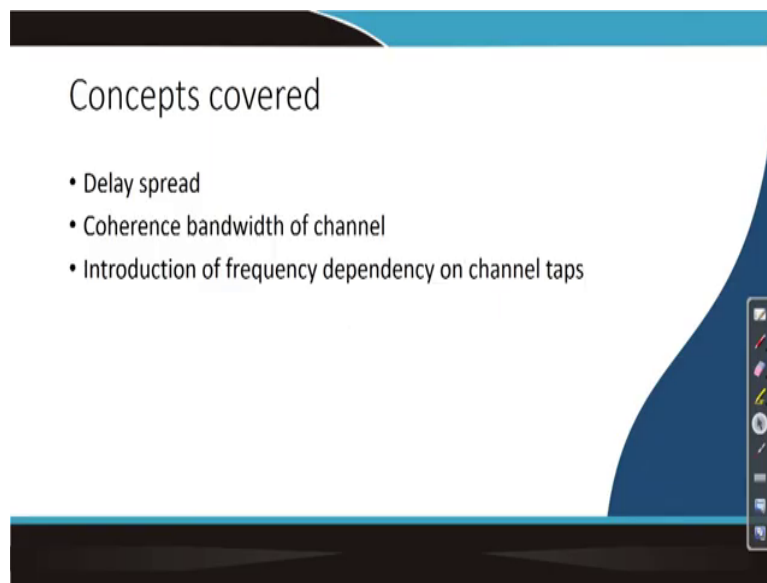


Signal Processing for mmWave Communication for 5G and Beyond
Prof. Amit Kumar Dutta
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Module - 02
Wireless Channel - A ray tracing model-Part-II
Lecture - 10
Wireless channel - A ray tracing model part - II (cont)

Welcome to Signal Processing for millimeter Wave in 5G and Beyond. We have learnt so far some of the characteristic of the digital channels. Today, we will be covering the lecture number 10 of the module 2.

(Refer Slide Time: 00:45)



Concepts covered

- Delay spread
- Coherence bandwidth of channel
- Introduction of frequency dependency on channel taps

The kind of topics that we will try to cover here are the following; delay spread of the channel and how it appears from analog to digital and what exactly the modification happens from RF analog to digital will be showing in details.

Then, the definition of coherence bandwidth of channel, very important parameter and then, we will touch upon probably, we will discuss this aspect, third aspect in later lectures; but it will be just an introduction of the frequency dependence on channel taps.

Now, we will be learning more into that some of the parameters that impact the channel performance. So, we have learnt in the last class the frequency flat fading channel, frequency selective channel; now, we will learn more on that.

(Refer Slide Time: 01:48)

Frequency selective channel $\rightarrow B_h < B_s$

Frequency flat fading $\rightarrow B_h > B_s$

τ_d

$H(z) = \sum_{l=0}^{L-1} h_l e^{-j\omega l}$ One tap

$\tau_d \approx$

$H_{A/RF}(f) = \sum \alpha_i e^{-j2\pi f \tau_i}$

The image shows a whiteboard with handwritten notes. At the top, it defines 'Frequency selective channel' as $B_h < B_s$ and 'Frequency flat fading' as $B_h > B_s$. Below this, the discrete-time transfer function $H(z) = \sum_{l=0}^{L-1} h_l e^{-j\omega l}$ is written, with a note 'One tap' and an arrow pointing to the summation index l . To the left of this equation, the delay spread τ_d is written. Below the discrete-time equation, the continuous-time transfer function $H_{A/RF}(f) = \sum \alpha_i e^{-j2\pi f \tau_i}$ is shown, with a bracket underneath it. To the left of this equation, $\tau_d \approx$ is written. In the bottom right corner of the whiteboard, there is a small video inset of a man speaking.

So, we are talking about the bandwidth; one is a frequency flat fading channel, frequency selective channel. So, if it is a frequency selective channel, say frequency selective channel, what is the bandwidth? I do not know, but at least I can guess B_h will be less than your B_s , that much I can know. We cannot say simply what should the bandwidth; but I know for sure, it is less than my transmission bandwidth.

Obviously, that is the reason it cut down some of the frequency component of my B_s and it creates the time domain aliasing or time domain interference, ISI. If it is a frequency flat fading, frequency flat fading channel, what will be the case? B_h will be B_s and you will have just one tap; you will not have multiple taps. If I have more than one tap; obviously, it get distortion. So, frequency flat fading except the amplitude distortion overall, it does not you know it does not discard any of the frequency component present in the B_s .

But then, the amplitude distortion; obviously, it is a filter. Then, we have phase distortion; obviously, because it is a filter ok. So, this is what we are learnt. Now, what is the bandwidth of it? So, now, let us concentrate on the bandwidth component. So, can I quantify this bandwidth?

Obviously, because it is Hz; it is a channel. It has a it has a you know all the coefficients. So, can I know its bandwidth? Of course, you know it. How? You will have you know all your h_l $e^{-j\omega l}$; l is equal to 0 to capital L minus 1.

So, this whole equation is given. This if the equation is given for a FIR filter, it is not easy to, it is not difficult to get a channel bandwidth. You just replace z is equal to $e^{-j\omega}$ and done deal. You can plot the whole thing and see what is the bandwidth coming in ok. That is one part. So, if I know h_l , I know the bandwidth. So, the question that comes does it have any correlation with the I mean does it have any correlation with my delay spread ok?

Now, there are two ways you can you can formulate it; one is that that is a maximum delay spread that we have already defined it and there is also an R.M.S value of delay spread ok,

that is the way we define the R.M.S value; but R.M.S value I am not talking of right now here. I will be talking next.

But here, I am just defining the kind of expected value of my delay spread and see, can I relate my bandwidth with the original channel bandwidth? Now, the question that happens is that this Hz is a digital version of your channel right. After the ADC sampling rate; but this channel inherently is coming from the analog and you know RF that also has a bandwidth.

What was that bandwidth? If you remember let us say this is analog or RF, what it was? It was summation of $\alpha_i b_j$; in b_j happens if it is an analog, if it is RF, it will be just α_i because it becomes complex number due to the delay, then $e^{-j 2 \pi f t}$, this is the one right. Remember this is the one for the bandwidth case.

Now, the question is that from here also this is a transfer function for the RF and analog case and this is the transfer function for the digital case. Do they produce the same bandwidth? Logically, yes. They have to because it is the same signal. I just digitize it.

The bandwidth does not increase or decrease ok. Due to my sampling rate that the bandwidth can be corrupted some of the thing because I may see some sort of an aliasing, that can happen; but bandwidth cannot increase it right. You cannot have a more extra component, just because you have done some sampling that does not matter, that does not happen. So, tentatively, the bandwidth from the digital side and from the analog side would remain constant. That will not change. It may be degraded here. Why?

Because I may do a sampling, where you know the analog bandwidth can be degraded. Why it can happen? Yeah, it depends on sampling right. Suppose, this channel, this analog wise or RF wise, it gives us a say 5 megahertz bandwidth. So, to reconstruct the signal, I need to sample at 10 megahertz. But at the receiver, I do not know what point I need to sample it right because I do not know the channel bandwidth.

So, I may sample at either high, but it is ok; I will not see any sort of degradation of bandwidth or if I do not know, I may sample at even at lower frequency right. Say instead of 5

megahertz, 10 megahertz, Nyquist state I may sample at 8 megahertz. What does it mean? I will see a 2 megahertz aliasing.

So, effectively it reduces my bandwidth because aliasing is just killing the part. So, it all depends; but at no point of time, I will see a larger bandwidth of it right, that cannot happen. I cannot create that extra bandwidth, but I may either maintain it or I can degrade it.

So, the point here is that can I guess the bandwidth ok; can I guess the bandwidth? And the good news is that I can guess the bandwidth. So, if you know your τ_d ; τ_d is your delay spread analog delay spread. So, it can be shown or it can be experimentally, you can show that this one almost equal to I have not written the same thing; I have not written equal to it will be rather I will put it this way.

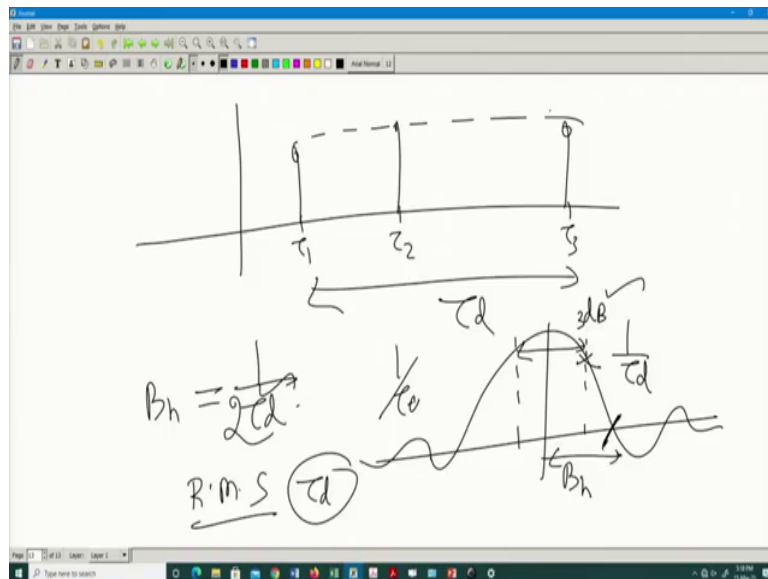
(Refer Slide Time: 08:16)

The image shows a whiteboard with handwritten notes. At the top left, there is a symbol τ_d . To its right, two lines of text describe channel types: "Frequency selective channel $\rightarrow B_h \ll B_s$ " and "Frequency flat fading $\rightarrow B_h \gg B_s$ ". Below these, the Z-transform of the channel impulse response is given as $H(z) = \sum_{l=0}^{L-1} h_l e^{-j\omega l}$, with a note "One tap" next to it. A box on the left contains the approximation $B_h \approx \frac{1}{2\tau_d}$. Below the Z-transform, the Fourier transform is given as $H(f) = \sum_{l=0}^{L-1} \alpha_l e^{-j2\pi f \tau_l}$, with a note " $H_{A/RF}$ " below it. A small video inset of a person is visible in the bottom right corner of the whiteboard area.

The bandwidth of the channel, this is the case. Tentatively, we can have some assignments obviously; we will have some assignment, where it where we will show that the bandwidth here whatever comes is very tentatively equal to $1/2$ of τ_d ok. But it makes some assumptions.

The assumption here is that every tap values in the RF will be almost equal. So, it is as if like some sort of a you know pulse impulse kind of things ok. So, those kind of basic assumption it, but it works tentatively, it comes into picture ok. Now, let us move on. So, this is just kind of a tentative. I cannot say that this is exactly equal because it has certain character, it has certain assumptions.

(Refer Slide Time: 09:21)



So, what is the assumption here? Assumption here is that if you have a τ say τ_1, τ_2, τ_3 , all of them will be almost kind of equal. So, it is as if like a sinc pulse of width τ

d ok. So, what is the bandwidth of such kind of sinc pulse? $1/\tau$. So, if I just say I take frequency domain of it, so what is it? $1/\tau$ right, that is precisely what it means.

So, here it should be B ; it is B . So, effectively my B should have been $1/\tau$ right. So, but there is a half here ok. I have reduced it just because I take the 3 dB point here. So, if I do not consider this point as my bandwidth point, that is the point where it is making 0, I just make a 3 dB.

So, what is that? The power should be just I just make it slightly up. So, instead of the concept complete τ , I just consider a half of it. So, that is why this two part comes into picture. So, this is my effective bandwidth; it is not a there is no proof on that. But this is more of an intuitive engineering kind of solution that is also relationship between the bandwidth as well as with its R.M.S value of it.

We will explain it later, what is R.M.S value of τ . So, this τ is more of an instantaneous τ . There could be an expected value also, we will explain it now. So, we will come to that point later; but this is the basic logic here. So, tentatively, it is as if like one you know impulse response is τ .

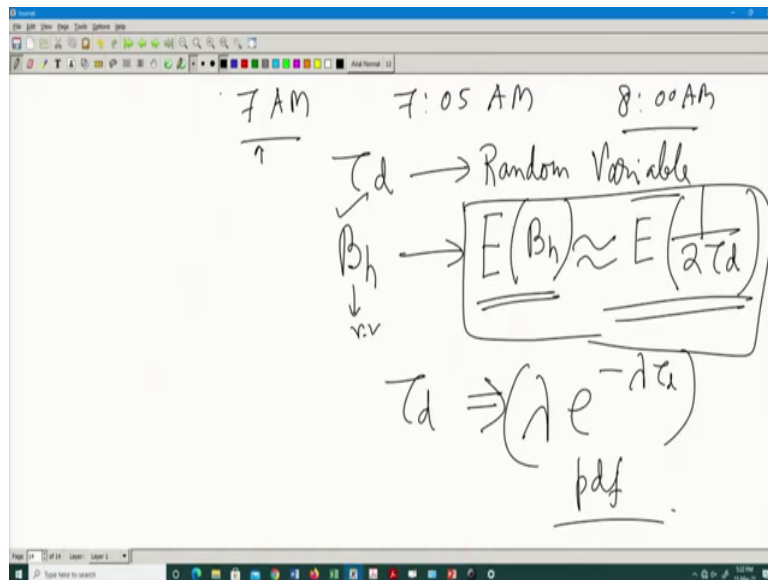
So, when I do a impulse response in the frequency domain, it just becomes a sinc. So, the point where it is 0, it is $1/\tau$ and that is basically, we consider it as our bandwidth; but to be conservative, what it does is that it takes you know 50 percent of the time and say that that is what my bandwidth is. So, this is one definition of the bandwidth.

But this is not you know this is not a very theoretical way of thinking. This is more of an engineering way of thinking it ok. Now, there is one more point here. The bandwidth, I have just shown you, it is an instantaneous. Whether it goes through R.M.S value of it or it goes through direct instantaneous value of it; but this is more of an instantaneous bandwidth ok.

But as you know your physical system can be a random system right. So, now, you are so far, I have assumed static. Now, we will moving slowly to the time domain cases, slowly we have assumed so far, we have assumed the static system. So, what does it mean?

Nothing moves out, everything is just constant; but let us take a different snapshot, though it is constant, let us not assume that things are moving continuous in time. Let us say different snapshot. For example, I am taking a snapshot at say 7 A.M. I want to know what my channel is. It is a snapshot.

(Refer Slide Time: 12:27)



Then, I say I am taking a snapshot at 7.05 A.M; after 5 minutes, I am taking another snapshot. Snapshot meaning I just transmit and see what my channel is and 8.00 A.M, I am taking different snapshot. So, practically, the channel will vary because it is not physically constant. So, far, we have assumed static; but practically, it would not be constant right. So, that mean

at 7.00 AM some channel will be there, at 7.05 AM different channel will be there because something would have moved, you may have moved, something would have moved.

But I am still not considering a continuous time movement. It is just like a you are taking a discrete snapshot and taking a different channel. How do they move? At 7.00 AM, you cannot guess at 8.00 A.M how the movement will be right. So, I can say at any snapshot, my channels are kind of a random variables right. Because you do not know the position of the reflectors and scatterer.

So, what you what does it translate to? The τ_d itself will be a random variable, itself will be a random variable. That mean at whatever time sample you take it, τ_d the delay spread would be a random variable. What does it translate to? The bandwidth of your channel B_h whatever we have discussed so far that would also be a random variable naturally.

So, that means there is no instantaneous bandwidth that we are talking of. I would say rather it is some sort of a expected value, some expected this is a random variable. This is also a random variable. So, the bandwidth that I am talking essentially is more of a expected value or a on an average, what is the bandwidth that system gives you ok.

So, what does it mean? It is not equal; but tentatively expected value of $1/2$ into τ_d that will be coming into picture. So, if this is a random variable, obviously the expected value of this one will be giving expected value of band. So, this is an important result.

For example, I give you a simple task, you can try it at your home. Say τ_d , it just takes it take some distribution. Let us say it is an exponential distribution. This is the pdf. Pdf of it, can I find out E of B_h ? You can, I am just giving you a hint; you can try it at your end.

(Refer Slide Time: 15:25)

The image shows a whiteboard with handwritten mathematical equations. The first equation is $E(B_h) = \int_0^{\infty} p(\tau_d) \frac{1}{2\tau_d} d\tau_d$. The second equation is $= \frac{1}{2} \int_0^{\infty} \lambda e^{-\lambda \tau_d} \cdot \frac{1}{\tau_d} d\tau_d$. Below these equations, there is a definition: $\tau_d = \max |t_i - t_j|$. The whiteboard also shows a Windows taskbar at the bottom with the date 15/02/20.

So, what do I need? I need to know what is the expected value of my bandwidth that comes. So, that is an expected value because there is a randomness here. So, what is that? It is $\frac{1}{2}$ by $\int_0^{\infty} p(\tau_d) \frac{1}{\tau_d} d\tau_d$ ok, the pdf of τ_d . So, it should be $p(\tau_d) \int_0^{\infty} \tau_d$, that will be coming into picture ok.

So, what is the limit of your integration? τ_d is it is a delay right. So, delay cannot be negative. So, 0 to it should be infinite of course, because you do not know where things are. So, it will be 0 to infinite $\lambda e^{-\lambda \tau_d} \frac{1}{\tau_d} d\tau_d$. I am not solving that problem ok. Solve it ok. You get some sort of a bandwidth concept.

So, these are you know these are all computer simulated issues because is not so easy to get the exact distribution of τ_d and also, sometime it becomes very complex. But it is a simple

case that I have assumed here because the distribution of this tau d is not so simple, it will be extremely compressive.

(Refer Slide Time: 16:57)

The image shows a whiteboard with handwritten mathematical expressions. At the top, the z-transform of an Lth-order FIR filter is given as:

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + \dots + h_{L-1} z^{-(L-1)}$$

Below this, the impulse response is shown as a sequence of coefficients:

$$h_0 \dots \dots h_{L-1}$$

Then, a specific example of the z-transform is provided:

$$H(z) = h_0 + h_3 z^{-3} + h_7 z^{-7}$$

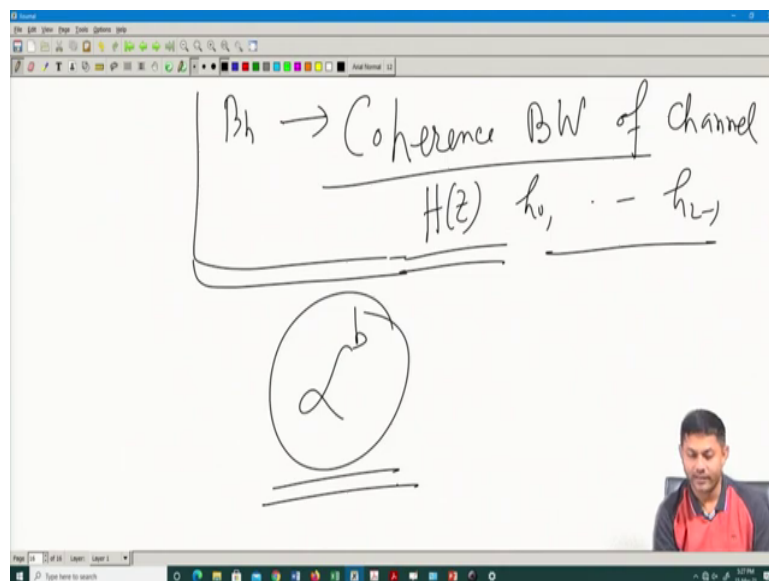
At the bottom, a stem plot is drawn on a horizontal axis. The axis is labeled with coefficients $h_0, h_1, h_2, h_3, h_4, h_5, h_6, h_7$. Vertical stems are drawn at each integer position from 0 to 7, with the height of the stem at position n corresponding to the coefficient h_n . The stems at $n=0, 3, 7$ are significantly taller than the others, indicating non-zero values for h_0, h_3, h_7 and zero values for h_1, h_2, h_4, h_5, h_6 .

In fact, if you look at the way the random variable is constructed, if you look at the way it is constructed ok, it is I do not know where it is the tau d defined as max max of tau i minus tau j. So, basically it is kind of an ordered statistics. It is tough to get the exact distribution of tau d ok.

Suppose, you have 20 such you know reflector and scatterer and each and every reflector and scatterer has a say Poisson distributedly movement, it is virtually impossible to find out this one ok. So, this can be done only in a computer kind of big simulation and you can try to get the distribution ok.

But this is just a simple problem, where you have just only two of them and I am telling you this is your distribution, you can find out the bandwidth ok. So, now, our key result is done. So, key result is this. The bandwidth is expected value of 1 by 2 of τd . So, this is an important result. Now, this bandwidth, the channel bandwidth has a very interesting name, it has a technical name ok.

(Refer Slide Time: 18:07)



So, this B_h or the bandwidth of your channel is also called coherence bandwidth; coherence bandwidth of the system of channel you can say, coherence standard. It is a very typical name that is there in 5G, 4G everywhere; coherence bandwidth. So, it is nothing but the bandwidth of your channel ok.

So, what is the best to define it? Well, you get the channel, you would get all the H_z . If you are not getting into the delay spread way of calculating the channel bandwidth, that is

absolutely fine; you get the channel taps, all your h_0, h_1 , blah blah h_{l-2} , estimate it, construct an FIR filter, then you get a bandwidth. This is the best way to get the bandwidth ok.

So, that will be your coherence bandwidth and that is the easy way to get it, if you know what is the delay-spread and expected value of $1/2$ delay spread will be your channel bandwidth ok. Now, so far, we have covered the channel in a static environment ok.

Now, before I get into the time domain variation of it, I would now like to have a more characteristics on the magnitude part of the channel. So, far, we have not done much that mean the characteristic of that α , this part, the one which is like the attenuation part, it is a complex number that is the only thing we have known. But is there anything else?

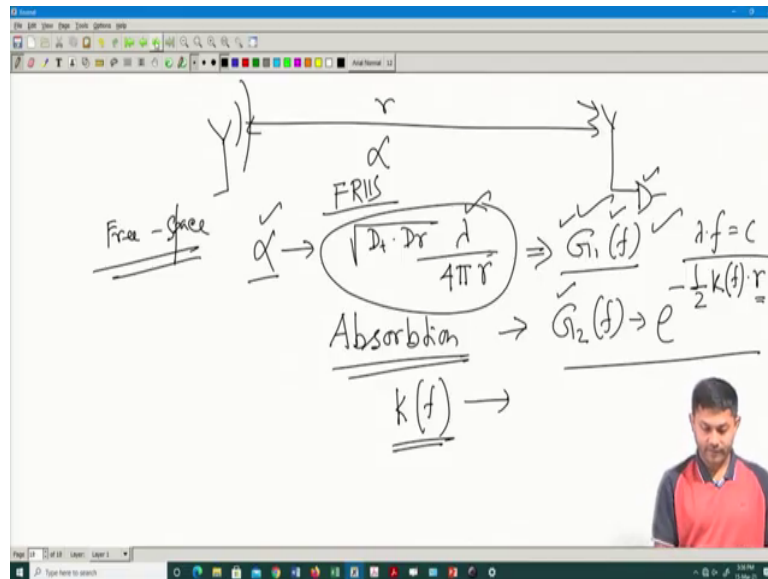
Because why I need to know because that derives the millimeter wave and terahertz kind of communication because this component is the key focus, once your system bandwidth becomes larger, once your RF becomes larger. So, because you are moving towards that millimeter wave right.

So, it is very important to know what the exact characteristic of it ok. So, let us try to understand it ok. After that, we will definitely come back again for the time varying part, which will be the next part of our lectures. So, we are now characterizing the α part; the attenuation part of my channel.

That when I say attenuation, does not mean that it will be only the air part right and you have explained multiple times, it will be everything LNA, power amplifier, all you know DAC, ADCs filter characteristic, everything you know all composite is your channel ok.

But here, we are only characterizing part of on a characteristic in the air part because air is something that which you may not know, only thing you have to do a measurement and kind of. So, air part only you are trying to characterize it here. So, so far, we have not concentrated much on the on that aspect.

(Refer Slide Time: 21:18)



So, now this is just a single antenna or single antenna system. Let us concentrate on that only. So, when I transmit the data here, it will be received here and we do not want to get into the electromagnetic wave definition. We want to go everything into the electrical domains ok; electrical engineering domains here. So, that mean I send an electrical signal, I receive an electrical signal and that is my way of viewing the channel. I do not get into the electromagnetic business here. So, here so the gain part alpha that we defined last time, we said that that is nothing but the path gain right; path loss you can say.

But as because there is an LNA here, it will be so just a path gain, this alpha and the complex part is coming because of the delay part. So, if when I say alpha, I am talking of only the RF part because RF 2 analog, it will be just $e^{-\alpha}$ to the power minus $j 2 \pi f \tau$ i the complex part will be added. What exactly this alpha?

The one which I have explained so far that is defined by a path loss equation right we know the path loss equation in a free space. So, that is defined by your Friis equation ok. So, what was that path gain? We have I think tentatively defined it; it was D_t into D_r into λ by $4\pi f$; $4\pi r$ if the distance is r ok. But so, this λ makes it kind of a frequency dependent.

So, I can say this part is some sort of a function of say let us call it $G_1(f)$, just an functional assumption. So, let us call this whole thing as $G_1(f)$. What is f ? Why f is coming? Because there is a λ here right and there is the relationship between λ and frequency right.

So, λ into your f that will be your C , so that part is there. So, let us call it $G_1(f)$ ok. So, that that is there; C is the speed of light ok. So, that means, this α is predominantly, we have already accepted and we have well-defined how it can be frequency dependent. Now, this component only assumes that you are in a free space because this Friis equation is only for free space.

But we are in the world, practical system and the next component. So, this is fine when you are in a 6 gigahertz kind of transmission that is typically your you know 4G system works 6 gigahertz; 5G also some of the components work in the 6G. But when you increase your RF say 30 gigahertz, 40 gigahertz, 60, 100 gigahertz and so on beyond, free space part is not the only component that is affecting your α . More and more other part will now come into picture ok, let us understand that. So, this α so far it was $G_1(f)$ that particular gain that was from the Friis.

Apart from that, it also has one more component called absorption. Why this absorption is coming into? Because we are now getting into slightly millimeter wave, though it is not the complete definition of millimeter wave, just the warm up how millimeter wave you know impact my channel, what exactly the what exactly it is meant by a millimeter wave channel. So, before I get into it this α definition has to be very thought through.

So, there is one more component called absorption. Why this absorption is coming? Because when you transmit the data, it goes through the medium right. Say for example, when I

transmit it goes through air, air is a medium. So, air will have lot of components. So, for example, carbon dioxide, oxygen, sulphur dioxide, water vapor, so many other invisible or some sort of a visible components like color.

So, all this component will act as an absorber for the RF. So, far, that was not a concern for us; but moment I get into say 30-40 gigahertz RF and beyond, the absorption part becomes predominant, it is not just the FRIIS, not just the free space that deteriorates the gain.

Absorption will now be coming big wave ok. So, now, there is a there are different model of absorption; one is the one of the key model, where the absorption is that it is basically called, so let us call that as a G^2 of f , this absorption part. So, this will be e to the power minus half of I will define it, what is what; something like that. What is $K f$? $K f$ has a well definition.

I am not going into details of it because that is not the intention of this particular lecture. Because we are more in the signal processing aspect, so this is more from the physics aspect. So, this is basically your absorption coefficient which is a function of frequencies and that also, depends on materials.

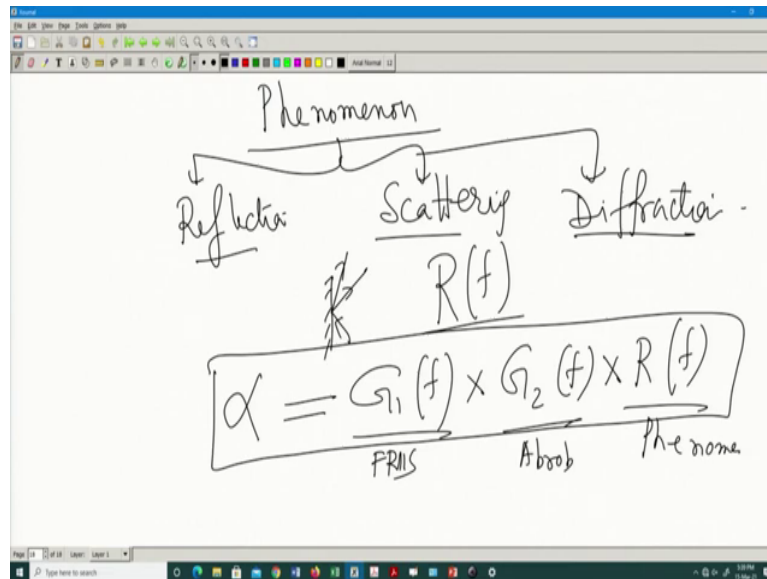
If it is a water vapor, it will have some $K f$. If it is some sort of a you know other materials like for example, wood or glass or anything, any materials everybody has its own absorption and everybody will now start absorbing some of the data, some of the power.

So, that mean if an RF of say 30 or 60 gigahertz goes or goes into the air, various component of the air like water or carbon dioxide or oxygen whatever, you will start doing a different level of absorptions ok and this $K f$ can vary; it can be multiple such things will be there and r is the distance.

So, that mean depending on the r , obviously your absorption will be larger and larger. So, this $K f$ has a very complicated definition, I am not getting into that. The paper reference paper that I have shown or I will also show you at the end of the lecture, it will have that paper and it will have a very big definition.

So, let not get into that; but here, it is more of a notion, what exactly it is happening ok. So, that means, this alpha is now $G_1 f$ which is the Friis part, $G_2 f$ which is basically your you know your absorption part; apart from that there are there is one more part ok.

(Refer Slide Time: 28:56)



And that part is called phenomena dependent; phenomenon. What is the phenomenon? What are the different phenomenon that happens when a ray is injected into the air? You have reflection, you have scattering and you have diffraction. These are the three phenomenon's that happens ok and each and every phenomenon will create a different physical activities on the array that comes into it incident onto it right.

Reflection is just a reflection; scattering is like a splashing; diffraction will slightly bend it and try to you know deviate the ray from the original direction. So, that creates that has a

frequency dependency that mean 6 gigahertz we will see a different type of reflection compared to 60 gigahertz ok. That is again, it is done by a physics.

So, the reason is that a surface the level of absorptions from the surface, the level of you know scattering from this surface will be dependent on the frequency. Why? Because the lambda will see a different level of roughness into the surface.

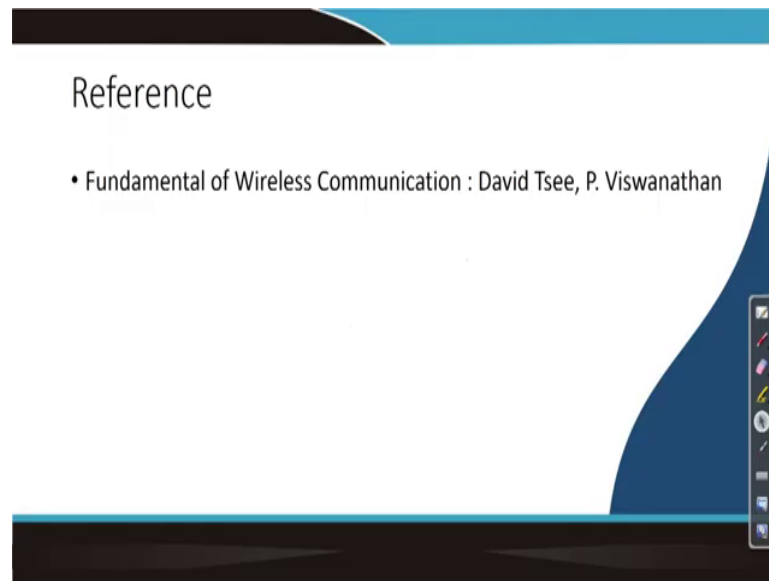
If the lambda is large, what will happen? A rough surface will become like a smooth surface, but even the lambda is smaller that is precisely even in a millimeter wave; 60 gigahertz in a millimeter wave. Each and every tiny things in the system, even a glass kind of things also, it may look like a scatterer because it may see that roughness there ok

So, depending on what frequency you are transmitting, this phenomena will be slightly varying in their nature. So, there is a coefficient called RF. It comes from reflection scattering and diffraction. Again, I am not getting into the details of this equation because again that is not the part of this discussion here.

So, this alpha whatever gain factor, we were talking, it will be that G_1 of f which is the plane FRIS, just the path loss multiplied by G_2 s; G_2 s; G_2 f which is the plane absorption part and multiplied by the phenomena related part. So, these three things defines what should be my alpha ok.

So, today we end this. We will explain more on the next class ok.

(Refer Slide Time: 31:49)



So, this is the reference; Fundamental of Wireless Communication by David Tsee and P. Viswanathan. This does not cover everything; but most of the things, you may find it.

Thank you.