Signal Processing for mmWave Communication for 5G and Beyond

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Module - 03
Understanding of various channel related parameter statistics. Narrow band and broadband aspect
Lecture - 14
Doppler impact on coherence BW

Welcome. So, today we will be covering the time series part and the Doppler part of the channel. Last classes we have discussed some of the impact of the Doppler on the spectrum and we have tentatively concluded not completely, but tentatively concluded that the spectrum will be somehow shifted either left or right, ok. So, let us get into the details even more.

This is the lecture number 14 of module number 3 we will be talking on the Doppler impact on coherence bandwidth.

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So, these are the concept that will be covering; Doppler and then its impact on the coherence bandwidth. So, see if it was a static environment what was the channel transfer function.

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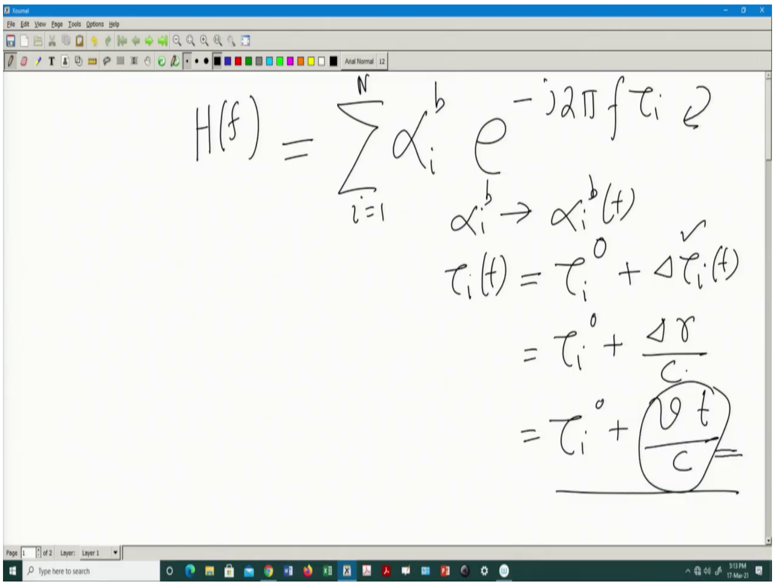

So, this was the channel transfer function, right. If it was a static, so there was no t term or a time term here. So, this was something like this. And then it was e to the power minus j 2 pi f tau i this was the case right.

Now we say that there will be a Doppler. So, what does it mean? It means that alpha i b will be a some function of t, but most important part is this part tau i t. So, that should be a function of time which will be some initial value plus some delta, whatever it is some delta value will be there in the time in if it is a function of t.

So, this delta will be a time function, ok. So, how. So, this will be a time function. So, it will be something like tau 0 plus some delta r. The r is the amount of distance that it covers; within a some whatever time frame you consider divided by c that comes around to be something like that v into t divided by c, right. Now t can be delta t the amount of time you consider it, ok. So, this is what we looked at it earlier.

See if I put this part into this equation what will I get? I get a transfer function which will be a function of time now, right.

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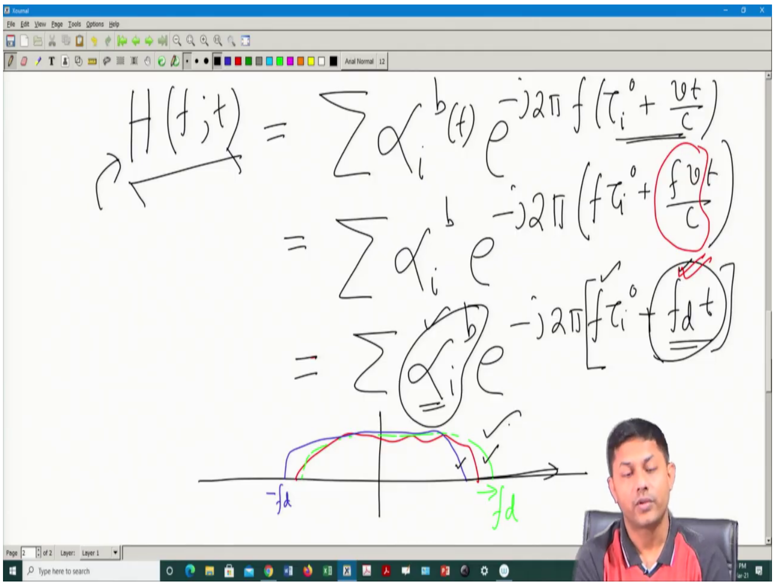

So, now I cannot write only this H of f, but rather it will be some function of time naturally, right. So, this would be alpha i b let it be a time function of t and this will be e to the power minus j 2 pi f; instead of tau I will write it like this. This will be what I have written v t by c, right. But I would prefer v t c we somehow some more conversion it will have let us see that. So, it will be v of t by c, correct.

Now, I just proceed further, now this is more of a some mathematical notion of what happens to the spectrum of the channel when there is a Doppler, that is what. But this may not be very complete picture because it has lot of assumptions ok. The actual picture of the spectrum what would happen will come from the time series which I will explain next.

But this is more of a kind of I would say a feel of exactly a feel of what a spectrum would be or rather a feel of what the spectrum would become if there is a Doppler. So, that is what I am trying to highlight it here. So, this is not even an accurate way of modeling, but this is more of a tentative way of modeling, but accurate way will be definitely time series or a you know random process way.

Now, I put it inside this f of tau i 0 plus f v t by c ok. Now what is this quantity? This quantity is nothing but your Doppler frequency, because f v by c that is precisely the Doppler frequency. So it would be alpha i b this is what we have wrote we have written last time; it will be minus j of 2 pi f of tau i 0 which was the earlier case, I would put it little bit here plus f of dt something like that, correct. This is what it is.

So, what does it mean? It simply means that the spectrum; what was the spectrum of this h t? It was some step of the spectrum was there. Now I see that this is the amount of shift which has happened to the spectrum. I can write it like that. So, what tentatively if I say that f d is equal for all of them; all of them suppose whatever components are there only one of them is moving so that with respect to all of them I can get the same you know Doppler. So, though it may not be the case, but this kind of a assumption.

So, I would say that the original spectrum where this f d was 0 versus the one where I will see a Doppler it is nothing but a shift of frequency; this much shift has happened this f d amount of shift has happened. So, if it was f earlier now there is a shift of f plus f d. So, which means if I had a spectrum of like that. So, say this is what my original spectrum because of Doppler; so that is an f d amount of shift would have happened.

So, there is a f d amount of shift may have happened here, ok. Now this f d can be positive or negative depending on the direction of the movement. So, it can be even this direction as well; I would say there is a left side or a negative side shift would have happened. So, this is exactly the you know the impact of the Doppler. So, Doppler is I mean it mainly what it does is a; is a shift of the spectrum.

Now, this is not a very accurate picture, why it is not an accurate picture? Because what I have not considered is that what happens to this fellow? This is not constant anymore because after time t this alpha would also change.

Why it would change? Because it may be that you may see a totally different reflector this one case, or even if not a different reflectors because there is a movement so the distance from transmitter to receiver via any reflector would also be different because there is a time; there is a time gap.

So, the distance would be different because everything is moving, right. So, this alpha if I see that is that is only a path loss there. So, everything I mean because it is related with r and if the r itself changes then alpha itself will be changing, because here also we have seen right, because the whole reason of changing delta I mean this delay is that there is a distance changing. So, naturally this alpha will also change.

Now, alpha is a complex number now if the complex coefficient of this particular filter changes what is the natural intention? I cannot say that this spectrum is just shifted right, they are slightly there may be a phase difference of the overall spectrum, there may be an amplitude difference of the overall spectrum. So, all things can happen.

So, I cannot say that this green curve, this green curve or this blue curve is just a shift of left and right with respect to this red curve. That may not happen. Yes, there is a shift, but the spectrum may not remain identical with respect to red, because there is a change in alpha ok there will be a change in alpha.

And this will be more prominent if this f d or the velocity will be very high. Then the shift will be much larger, then alpha will also be larger; if the alpha becomes larger you cannot say that, that this blue curve or this you know this green curve will be exactly identical with respect to red curve.

So, the in conclusion when there exists a Doppler, the spectrum this spectrum is actually a time varying spectrum; why it is a time varying spectrum? Because now you can see this movement this right shift or the left shift is dependent on time; whatever at any particular time whatever velocity you have depending on that the movement can happen right. So, this is a time varying frequency kind of things.

And second thing is that the shift may not be identical. In the sense that it is not the same spectrum getting just shifted left or right that may not be may not be the case provided the Doppler is very high. But if it is if the Doppler is kind of within a limit yes more or less the spectrum is just shifted left or right. So, this is very interesting phenomena that Doppler can introduce.

So, in a sense or in a simple conclusion I can say that whenever you have a Doppler, the spectrum is just wobbling it will be just wobbling mean vibrating is kind of a it just like a you know it just vibrates.

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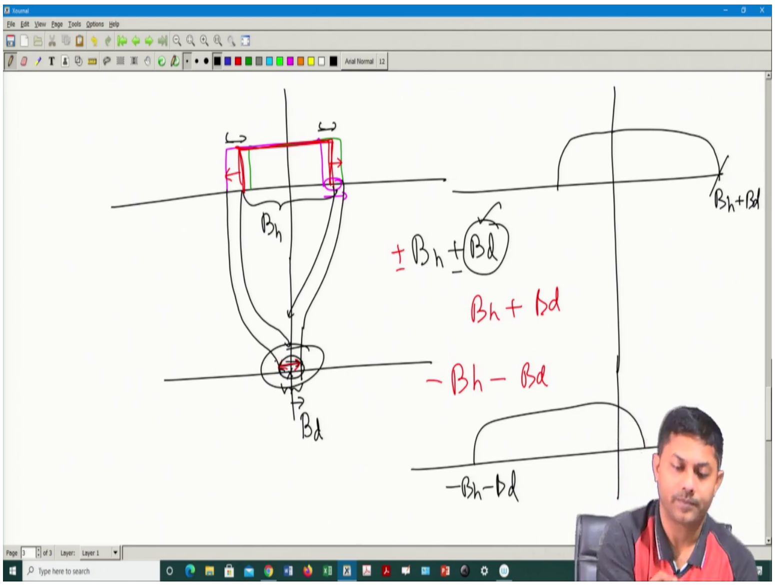

So, which means that as if like you have a spectrum this red spectrum ok, it just keeps sliding left and right; left and right depending on how exactly the Doppler is there. So, there will be left shift or there may be there may be right shift or there may be a left shift ok. How much it will shift? That depends on the on the velocity of the system.

Now, the question that we would like to answer here is that, is this shift a constant shift it is not right because it is a time varying shift, the velocity itself is a random variable; why it is random variable? The reason is that your actual scenario or the physical scenario is a completely random scenarios, right.

Even if you think of yourself, if you take a mobile and move do you guarantee that you will be always moving in the same direction with the same speed, it never happens, right.

If you drive a car the speed of the car will never be constant especially if you are driving in a city right, depending on the traffic the speed will be constantly changing that. So, you will have a different level of acceleration. Secondly, if you have a instead of you being the receiver suppose you have a lot of reflectors in your area.

Say for example, you are in a city the reflectors could be the cars or moving vehicles or a you know any flying objects anything can be there reflectors or scatterer or maybe a tree, ok. So, tree branches will be you know vibrating due to the air.

So, how can you ensure that everybody is moving at a very constant speed or how can you ensure that everybody even moving in a constant direction? So, that is a completely random in nature, right.

So, every individual reflectors, scatterer maybe even a transmitter and you being the receiver everything; everything can move in all direction and with a you know with a random movement. In that case it is extremely difficult to even predict how much this shift would be right, because this itself is a random variable right. See this cannot be predicted you can predict only if it is a constant.

So, can I have some sort of a you know average value of shift? Because when something is random and I want to know how much its impact is, right. For example, I am transmitting a power and that is random in nature and I want to know what is the power. So, I never can I will never be interested in an instantaneous power rather I will be interested in knowing what is the average power.

So, when something is random similarly I will also be interested to know what is the average such shift happens. So, if I make a conclusion something like this that the amount of shift in the spectrum I just take that fellow apart. So, that mean this is your red spectrum, whatever shift it happens in left and right I just take that as another signal.

So, for example, I am interested to know how much this shift has happened. So, I put it back into the 0 and I said this is. So, this is the amount of; this is the amount of shift I am seeing with respect to that. So, that mean this is the data B h, so this one is B h this red is B h ok and this one is say B of d, ok. Now on an average I can say my spectrum moves with B h plus B d. So, either it can be B h plus B d or B h minus B d. So, that is the on an average it can have.

Now, this B d itself is a random number. So, how much the shift would be right, because nothing is moving at a constant speed it will be very random in nature. So that means, I am now interested to know how much exactly the bandwidth change happened.

So, the changed bandwidth, the amount of bandwidth that gets changed; in this case the bandwidth does not get changed rather how much the bandwidth gets shifted by. So, that particular bandwidth I am referring to as a Doppler bandwidth.

That means, that is the bandwidth which is arising due to the Doppler, right. Because as you know if the Doppler is 0, this band this particular band I am talking will also be 0. So, which means if you have this red color original spectrum I would like to know how much this particular shift how much this particular shift left and right.

And based on that I am just taking only the extra part. So, which means that overall the spectrum will be either B h plus B d or this side it can be even. So, it will be B h plus B d amount it moves in the right side or it could be minus B h minus B d amount moves in the left side. So, I should have written plus or minus because both side it can move, right.

So, which means that after the Doppler, I may have a scenario where the bandwidth would have moved here, B h plus B d amount. So, there may be a smaller amount here if it has shifted in the right side. So, that mean its bandwidth is now B h plus B d or its bandwidth can be this minus B h minus B d, right.

So, you can say tentatively that its bandwidth has actually you know because of this shift the baseband bandwidth got shifted, so it has increased. So, it just added up right this extra bandwidth comes into picture in the system. Now you want to quantify that. That is the job of our; that is the job of analysis how do you quantified?

As you said as everything is moving in a random nature, it is virtually impossible to quantify that. So, we can all what all we can do is to quantify in a statistical sense. So, what is the mathematical tool that is available for us that can guide us to quantify that particular amount of bandwidth shift that can happen? So, this bandwidth I am interested in this particular bandwidth I am interested, ok. So, let us see.

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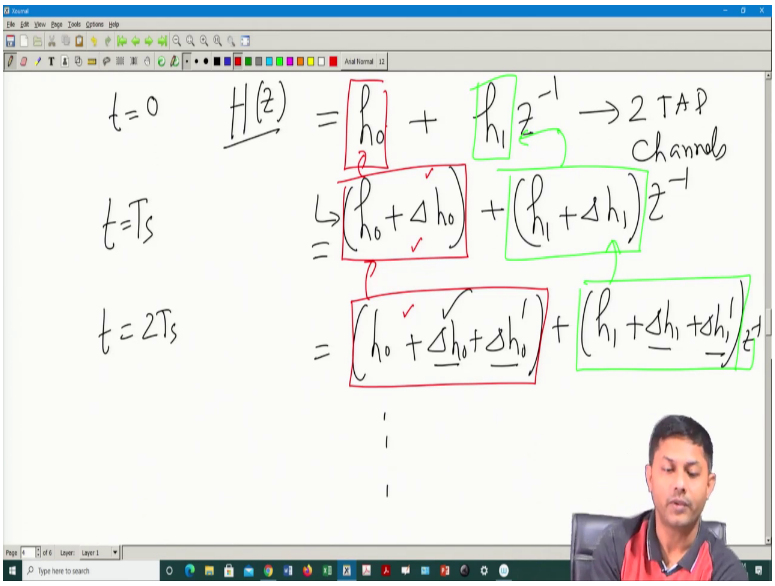

Now if I just draw our original channel. So, let us say this is our original channel z transformation of my channel what was that? It was same just take an example let us say I have just 2 tap channel simple 2 tap channels. So, I would say this is a 2 tap channel that we have observed just a simple 2 tap channels we have observed it at a particular time, this is all just static environment.

Now this is say for example, at t is equal to some because we are sampling it we are in a mode of DSP. So, this is say 0 at t equal to 0 and let us say my ADC sampling rate is T s. So, my t is equal to T s what would happen? This band this channel will no longer be the same channel it will be modified to h 0 plus extra part will be coming.

This part we have already discussed it why the channel will changes plus this h 1 would also be something, ok. Now t is equal to 2 T s again some changes will happen originally h 0 plus delta h 0 plus delta h 0 dash may not be the same amount of changes right, plus here also there will be changes delta h 1 plus delta h 1 dash z inverse and so on so forth correct.

So now, how do I analyze that? Now you can see that this is a random variable the amount of change that happens is random variable; why? Because the Doppler itself is not a known quantity to me its a random variable to me. So obviously, delta h will be a random quantity for me. Naturally delta h dash will also random this is also random this is also random correct.

But you notice one thing if I consider h 0 and h 0 plus delta h 0 are they correlated? Naturally correlated because this is h 0 now here h 0 plus delta h 0. So, given or given the fact that you know h 0 the only randomness comes from the delta h 0.

Now if h 0 is a random variable. So, h 0 plus delta h 0 is also another random variable, but between h 0 and this two quantity it is a correlated random variable. Similarly, this one is also correlated with the previous one and this one is correlated with this one. Similarly I have a similar conclusion this quantity is correlated with this quantity. Similarly this one is correlated with this quantity.

So, every time I have a change there is a randomness, but there is a correlation, ok. So, given that fact can I do a better modeling? Ok. So, let us see that.

So, how I can view it? It is all depends on how do I view my whole system.

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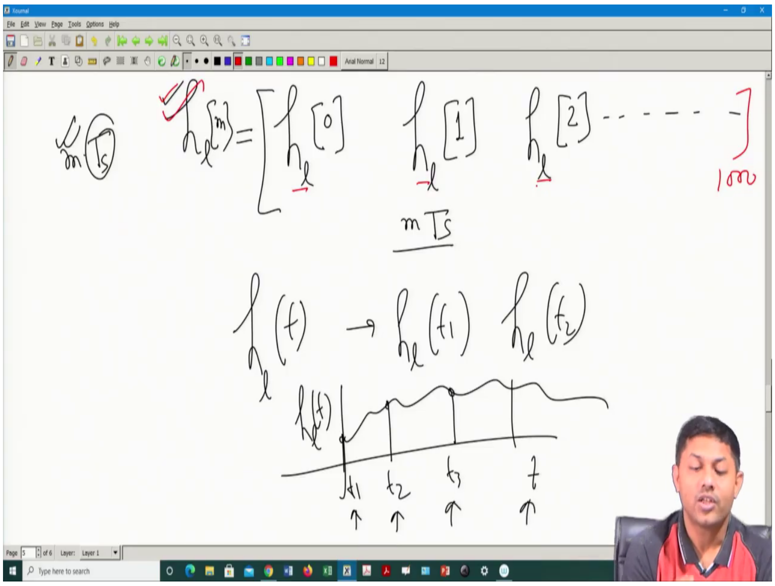

So, let us say I am only viewing h 0 or h l l-th tap in a sense ok let us not get into only h 0 or h 1 l-th tap. So, if I observe l-th tap what do I say? This is h l I am observing at time 0 this is h l I am observing at time 1; 1 meaning m into T s is m into T s. So, T s is known is just this integer quantity.

So, next I observe. So, I can think of it like that; I can think of it like that if I keep on observing at m T s time, I can see that at different-different time I am seeing different-different observations, ok. And all are correlated, this is what I know. That mean h l of 0, h l of 1, h l of 2 and h l of whatever time they are all correlated with either the previous value or some other value and that is where the modeling comes; how you actually model your correlation, ok.

So, this one I can think of it or I can view it as a random process; I can think of it like a because at different time. So that means, h of l if there exist a random process. I would say h of t 1, h of t 2 if I sample at different as if like as if it just like a visualization this is what say my h of l t; continuous time how it is varying. If I imagine some sort of an envelope over the time I would say this is my t 1 time this is my t 2 time I am observing this is my t 3 time I am observing and so on.

So, I can think of it some sort of a random process because h l t is a random process because it is varying over time and each and every instances itself is a random variable. So, which means that at every instances of the data of the h l is actually a random variable ok, but notice this is a correlated random variable.

So, I can say this itself this h of l or rather m I should put it here this h l of m I can view it as a random process that mean if I observe it for several times I would see my hl the l-th tap will keep changing with and there exist a randomness at every time. So, it is a random process, ok.

One part of the story. Second part I suddenly discover that they are all correlated. We need to figure out how much the correlation is we need to mathematically model it, but there is a correlation. If that is what the case that means, a random process and there is a correlation and if I know the correlation can I do a better modeling of my channels time behavior time changing behavior.

So, what is the ultimate goal? Ultimate goal is to say what is the randomness or what is my you know the statistical correlation and those kind of property for different-different times. Because that is precisely what is required for me because I may have a channel and I want to know what is my channels coefficient at different-different time, ok. I need to know the modeling first, because I need to know how much the bandwidth will be shifted by because I need to know that amount, ok.

So, these are the kind of situation where we are in. Now what can I say? It means that I can generalize this model.

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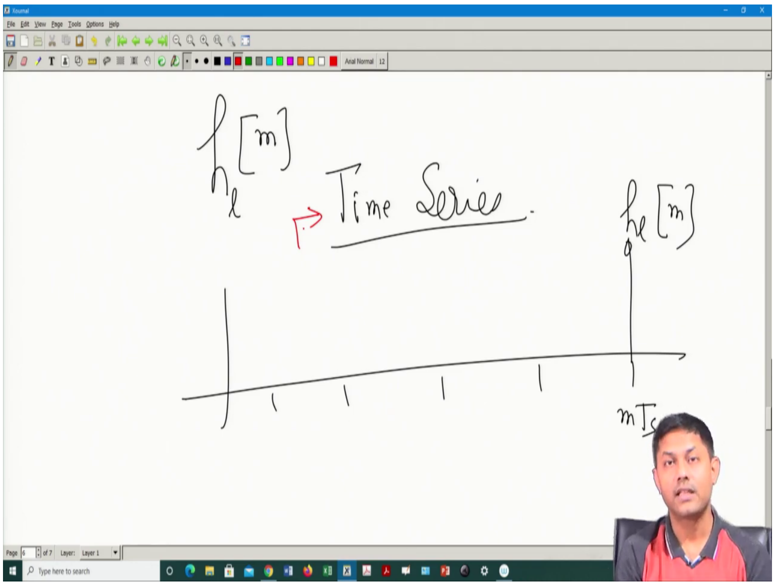

I can say that h of l at some time say m that m-th time that mean if I observe it at uniform distance. So, this is at m T s time whatever you know h l I am getting. So, I call it h l m that is the value, ok.

Now, what are my job is to model this h l of m that mean h l at the m-th time I want to model the random variable its relationship so and so forth with respect to the other variables which are predominantly the h l of other times. So, this kind of modeling when I want to when I am interested to even know this kind of modeling a good mathematical tool is the time series; a time series.

So, times series is a very interesting tool mathematical statistical tool I would say that can allow us to model the correlation of that h l at time m with respect to its previous value.

Because this is what my interest is, ultimately what am I interested here. I want to know what is the correlation at this point with respect to the second one previous one or rather what is the correlation of this fellow with respect to its parabola; I want to know that information. This is what I am need I am required to know; that means, unless I know that I cannot figure out how much my you know how much my bandwidth that would be coming into picture here, ok. This is what I am interested to know that.

Second point is, this one motivation why I should go for the time series modeling for my h l m a; second one. Ultimately at the end of the day I am getting a sequence right I am getting a sequence for my l-th tap different time sequence right.

So, it is a sequence of data ok and what is my final goal? Is to know how much my Doppler bandwidth. That mean how much this sequence is creating a bandwidth for me, but there is, but they are all random sequence. This is actually a random sequence right and this is a random process I would say.

Now, when somebody gives you sequence a random process and wants to know how much the; on an average this particular sequence would occupy on an average how much bandwidth this particular sequence will occupy.

What is the natural question that I would like to have it? That mean if I take if I know this sequence immediately I want to do what? I want to do the DTFT operation and I try to find out what is this bandwidth of it. But my dear friend this cannot be done here, because they are all you know they are all random numbers.

So, DTFT will only give you an instantaneous value of the spectrum, but I want to know on an average what is the spectrum comes into picture, because this is a random sequence, right. So, the best way to know a particular sequence how much this randomness generates a bandwidth is by what? Is to get an autocorrelation function. And thereby if I know the autocorrelation function I do the Fourier transformation I get a average spectrum on that.

See if I just take this sequence I will take thousand values of that sequence and I do a DTFT on that will I get a spectrum naturally I will get a spectrum. But that spectrum is a instantaneous spectrum, that is with respect to a particular value of them. But that will not be the case because this is a random sequence, you may be observing right away this particular sequence. If you change the time you may see a different sequences, right.

So, DDFT will not give you perfect value of your you know perfect value of your spectrum because that is instantaneous. So, I need to take a different route to get a spectrum of it. Now time series obviously come into picture.

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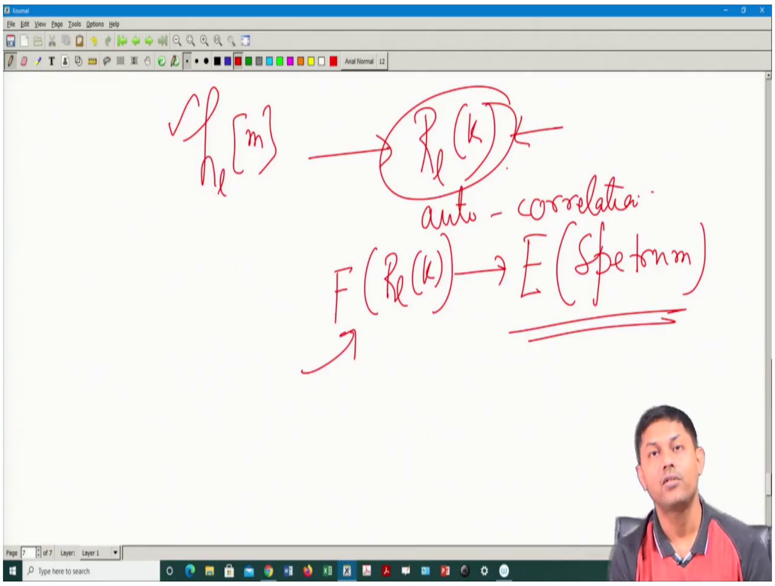

So, that answer can be given if I have a sequence some sequence and if I take the auto correlation of the sequence ok. If I take an auto correlation of that sequence, so this say this is auto correlation.

So if I get auto correlation just take a Fourier transformation of this auto correlation I get the average value of my spectrum and this is precisely what I know. So, this is your that Doppler spectrum the extra shift comes into picture generated by one tap got it. So, this is my ultimate goal. So, that I need to get some sort of an auto correlation function, finally I have to do some Fourier transformation I get the spectrum. This is the ultimate goal, so that mean I need to know the auto correlation.

Now, to know the autocorrelation you have to first define the sequence, because unless you define the sequence properly you cannot take the autocorrelation. And that is where another motivation why time series would help us? Because time series well defines the correlation among the sequence point. And you can get very easily the autocorrelation generation and hence you can get the average spectrum. So, this is the ultimate goal why time series needs to be known here, ok.

So, we stop it for this class. Next class we will be talking more about the time series part, ok.

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So, this is the reference; Fundamental of Wireless Communication same book which we have communicated earlier as well.

Thank you.