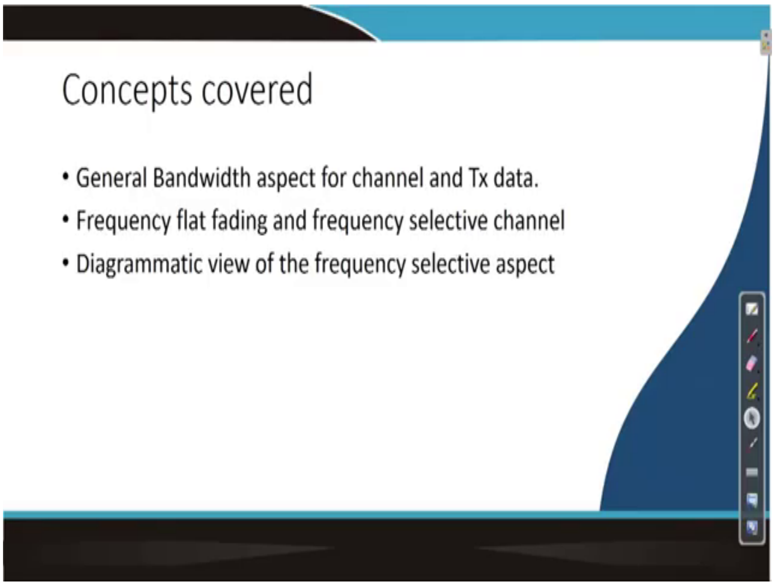
Signal Processing for mmWave Communication for 5G and Beyond
Prof. Amit Kumar Dutta
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Module - 02
Wireless Channel - A ray tracing model Part-II
Lecture - 11
Wireless channel-A ray tracing model part-II (cont)

Welcome to the Signal Processing for 5G and Beyond in millimeter wave context. We covering the lecture number 9 for the Wireless Channel A ray tracing model part 2.

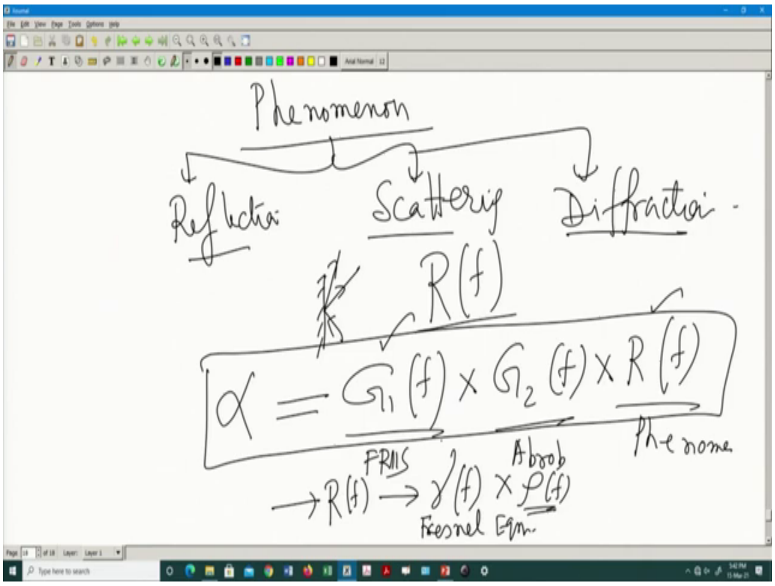
(Refer Slide Time: 00:43)



Things that will be covering are the following General bandwidth aspect of channel and Tx transmitter data. So, we will be showing how exactly they will be related and what happens when one of them is greater another one is smaller, so that kind of conceptual aspect.

Then I will be talking about frequency flat fading and frequency selective channels and then some diagrammatic view of the frequency selective aspect. So, what exactly it means for it. For example, there will be FIR filter, IR filter of the channel models that will be coming into picture here.

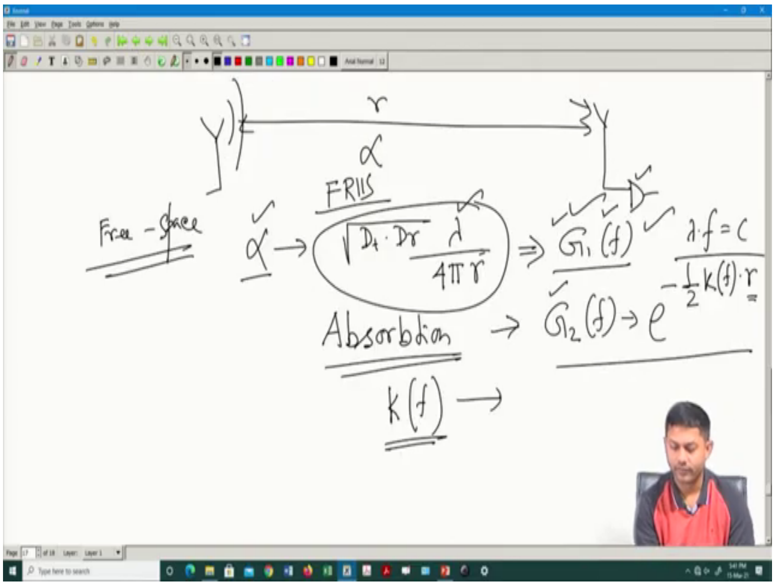
(Refer Slide Time: 01:25)



So, far we have explained some of the characteristic of our alpha right. So, we have seen that alpha has you know predominantly 3 components ok.

One is your path loss component which is defined by the well known Friis equation. Second one was the absorption related things, the third one was phenomena, dependent this 3 thing reflection scattering and diffraction these 3 phenomenon’s that will come into picture and this Friis part we have explained in detail what is the exact equation.

(Refer Slide Time: 02:01)



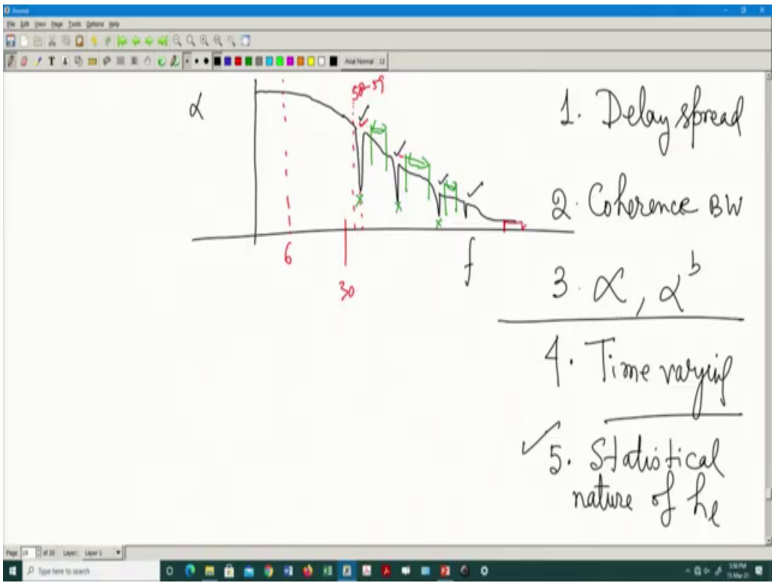
The absorptions we have kind of shown you exactly e to the power minus 1/2 k of r, but I am not getting into the k f part because this is more of the physics related equation. But you can have a look at the reference point and get a big equations because the intention is more of here signal processing part here.

And the third part was the reflection part this reflection part has 2 component this Rf. This Rf will have one component is called is actually the reflection or scattering. So, that is basically the physical part of the activity see if it is a reflection if it is a scattering or a diffraction there is a physical activity. So, it is called the Fresnel equation.

Second one rho of f this is basically called the roughness surface roughness that is also frequency dependent. So, this is only two now exactly how the gamma f is that depends again that is complicated equation let me not get into that ok. We may off we may get off tracked by that again the same paper whatever reference we have given the big equations are given. So, that is not our interest is.

So, here we want more of a discussion and signal processing part of it. So, here what are the what are the aspect that you need to look for when you are in a alpha. Now because that drives our millimeter wave you know that drives our millimeter wave channel model and some of the aspect that we want to bring it out here.

(Refer Slide Time: 03:44)



So, now we know for sure one part that this alpha has a frequency dependency we explained it earlier through the path loss part. But apart from that it has 3 dependency frequency dependency; definitely, but it also depends on the material through which it is going correct right.

Because if you look at this G 2 this component G2 that was an absorption component, that mean the material it gets hit or it goes through say air is a air is a composite you know composite materials right it has. So, many materials there or if it say hit by a wood or if it is hit by glass or if it is hit by say some plastic body or it can be human body also anything that it passes through or hit by. So, G2 f plays a very important role.

So, if it is hit by a glass if it is hit by a wood the amount of absorption that will be going on is completely different. Or if it goes through air normal air for example and the amount of material that is present inside the air that also get a different level of absorptions ok. So, if I plot it if I plot the alpha or the gain factor of my you know gain factor of my channel somehow. So, this could be alpha some of the alpha meaning as a whole the gain I am talking and this is the frequency.

I will see that the gain will be predominantly going low as I increase my Rf; obviously, right because if I increase my R f the Friis will put it down. But apart from Friis equation or the Friis space equation other components are also coming into picture and that is important. So, I will suddenly see it is going down obviously, but suddenly I will see some sort of a dip like that it will go ok. Why these dips are coming?

So, the these dips are mainly because of G2 absorption phenomena ok. So, these are called you know the resonance frequencies of those materials. So, suppose for example in the air there exist some material something whatever it is may be H2O maybe carbon dioxide may be oxygen which has a resonance frequency of say around 60 gigahertz. What does it mean? It means that whenever I make a 60 gigahertz transmission that particular material whatever it is will absorb the Rf power most of the time it will just you know kill the power of the Rf.

So, this kind of phenomenons are very predominant in the Rf spectrum. Now we have seen this was around 58 59, so around you know just after 60 gigahertz. You will see one dip and there are some more it is around 80 and there are many this dips are there everywhere this dips are there. So, that mean in the air there are materials for example H2O or carbon dioxide or oxygen which has a resonance frequency same as that particular frequency of Rf.

So, it just kills that Rf, so you cannot make any transmission at that particular frequency. So, this is just a guideline kind of now such kind of things happens only when you go in the higher Rf. But if you are in the you know if you are in the 6 gigahertz or 10 gigahertz it is kind of flat it is not an issue.

But moment you go 30 megahertz 30 gigahertz or 40 gigahertz beyond it such phenomenon’s are very common phenomenon’s ok. So, this is limiting your you know transmission bandwidth. So, you can say that I can transmit only this region I can transmit within this region I can transmit within this region and so and so forth.

So, you have to look at that curve and that curve is a very standard curve and that shows that what should be your transmission region, such that you won’t see such kind of dips. The dips have to be avoided because you have to pump more power to get some say signal strength ok. So, this is one of the one of the impact of this G2 f ok.

Now, Rf is this part the phenomena dependent part is again dependent on the phenomena how many reflectors are present, how many scatterers are present that defines how that particular Rf will be there. Of course, this has impact but this G2 f has a really good impact on the material part ok.

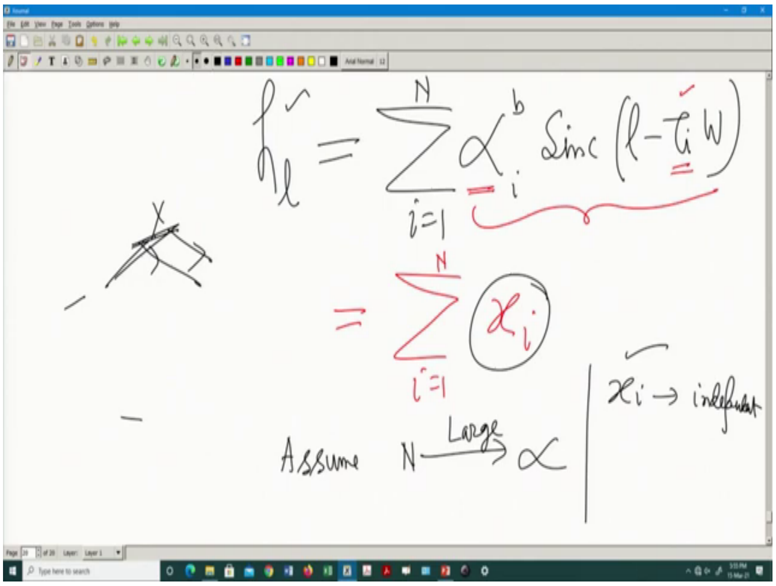
So, now let us so this part is more of the millimeter side yeah this will be more predominant and that determines how the bandwidth will be there. Now apart from this parts now let us get into our time varying issues. Because now we are slowly getting into the millimetre wave and now we have to understand what are the other aspect on which our channel coefficient depend on ok.

So, one part we have defined from the channel side one is the Delay spread, then you have defined the Coherence bandwidth, then third what we have defined the characteristic of alpha 3 things we have defined it. Alpha or alpha beta both are same it is just that this becomes a complex number due to the delay element which we have already shown it here.

Now 4h point now we are getting in the time varying part is the Doppler or time varying time varying part. How exactly a time varying issues can be addressed in our channel model. So, far we have not consider so far you have assume everything is just a static ok. One more part I would like to have it the statistical characteristic, statistical nature of individual hl. So, we have the channel taps what is the statistical nature of this channel, because we have seen that this can be a random variable.

So, what is the statistical nature of the channel you need to understand that part ok. Now we have finished this three before I get into 5 4 let me explain quickly the statistical nature of h l ok. After that we will get into the time varying the time varying takes some time because it is time varying ok.

(Refer Slide Time: 11:22)



So, let us get into the statistical part. What was the equation for h l? The equation for h l was summation of alpha b i sinc l minus tau i into w this was the equation, how many such data are present it depends it depends on the reflector; scatterer it depends totally it depends right.

So, this was my digital view of my channel I repeatedly say this is a digital view of my channel this is the lth tap ok. Now what is the random number here in this whole case in this whole equation, can I say this alpha is a random number; obviously, why it is a random number because I do not know where the reflectors are present right. It is completely unknown suppose I am at the receiver side I really do not know where the reflectors are present it is invisible right, you do not know [Laughter] through which reflector it really comes to you right.

So, you alpha is completely an unknown to you it is a random number to you not unknown I would say definitely it is unknown, but naturally it is a random number random variable. What else is the random variable tau i is also random variable why it is a random variable because alpha I do not know why I do not know alpha, because I do not know the distance from transmitter to receiver through which it comes back. So, if I do not know the distance naturally my tau i is also random variable alpha is also random variable.

So, as a whole this whole becomes a random variable ok. So now, I can say this is some random variable i equal to 1 to n some random variable let us call it x i l x i ok. Now let us make some small assumptions. What is the assumption? Assumption is that assume N is very large now how large it is let us say N tends to infinity N is very large very large.

So, I would say it is a large second assumption can my x i be this whole x i what I am talking this individual x i be independent can it be independent yes it will be independent.

Why? If I assume that each and every reflectors or scatterers are independent if I assume and which is not a very bad assumptions ok. But that also won’t hold true all the time because it may happen that a same material one side both the side of a same material like a it is a larger material this will act as a reflector this will also act as a reflectors.

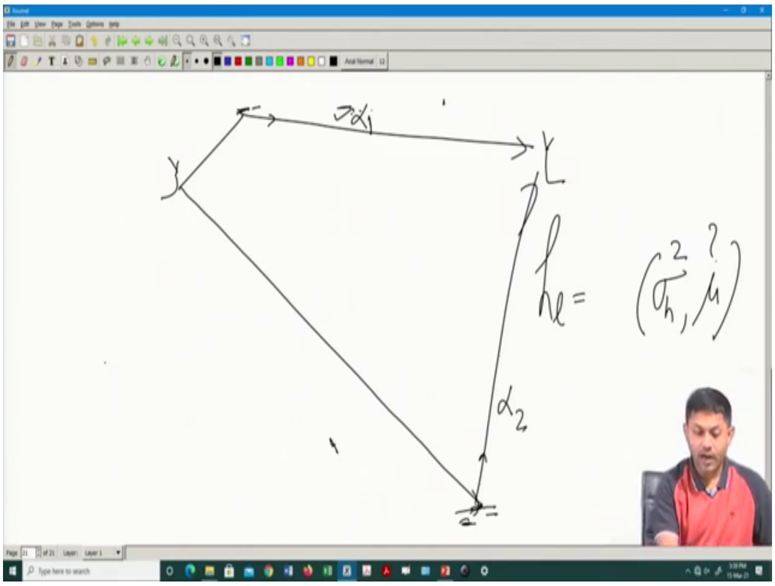
So, they are correlated kind of things right because the surface density surface roughness may be equal and path wise also may not be so much different slight difference so there may be. But apart from those kind of you know issues which will make the millimetre wave more interesting where channel becomes slightly correlated, this individual path will itself will be correlated.

But in a sub 6 gigahertz I would safely assume that individual x i itself is correlated, because individual reflector scatterers their positions are independent of each other and one reflector is one reflector. I mean we do not have multiple such cases parallel cases ok.

So, I can say this alpha is independent and hence the tau i is independent, hence the whole x i is independent I can make that assumption ok. So, all x i are independent is independent ok. So, I have made 2 assumption n is very large x i is independent, 3 rd assumption is this x i identically distributed.

In fact, that is a very crucial assumption I would say that is not true correct. Why it is not true? Because you cannot say that every reflectors are of their power everything will be almost equally distributed from a reflection point of view that is not possible. Why it is not possible look at that that logic is the following.

(Refer Slide Time: 16:08)



The logic is that you have a transmitter here you have a receiver here you may have a reflector here you may have a reflector somewhere here bottom of my screen. So, the guy which is closer to it will always pump you larger power compared to the one which is here right. So, this will be a kind of a weak fellow this will be kind of a bigger fellow.

Now, even it moves little bit here and there even it moves little bit here and there I cannot say that the distribution of or that alpha i and this alpha this is alpha 1 or this alpha 2 will have the equal distribution I cannot say that right.

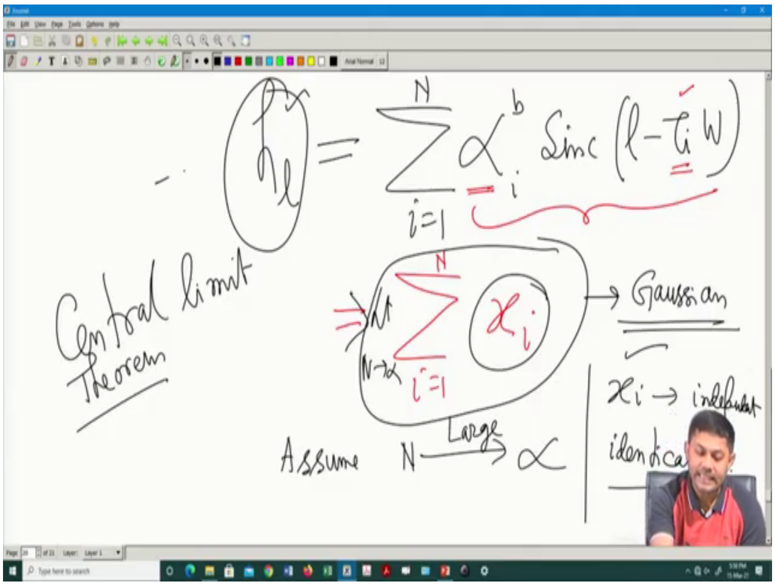
But here I make a small assumption again here the assumption is that though the alpha i can have a different positions. But all have a chance to be distributed in a fashion that makes the alpha I all kind of a same kind of distribution not same distribution

I cannot say that this alpha one and this alpha two is same that I am not assuming, but the distribution can be same kind of that. How does it possible? That mean this reflector will be somewhere here or these reflectors can have a chance to go back here or this reflectors can have a chance to go back here.

So, it is like a uniformly you can think of it is like a uniformly distributed those reflectors and scatterer. So, the power which is coming out of them reflected or scattered power will have the equal distributions somehow.

So, and hence the tau i is also equal distribution ok. So, it is not like a one is a very large one is a small everybody has the equal chance to be smaller and bigger. So, that kind of a though not a very correct assumption, but just for the benefit of you know benefit of this analysis. We can assume that this is independent at the same time this is identical distributed this is identically distributed ok.

(Refer Slide Time: 18:10)



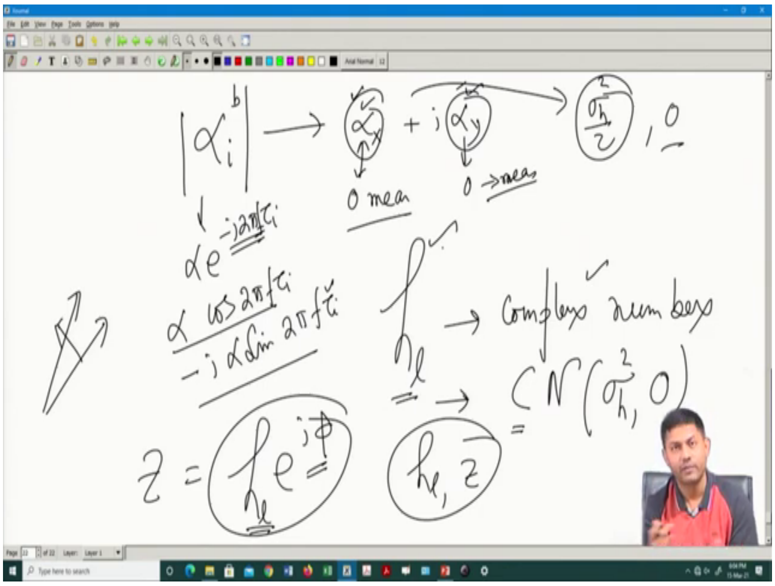
So, it is an independent identically distributed. So, what does it mean it means that this hl I am coming up with a summation of identical and independent variables. Let us say assume that this is somehow normalized, so that is not an issue. So, what is the total distribution of this gentleman if N tends to infinity ok?

There is a normalization factor is assumed there? So, if I assume some sort of a normalization with respect to it is mean and variance I can say that no matter what the individual x i is that the actual distribution of this h l eventually it becomes a Gaussian distribution it becomes a Gaussian distribution ok.

Because that is the this theorem is called Central limit theorem. So, these are some of the key assumptions that I had to make which may not be true, but that makes my job slightly easier and done. So, this whole hl would become a Gaussian distribution and that is the statistics of it.

Now what is the mean what is the variance well that comes next. So that means, some variance will be there it depends on individual x i and this N some you know composite variance will be there. So, moment I say it is Gaussian what does it mean? It means that h l will have a Gaussian distribution with some you know variance sigma. What about the mu what about the mu we need to understand that ok.

(Refer Slide Time: 20:20)



Now, let us see your alpha your alpha i b it is a complex number. So, let us say alpha x part plus j alpha y part it is a complex number right. You do not know how the phase tunes it right. So, this can be positive and negative both because it is a complex number, the mod of this fellow will be; obviously, a positive number. But individually once it becomes a complex number individual real and imaginary can be positive and negative both ok.

So, when you have that so it means it can be either it can be positive it can be negative. So, it is easier to think of this whole thing as a 0 mean, because both can it can take positive as well as negative. Because you do not know how the e to the power j tau because this what is this; this will be alpha in e to the power minus j 2 pi f of tau.

So, you do not know how that will be you know impacting. So, it will be alpha cos 2 pi f tau i minus j alpha you know sin of 2 alpha f of tau i right. So, it depends on how the tau i and l f are so depending on that this whole thing can be either positive negative depends right.

So, I can say this is a positive number negative number this is also both side positive negative. So, it is safe to assume that they may have a 0 mean this will also be having a 0 mean ok. So, that hl which is just a complex number it is a complex number it just a complex number but it is a random variable right. So this whole h l is a Gaussian random variable ok.

So, it is a Gaussian assumption is that it has a variance sigma square h and mu is 0. So, this is your Gaussian now the question is that is there anything else that I need to assume is there anything else that can happen ok, more things can happen this real part and this imaginary part ok.

What does it mean? It means that they have can I say one of them has a larger variance or one of them has a lesser variance no right that cannot happen, because both are equally impacted by this delay. That means, both of them I can say for a safe assumption both of them can have equal variance both of them has mean 0. So, that mean I have a complex number whose real component imaginary component has mean 0, because overall the mean is 0 at the same time both of them have equal variance..

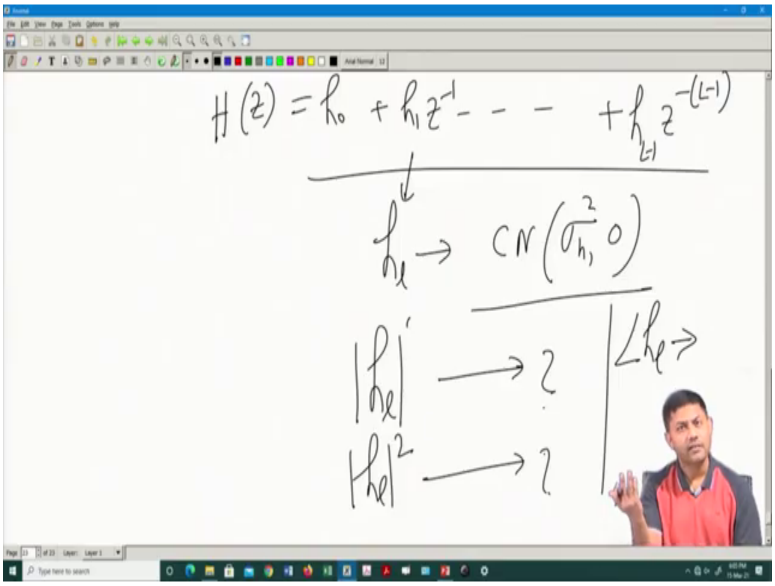
So, I can say that this whole complex number will be cyclic symmetric Gaussian. So, what does it mean it means that if hl is a complex Gaussian number. If I multiplied this function with some e to the power j phi and at that depend that becomes a new random variable. The distribution of these and the distribution of this whole that the distribution of h l and the distribution of z they do not vary if it is Gaussian, both of them remain Gaussian if the same means same variance, so that is your cyclic symmetry.

That mean, if I have a vector Gaussian vector cyclic symmetry meaning if I just rotate that by e to the power j phi without changing the amplitude and it becomes somewhere here the distribution does not change, it is a cyclic symmetry kind of things. So, this number h l is actually a cyclic symmetric Gaussian and we denote it like that C of N it say that cyclically Gaussian cyclic symmetric Gaussian distribution.

So, what does it mean? It means that if I multiply e to the power j phi the distribution does not change. And naturally if this alpha x is alpha x and alpha y they are independent component and that hl is a Gaussian, so that mean individual is also a Gaussian distributed. And I am saying that both of them has the equal mean both of them are Gaussian both of them are independent and both of them has equal variance, because this tau I does not make any discrimination between real and imaginary. So, both of them can have the equal.

So, that forces this hl to be a complex number obviously cyclic symmetric Gaussian, because cyclic symmetric Gaussian meaning it will have individual real and imaginary will have the 0 mean and it will have the equal amount of variance and that is predominantly I can see that ok. There is no discrimination between the real and imaginary and both of them have the equal thing.

(Refer Slide Time: 25:17)



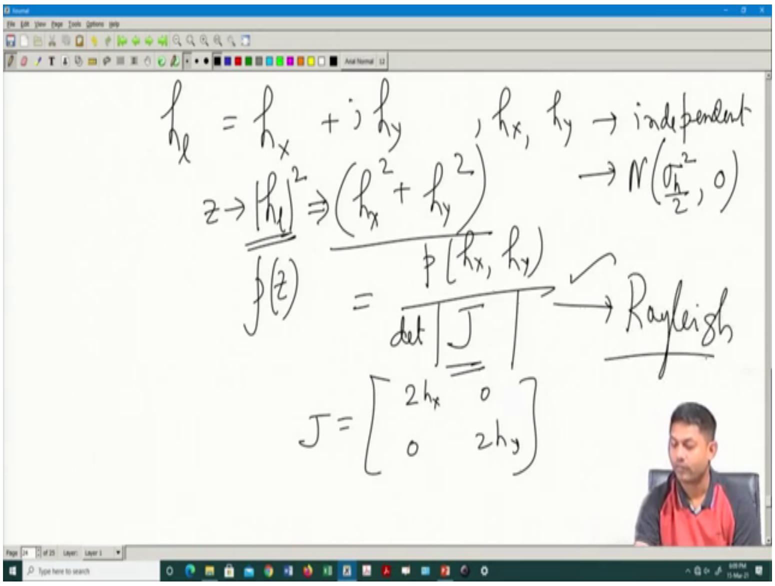
So, in a conclusion in a conclusion I can say that this whole channel whatever we have observed.

So, far h 0 plus say h 1 z inverse plus say h of L minus 1 z inverse minus L minus 1 this whole channel it has how many taps are present L taps are present digitally I am seeing that.

Every individual taps that h l tap it has a distribution PDF is a complex Gaussian PDF with 0 mean and real and imaginary both of them has equal distribution ok. Now quick questions come what happens when I take a mod or the absolute value ok. What is the distribution of it, what happens if I take mod square of it, what is the distribution of it, what about the phase of h l.

What is the distribution of it ok, now these 3 answers can be easily found out I just give you again a hints probably that you can take it as a assignment or I am not saying it is an assignment as a as a brain teasing task you can think of.

(Refer Slide Time: 26:38)



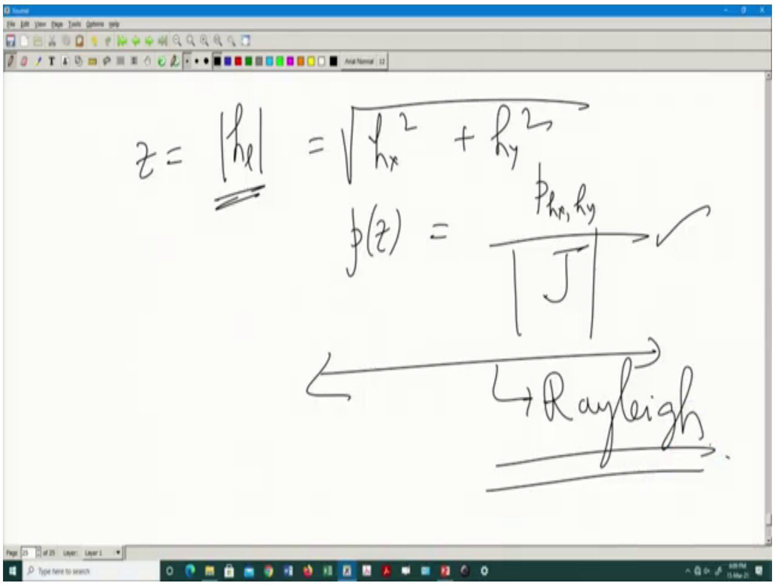
Say for example h l is a complex number which has 2 component h of x plus j h of y ok. So, what I am trying to find out an individual h of x and h of y are independent let us assume that ok. Individual is a Gaussian and they are real components. So I need not to say C of N Gaussian with variance sigma square h by 2 and this is 0 variance is half that we have discussed it.

Point that we are trying to understand here is what is the distribution of h of l mod square and mod? What is this this is nothing but h of x square plus h of y square right. So, what is the distribution of this random variable when I know individual h of x ok. So, how do you solve such kind of problem? So, usually the way you can solve it is that distribution of z you have to get into the joint distribution of p of h x p of h y and then you have to find out some sort of a Jacobean ok. So divided by determinant of this Jacobean matrix.

So, how do you get a Jacobin matrix? So, Jacobean matrix is basically how do you get it. So, it is a is basically it will be like a this whole parameter you do a partial derivative with respect to one variable and then again with respect to the other variable. So, if I do a partial derivative with respect to h x it will be 2 of h x then it will be 0 then it will be 0 here 0 here 2 of h y and you try to solve it.

So, there are ways to you know get into that. So, then you can find out what should be the distribution of p z. So, this just an hint kind of things.

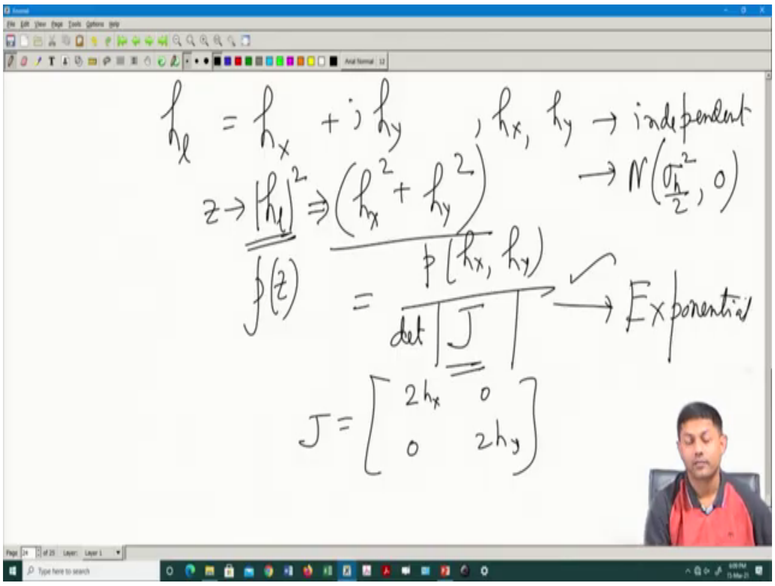
(Refer Slide Time: 29:02)



Similarly, when you have mod of h l just mod of h l not the square, so what it will be it will be h of x square plus h of y square. Again the distribution of such things will be you know p of h of x h of y joint distribution divided by the Jacobean ok. So, try it right at your end what should be the distribution and figure out for those two cases what should be the distribution of this quantity what is the distribution of this quantity.

I can only tell you the answer this will be Rayleigh distribution and sorry opposite this will be exponential distribution ok.

(Refer Slide Time: 29:52)



And this will have a Rayleigh distribution, the Rayleigh distribution try it at your end I am not getting into that, this will be straight forward and simple. Because I have given most of the hints you can try it at your as a task you can try it at your end ok. So, I stop this today and we will now get into the time varying channel in the next class ok.

(Refer Slide Time: 30:28)

So, this is the reference same book Fundamental of Wireless Communication.

Thank you.