Understanding of various channel related parameter statistics. Narrow band and broadband aspect
Lecture - 15
Introduction to time series

Welcome to the Signal Processing aspect of from millimetre wave. So, we will be continuing the time series part today. So, today we will be covering the time series part, starting the lecture number 15 where we will be introducing the time series part. And, that will be very much important, when you go for the time sequence analysis especially, when you go for the Doppler.

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So, these are the concept that which we will be covering here, introduction to the time series, then related parameters like auto correlation function and then Power Spectral Density or PSD. So, these are the terms which we will be using very often in our discussion.

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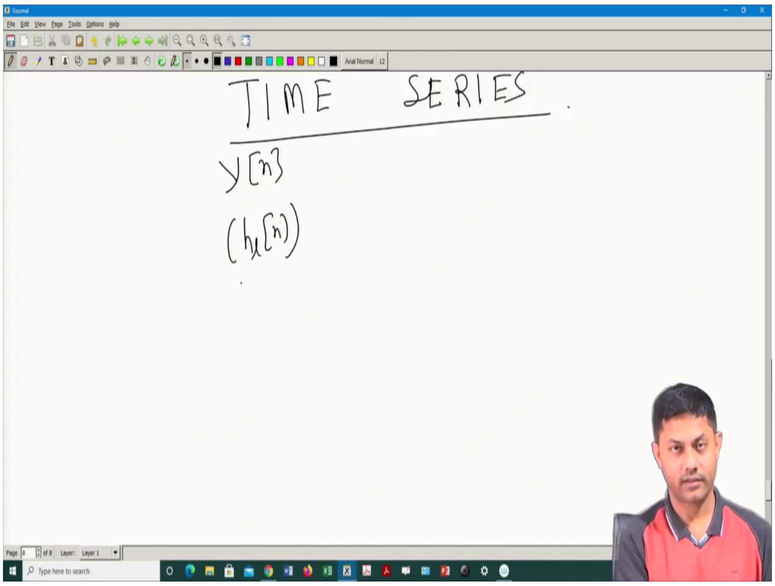

Time series part ok. Now, let us define in very general way then we will map it to the channel part ok. So, in this case, what we are interested is basically a stationary time series. So, we are assuming that, let us assume that there is a sequence, let us say the sequence is y n, which was basically h l n in our case ok, just drawing an analogy. So, I am defining a sequence called y n. So, y n is some sequence maybe a time series may be a channeled l th tap can be one of the such sequences ok.

Now, this sequence it is a correlated sequence and that is precisely why I am interested to know the time series here.

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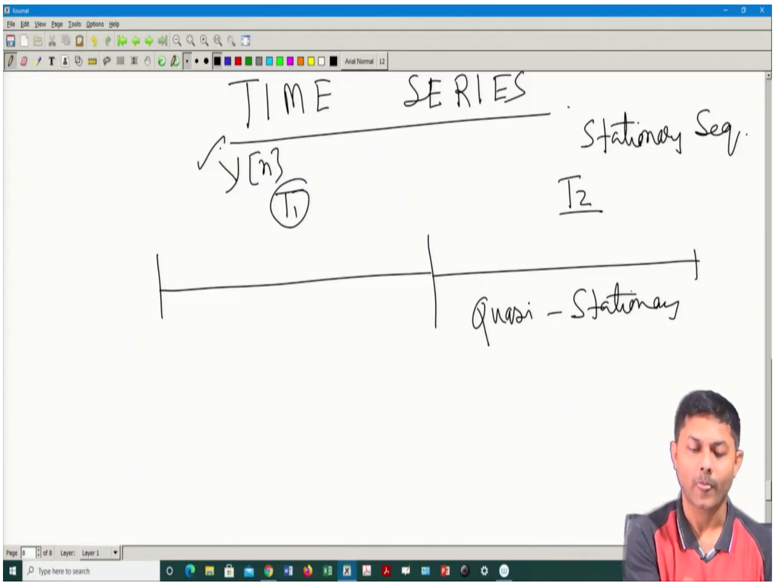

We are assuming in this case that this sequence is a stationary sequence; it is a stationary sequence ok. Now, we are not modeling the non stationary sequence here, what is non stationary sequence? A sequence whose statistics completely changes after some time, which is not constant ok? If it is a stationary sequence the statistics will never change ok. The PDF the joint PDF of you know any number of sequences will never change, that is precisely the definition of a stationary sequences.

If it is a non stationary y n minus 1, if I take the joint PDF they keep changing I mean, you cannot say that y n minus 1 and y n minus 2 and y n minus 3 they will remain constant we cannot guarantee, that mean time to time the statistics would change. In this case we are assuming it is a stationary sequence.

Now, practically is my channel a stationary channel, practically not practically it is completely a non stationary sequence ah, but the, but the advantage that we are taking there, the channel whatever we are considering it, we are considering for a large time.

And what is that large time it is difficult to answer it can be 1 minute, it can be 2 minutes or it can even be a 1 second also. So, it depends on the Doppler, context of Doppler ok. So, there is a Doppler and there is something else we will be discussing it and there is a something called a coherence time we will discuss that later. But, let us assume that I am considering a time over which my channels statistics do not change, channel may change ok. Channel may change because something moves here and there because of the Doppler channel may change.

But, statistics do not change ok. When it can be a can we assume some scenario where we can think of the statistics do not change and statistics changes completely. Say for example, we are walking in a busy street of a road ok, busy street of say Kolkata we are walking.

For 10 minutes 15 minutes for 10 15 minutes, the nature of the environment would remain almost same right. The number of cars, number of people, houses and all more or less I mean exactly they may change, but more or less it is like a statistically they might remain same.

So, I can say, if the time occurs, when I walking when I am walking through such kind of environment my channel content will be changing, but would the statistics of my channel change need not be, because more or less it is the same.

But, now you assume a scenario you are walking through that street of a busy street of a particular city. And, then after walking for say 1 hour you suddenly enter into a village ok, or suddenly enter into a very enter into a region where there is no houses ok, not many cars are also there. Now, think of that scenario.

So, here of course, channel values are changing, but channel statistics may also change in that case, because the environment completely changes statistically in a very very drastic way.

So, such kind of scenario I can think of that the such can in such kind of scenario, you can think of a channel statistics changes from time to time. So, now, you can imagine how often such things happen. Does it happen so often, does it happen just after 1 micro second my channel statistics changes completely, that mean after 1 micro second you have to enter into a region?

Where you know objects, the nature of the objects, the number of objects, will be completely drastically changed, that does not happen. We do not you know we do not move in a such a high speed, we are not moving close to the speed of light. We are moving in a normal, even if you are in a train probably high speed train could be some 250 kilometer per hour or something like that, in a general train 100 kilometer per hour or 80 kilometer per hour or a car.

So, in there at least for some significant amount of time, I can say my channel statistic would not change so much ok, because the regions geographic regions are not so, small where things will change so, fast. So, which means that though our actual system, our actual channel is a non stationary system, but for a considerable amount of time, I can assume my channel statistic remains same; that mean, it is a piecewise stationary. So, such kind of real system is called a quasi stationary.

So, our consideration of channel is basically that. So, you have a long time. So, how long it is difficult to even quantify that, tentatively is sometime. So, let us say a large time, I do not know maybe 10 minutes 15 minutes depends on again your Doppler right. If you are traveling in a trend probably after 5 minutes you may encounter a different geographic locations ok or maybe half an hour good enough.

If it is a plane probably not in the sky, but at least when it is in the ground it encounters changes very fast. So, it all depends on the Doppler. So, let us assume that I have a time where my channel do not change the statistics. So, I can say during this T time or rather at T 1 time it is a stationary process.

Again for T 2 the channel statistics do not change, channel values may change, but not the statistic that what is mean by statistics that PDF, we have shown earlier right. The channel is more or less a complex Gaussian nature with some mean and variance right, mean was supposed to be 0 and some variance.

I am talking of that quantity say mean becomes 0 let it remain 0, but variance do not change, if it is a Gaussian. So, I am saying that throughout T 1, the variance of the channel would remain constant. Throughout T 2 the variance of the channel will be a something else, but it remains constant so, that way. So, this kind of thing I call it a piecewise stationary or rather quasi stationary. So, this is a Quasi-stationary.

Now, in this time series whatever we are going to explain it, we are considering a stationary system. Meaning we are assuming that we have enough time for which my statistics do not change. So, under a stationary sequence, what should be my channel modeling in terms of time series. So, now, let us say y n is such random sequence that could be lth tap itself at a time n ok. So, there are three ways we can model the time series.

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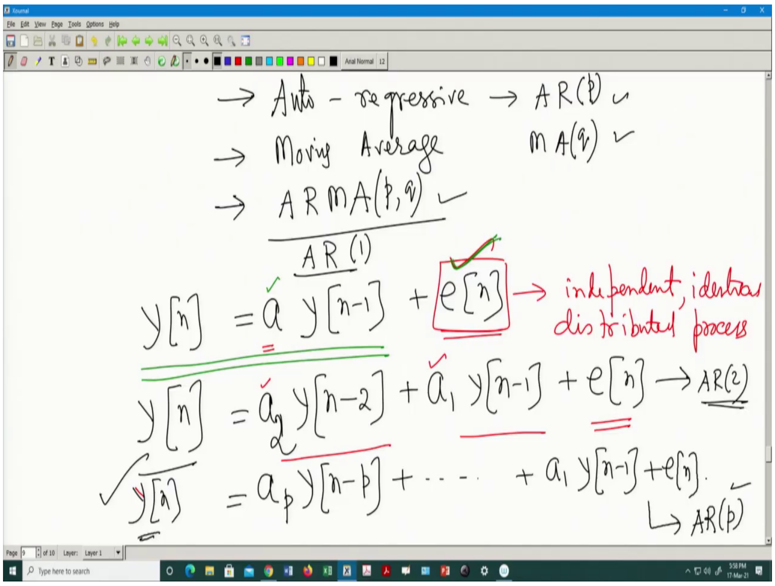

One way is called auto regressive or it is called AR ok. Another one is called moving average ok. So, this is called MA. And, the third one is a mix of both. So, it is auto regressive moving average, rather we call it ARMA ok. So, there is an order whenever we talk about the time series, there is an order associated with just like your polynomial every polynomial has an order right. So, AR, when we say AR it should have an order of what of our order, what order.

So, I will say order p order. So, say AR times the AR model of time series with order p this m a order of time series with order Q, it is an ARMA order of time series order p and q both will be there ok.

So, we will explain, it in a very simple mathematical logic what exactly auto regressive, what is moving average, what is ARMA process? Ok. So, these are the three ways you can define a time series. So, basically it is the; it is the three ways you can define the correlation.

Correlation between what or among what among the sequences that you already have it, at say at the mth sequence with respect to it is previous sequences. So, that is exactly what we wanted to have it here right.

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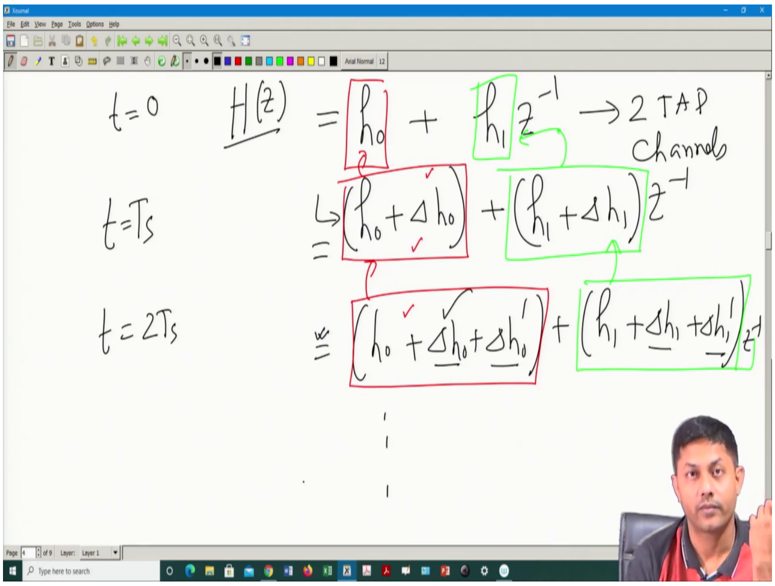

That this is precisely, what we wanted to have it here now? So, this is the channel at so, this is the channel at say m plus 1 or rather 2 m equal to 2, I want to know what is it is correlation with m equal to 1 and m equal to 0 and so and so forth ok like that. So, here also we want to have that definition. So, let us say what is the AR model? AR model auto regressive model; see if I have a sequence y n ok.

So, if it is AR 1 order 1 what is the definition? It is defined something like that, a y n minus 1 plus e of n ok. So, a is basically some sort of a known coefficient, e n is completely an independent random process. So, this is a random process, not random process it is independent it can seem in an identical process also I do not mind, identically distributed process.

And, this introduce that randomness we are talking, that between m equal to 1, to m equal to 2 the channel how they are correlated they are correlated, but there is a delta h we are talking right. So, that is the extra part which we want to model it as some data by randomness. So, this is basically the delta h part ok. We are assuming that that particular delta h is some sort of a completely an independent process going on.

So, this e n is basically an independent process identically you can assume independent identically distributed process IID process ok. And, now you look at the correlation. So, this is this defines or this creates that correlation factor. So, it is y n is correlated with y n minus 1 through this quantity a. So, this kind of so, basically it is a regressive model right. If, you have learnt what is called IR filter, it is what in basically its kind of a y n is dependent on y n minus 1 and so and so forth right.

So, these are all the regressive process, so, here also the same thing kind of happening here. So, y n depends on the previous y n minus 1 through a constant called a and a is known to you that is not a random variable. Now, the only random part that is getting introduced here is the e n part. So, through e n y n becomes the random quantity, but y n and y n minus 1 is correlated ok. So, this is called a AR 1 modeling.

Similarly, I can you know I can generalize it say I want to have it two say, in fact, I want to have say 2 I mean AR order 2. So, how can I do a 1 say y n minus 1 or I put it this way plus a

1 y n minus 1 plus e n. So, this is your AR 2 model, I can even generalize it. Now, I can I will introduce the AR p.

So, now what is the AR 2 model? AR 2 model meaning the y n depends on 2 previous data, 2 previous data. One previous data is this, another previous data is this; y n minus 1 and y n minus 2, it depends on that ok. And, these two are constant, this is the known constant. So, that creates the correlation.

And, this one creates the randomness inside the y n. So, this is called AR 2 process. Now, what is the natural extension of this process? So, let us say, it is AR p model, let us say I have AR p model now pth order AR p. So, what can I this is a of p right. And, this will be y of n minus p happening plus dot dot dot. Finally, you will get a 1 y n minus 1 plus n. So, this is your AR p model. So, AR p model.

So, that; that means, the y n depends on p number of previous values; p previous time values will be there y n currently depends on all of them ok. So, this is kind of a AR p model ok. Next one ok, so, this is clear so, AR p model. Now, before I get into the moving average model, I want to introduce some more operator ok.

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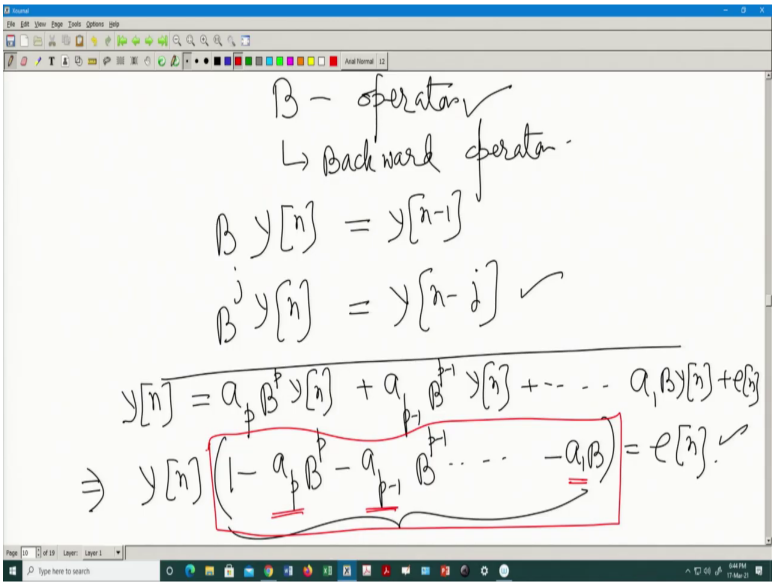

So, there is an operator called B operator, its easy for us to introduce this B operator, because it will make our spectrum calculation very easy ok. So, this is the B operator. Ultimately that is what we want right we want to get the spectrum. How much extra spectrum, that appears for me due to the due to the Doppler, that is precisely my goal is right. So, this is one case B operator ok. So, let us introduce that B operator.

So, B operator is called a backward operator. So, this is also called backward operator ok, is called a backward operator. So, the definition of this one is that if y n is a sequence and I give an operator. Operator is some sort of an you can say, it is an operation to be done on a particular mathematical notation. So, that is an operator.

So, it means that if it is a B y n it is nothing, but B y n minus 1. It is just an you can say it is a notational way of say expressing it. And, if it is B to the power j y n, it means it is y n minus j just a definition is an operator ok. Now, with this operator I can express my AR p process very simply. In a very simple way, I can express my AR p process. Similarly, I can express my M A Q process or AR ARMA p of q process can also be expressed in a very very simplistic manner by this operator.

Now, there is a very good advantage of this operator. The advantage of this operator is that, you really do not have to go through the pain of auto correlational function generation for spectrum evaluation. This makes, this life, of this makes our life of generating the spectrum very easy if we use a operator ok. That is why I directly get into the operator rather going into the auto correlation the, but I will define autocorrelation, the autocorrelation will show some other properties ok.

But, let us first close our main intention of spectrum part. So, this is an operator ok. Now, let us see how I can express my AR p process using the operator ok. So, what can I say, if I look at this y n and AR p process, I can always write it?. So, finally, can I write it like that if it is an AR p process this is a of p right B to the power p y n, plus a of p minus 1 B to the power p minus 1 y n and so and so forth right.

What it would be a 1, B y n plus e n, this is what I will get, if I use an operator ok. I just do some mathematical jugglery here, can I write it like that, I just put the y n this side, left side ok, this is precisely what I have done.

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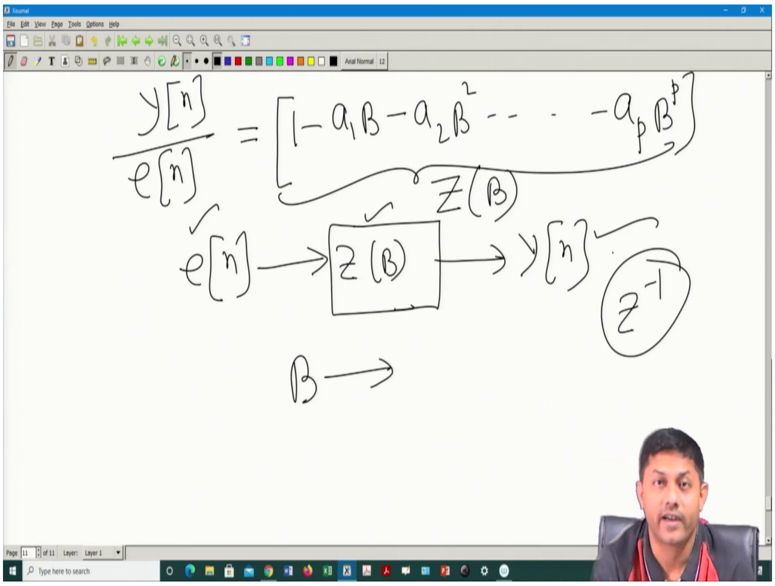

Now, what is this some sort of polynomial right, standard polynomial I get it? So, can I write it like that, see if I say y n by e n right? So, what can I say, see what is written here 1 a minus B and so on so forth I will write it like 1 minus a 1 by a 1 B minus a 2 B square dot dot minus a of p B to the power p. This is what I can write it correct; this is what I can write it right.

As if like again I am having some sort of a FIR filter this A R model right, where I am giving e n as an input to the system. So, let us this whole thing I call it say some notation say Z of B. So, it is as if like I am giving e n as a sequence or input sequence to a system linear system B and I am getting an output y of n, where this Z B itself is an FIR filter of order p I can think of that way.

So, this whole AR p process I converted to a simple input output relationship, simple input output system, where I have some sort of a you know filter kind of things. So, now, what is my job?. So, it is as if like I am giving e n I am getting y n, what is my job? My job is to know, what is the spectrum of Z B? That is precisely, what I want to know right? So; that means, what is the spectrum this particular fellow gives me I can have two ways of evaluation.

One is this B operator. This B operator is simply saying this is nothing, but some sort of a FIR filter right. So, now, what is my job? I want to know the spectrum of FIR filter, how do I get it. Spectrum of FIR filter well, you replace B, because what is B? B is a backward operator. Now, how do you replace, how do you imagine a backward operator in your signal processing, it is like a Z operator right.

So, Z inverse you have seen in the context of DSP, this is Z inverse is nothing, but a backward operator right. It is basically a delay operator right. So, B is also something like that basically a delay operator.

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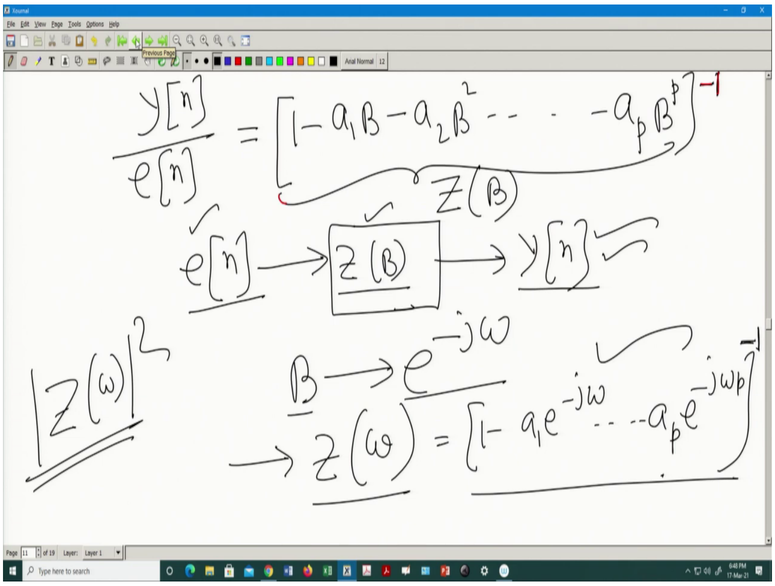

So, which means you just replace B is equal to e to the power minus j omega into this filter, what will I get. I will get Z of you know Z of omega, whatever it will be; it will be 1 minus a 1 e to the power minus j omega dot dot a p e to the power minus j omega p is what it will be coming right. I want to know the spectrum of it. So, what should I do, if I want to know the spectrum?

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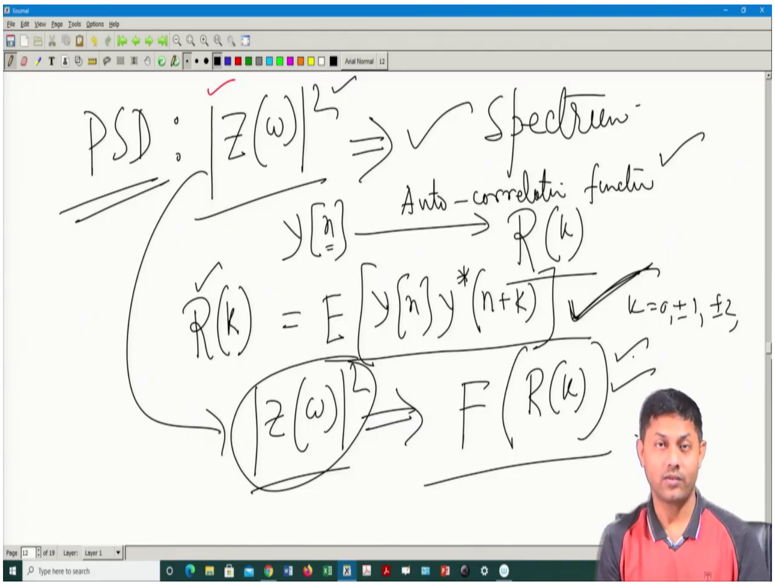

Naturally, I want to know how this is plotted. So, that gives me the actual spectrum or power spectral density basic this is the power spectral density right. So, this will give me the power spectral density, this is the my power spectral density, because that is what I want to know, how do I get a spectrum only from power spectral density. So, you plot the power spectral density see where the power is existing, that is your spectrum ok. So, I want to; I want to calculate that.

So, can I write it like this? So, you can calculate it. So, here you know what it is. So, take Z omega and then take the conjugate of it you are done ok. So, that is a very simple way of calculating the power spectral density or the power rather not power spectral density, which will eventually give you the amount of you know spectrum that, this particular you know

Doppler holds it right ok. So, that is the very easy way of getting it, there is a second way of getting it ok.

So, this is one way of calculating my spectrum. So, from here you can you know what is your spectrum, you can evaluate your spectrum, but here the catch here is you have to know all these all these coefficients, all these coefficients have to be known, otherwise it is not possible.

These coefficients have to be known ok. So, I do not have to do any you know Fourier transformation or anything like that, just take the B operator, create an input output relationship, replace B equal to the power minus j omega and take the mod of it, that whole ones mod of mod square that gives you the power spectral density. Is there a is there any other way to come to the power spectral density yes.

So, if the sequence is say y n, it is a random sequence what I can do. I can create an auto correlation function; let us call it R of k. Now, we have assumed it is a stationary process. So, it really does not depend on n that is the advantage we are getting. So, what is R k how do you define R k? So, R of k you define it is expectation of your y of n and y star of n plus k ok.

So, you can define it. Now, if it is an you know if it is a stationary process, then R k and R minus k both will be equal, in that case you find out what it is. Then what how do I get a power spectral density of it? So, the same thing this mod Z square right, the same mod of Z omega square, you get it by take a Fourier transformation of your R k take a Fourier transformation R k and you get Z omega square.

But, you know there is a slight difficulty here, because first you have to get the autocorrelation function. Once, you get the autocorrelation function, you get the you get a sequence ok, you get a even sequence k minus and plus both side you get a sequence. Then, you have to take a you know Fourier transformation on that ok.

And, then you just get a, in this case you cannot get a Fourier transformation, it is a DTFT, just take a DTFT on that, you will get Z omega square, because it is a discrete sequence you get Z omega square. So, this is another way of finally, another way of getting your spectrum or power spectral density yeah. So, from there you can get a spectrum. So, it is a another way of getting your power spectral density.

Now, we will try to do some example on the auto correlation aspect here and see what exactly our power spectral density comes into picture here ok. So, this was the model that we get it after the B operator. So, this is the input out. Now, you think that that is kind of an input output relationship right using the B operator. So, this is y n is equal to you know some polynomial whatever I have written here and then e n.

So, what I can think of I can think of it something like that. I can think that if y n comes here y n by e n I put it is equal to this whole thing, whole inverse I will put it like that right. Because, this is what it is. It is y n is in the left side, e n is the right side, and there is the this whole polynomial part is present here pth order polynomial present.

Now, y n by e n you take the other side the whatever is there inside the red part you take it in the other side right side. So, what will I get? This y n by e n is equal to this whole thing inverse, this whole thing inverse will be there ok. Now, what can I think of this. This whole thing I can think of it as a input output relationship with some linear model right.

Now, in this case let us call this as a Z B this whole thing, this whole thing I call it as a Z B, some polynomial factor right, I mean you can think of it is some sort of a transfer function. What does this transfer function represent? You think that, this transfer function is representing a it is like an IIR filter, infinite impulse response right, because this is whole to the power inverse.

So, now, I can say this whole thing can be modeled as if like I have an input e n, which gives me an output y n, it goes through a IIR filter called Z B, which has a transfer function 1 minus a 1 B 2 a p B to the power p whole to the power inverse. So, that is kind of an IR filter you can think of and e n is going through that IR filter and I am getting that y n I can think of that way right.

Then, how do I get a spectrum? I get a spectrum, how do I get a spectrum of it or how do I get a power spectral density of such kind of; such kind of transfer function. Just replace the Z operator in a normal case right, in a normal filter you replace the Z operator by e to the power minus e to the power j omega. Similar thing you do it. Here B is equivalent to that Z inverse, because it was a delay element right.

So, just replace B by e to the power minus j omega. What will you get you will get some j Z omega you will get that particular this particular things you will get it, 1 minus whole inversion that, because it is just an IR modeling.

And, then what you get you get? A Z omega square and everything you will get it, as it is you get actually a power spectral density right, because if you know what is Z omega, what will you get, you get everything, you get the power spectral density, how do you get a power spectral density?

You replace the B by e to the power j omega you get the omega domain data, and just take the conjugate and multiply it you get the power spectral, you get the power spectral density. So, what is power spectral density? It is nothing, but mod of Z omega square, that is exactly your power spectral density, you plot it you get a power spectral density. This is same thing we have done it here you get a power spectral density using Z omega square, you get spectrum and so on so forth.

Now, same thing you can use auto correlation ok. So, what is the definition of auto correlation of a sequence? Auto correlation of a sequence, if it is a stationary sequence, it will be defined as expectation of y n into expectation y n into y star n plus k.

So, that is the definition we will do some mathematics we will do some examples, how to get an auto correlation of a simple sequence we will do that, but that is the definition of an auto correlation. So, once you get an auto correlation, again I am kind of half done for power spectral density estimation, because to get the power spectral density you have to take a Fourier transformation of it.

Now, here you cannot do a Fourier transformation you can take a DTFT of that because it is a discrete sequence right, because it is a discrete sequence the auto correlation will also be a discrete sequence, because k is kind of you know it is a sequence. So, plus 0 plus minus 1 plus minus 2 and so on it is an integer correlation its a sequence right. So, how do you get a spectrum of a sequence take a DTFT as simple as that and it will give the same Z omega square.

Now, there are problems here. What is the problem? First you have to get the auto correlation ok, which may not be so, easy to get it, but for a given sequence probably it may be easier, but you have to do lot of expectation operation and so and so forth. And, then you have to take a DTFT operation ok.

Now, this is one approach of getting power spectral density. And, other one which is the very very simplistic approach is the one which I have just shown, use B operator, this is like a filter. Either you get a FIR filter or get an IR filter or get a mix and match, is like a your p and q both everything its like IR or FIR filter you get it. And, the job is to get the spectrum of the IR filter. So, you replace B by to the power minus j omega done you get a power spectral density ok.

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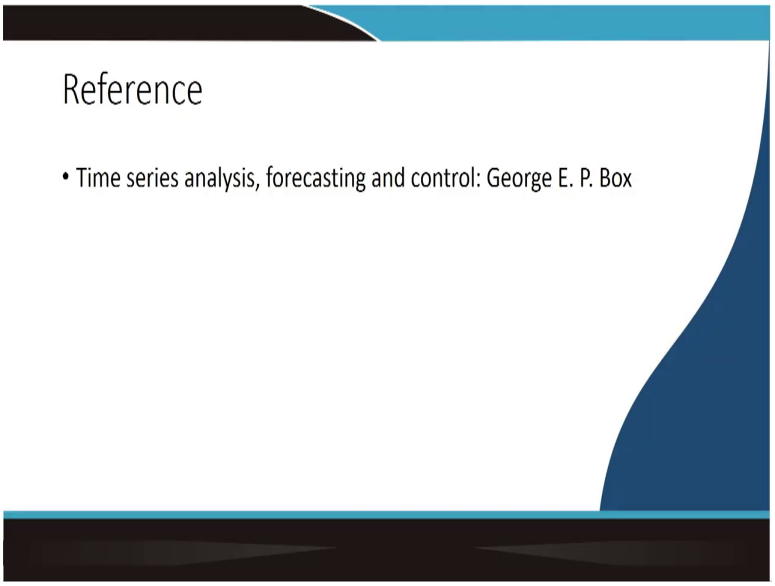

So, that is the reference book Time series analysis, forecasting and control by George E. P. Box.

Thank you.