Power System Protection
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Lecture 08: Least Square Technique

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Welcome to module two on phasor estimation and we will continue with the least square technique. So in this lecture, our focus will be on how we can formulate the least square estimation technique and how this can be applied to phasor estimation perspective and in particular, the fundamental component which we like to estimate for the relay. So with this target we will go with this.

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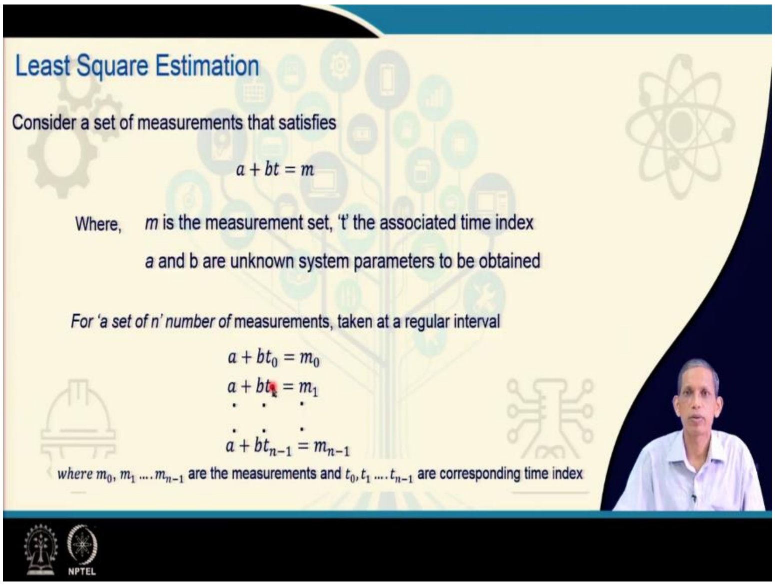

So let us first understand what least square estimation technique is before applying it to the relay applications. We consider a set of measurements that satisfies

$$a + bt = m$$

Where t stands for the time index and m for the measurements. Now *a* and *b* are the system parameters which happen to be not known to us, and these unknowns are to be estimated in this process. Such least square estimation technique you might have come across in the curve fitting techniques or as a tool in signal processing, we are addressing the same thing here. Now here in this case, we have a set of measurements and the corresponding different time are considered. Now let us say that we have n number of measurements for this purpose.

$$a + bt_0 = m_0$$

$$a + bt_1 = m_1$$

.

$$a + bt_{n-1} = m_{n-1}$$

Therefore that m_0, so we can say that m_{n-1} and the corresponding time where we took these measurements are the t_0 to t_{n-1}. So for this you can say that individual measurement and the associated time we can write the for the corresponding system.

So we will have we can say that a set of n equations for this. So in this case, we can say that these corresponding measurements are associated the corresponding time, note that the time index we can say that the time index you can say that t_0 through t_{n-1} are also known to us, and so we can say the corresponding measurements are also known to us through which we like to find the corresponding *a* and *b* values for the system as system parameters. These you can say that time intervals between the two consecutive measurement points generally remain to be fixed and that is what we will see in all relay applications.

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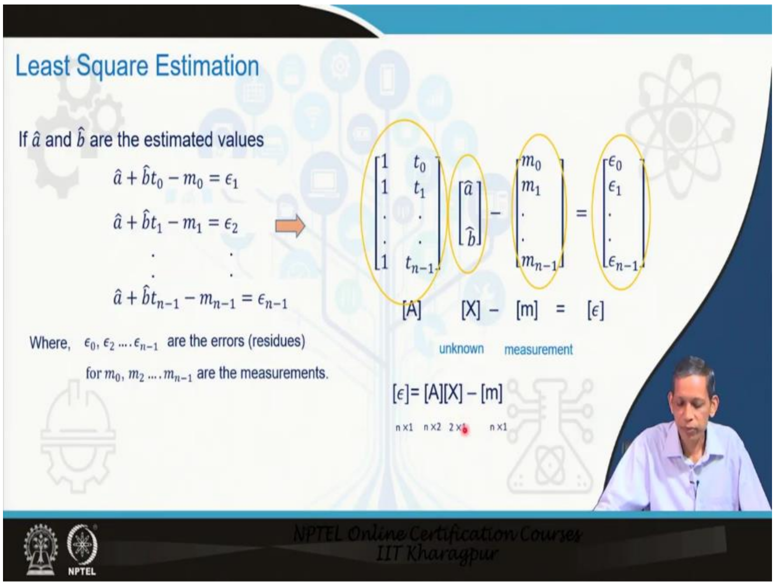

Now considering that the a head and the b head as our estimated values, so in the same equation, we can fit you can say that for the system equation like this,

$$\hat{a} + \hat{b}t_0 - m_0 = \epsilon_1$$

$$\hat{a} + \hat{b}t_1 - m_1 = \epsilon_2$$

.

$$\hat{a} + \hat{b}t_{n-1} - m_{n-1} = \epsilon_{n-1}$$

Error represented by ε which will be there because the estimated value not exactly match with the $\hat{a}$ and $\hat{b}$ So that will be ε_1 and like that you can say that for the other measurements also you can say that we write down ε_2, ε_3 like this. So if we have the set of measurements here m_0 through m_{n-1} or m_1 through m_n, t_0 through t_{n-1} or t_1 through t_n,. The set of measurements we n measurements. So ε_1 through ε_{n-1}, so this will be we can say that have *n* errors. So this inner errors or otherwise we call residues also, and m_0 through m_{n-1} are the measurements. So we have the estimated value $\hat{a}$ and $\hat{b}$ for this particular system. Then we can write this corresponding equation in matrix form like this,

$$\begin{bmatrix} 1 & t_0 \\ 1 & t_1 \\ . & . \\ . & . \\ 1 & t_{n-1} \end{bmatrix} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} - \begin{bmatrix} m_0 \\ m_1 \\ . \\ . \\ m_{n-1} \end{bmatrix} = \begin{bmatrix} \epsilon_0 \\ \epsilon_1 \\ . \\ . \\ \epsilon_{n-1} \end{bmatrix}$$

These are nothing but we can designate them as matrices as

$$[A][X] - [m] = [\in]$$

So the [X] here is our unknown that is a and b are to be obtained and m are our measurements, m_0 through m_{n-1} and then we can say that we can fit the corresponding equation also. In the other way, we can write down that as

$$[\in] = [A][X] - [m]$$

These t's are under the observation. So that is why these are known to us, the time index you can say that are considered to be known to us. So if we say that for these n sets of measurements $n \times 1$, so we have $n \times 1$ for the ε and for A you can say that if we see here, $n \times 2$ and the X we can say that 2×1. So you can say the matrices are matched in this way.

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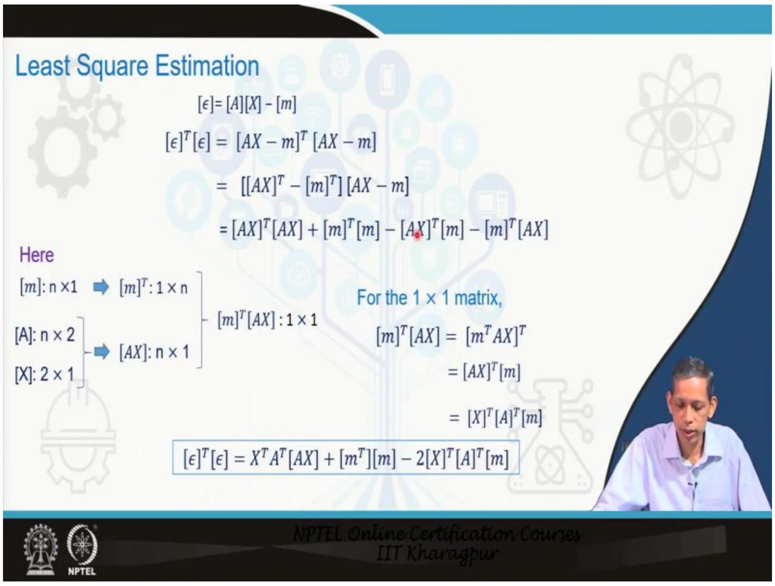

Now we will try to formulate the least square estimation technique. So the ε, the error matrix that becomes equals to

$$[\in] = [A][X] - [m]$$

If you multiply this [ε] with its transpose $[\varepsilon]^T$ that becomes

$$[\in]^T[\in] = [AX - m]^T[AX - m]$$

$$[\in]^T[\in] = [[AX]^T - [m]^T][AX - m]$$

$$= [AX]^T[AX] + [m]^T[m] - [AX]^T[m] - [m]^T[AX]$$

Now we will see in this we can say that these terms. If we see, we can say that here the m is n×1, so therefore m transpose will be 1× n and the A is n×2, we have already seen earlier slide also and the X is you can say that 2×1, a and b are the two unknowns. So therefore, the AX gives us you can say that n ×1, n ×2 into 2×1 that is n×1 and the $[m]^T$ is nothing but 1×n.

So therefore,

$$[m]^T[AX]: 1 \times 1$$

We can write also because transpose you can say that of the one element matrix that becomes itself the element and the matrix. Therefore, for this 1×1 matrix

$$[m]^T[AX] = [m^T AX]^T = [AX]^T[m] = [X]^T[A]^T[m]$$

And therefore, if you see, you can say that this equation, we can represent this equation

$$[\in]^T[\in] = [X]^T[A]^T[AX] + [m]^T[m] - 2[X]^T[A]^T[m]$$

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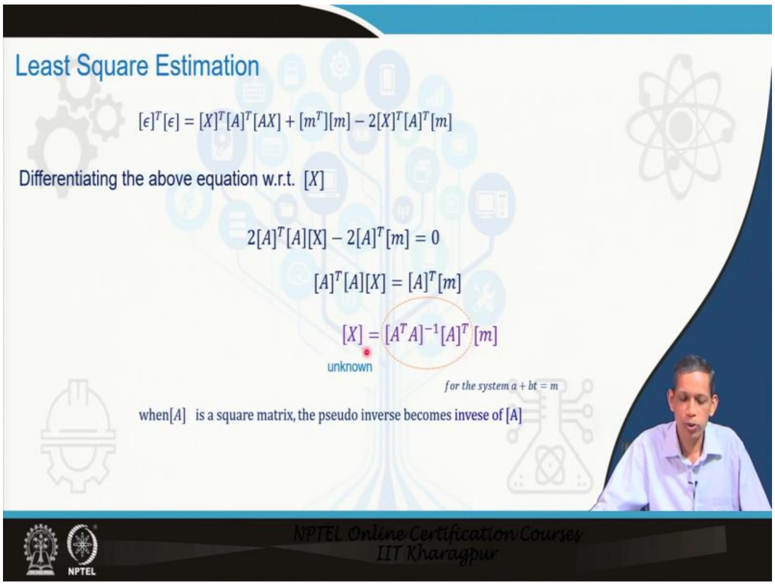

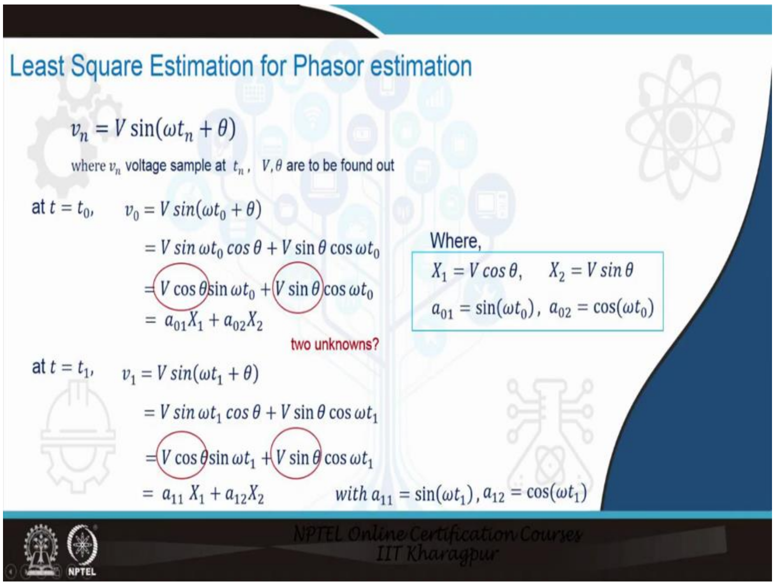

Now leading to that borrowing that equation from the earlier slide we will differentiate this equation with respect to X, the unknown vector

$$2[A]^T[A][X] - 2[A]^T[m] = 0$$

From this relation we get the least square of error that can be rewritten as

$$[X] = [A^TA]^{-1}[A]^T[m]$$

So this gives us that least or minimum error, we can say minimization problem. This is the solution to obtain the corresponding unknown parameters of the system, a and b as we have defined for the system

$$a + bt = m$$

For m is set of measurement, a is that we know this 1 and t's it contains and then a and b are the unknown. So in this case, A matrix is also known to us and so also the measurement you can say that matrix is also known to us. Therefore, the right hand things are available to us, so we can find out this X vector that contains a and b, the unknown parameters of the system. So this you can say that $[A^TA]^{-1}[A]^T$ is called the pseudo inverse or left pseudo inverse in the literature. So once we have this inverse part and then we multiply the measurement side, we can find the corresponding X perspective. Now see here that matrix A becomes a square matrix then the whole we can say the term becomes and simple $[A]^{-1}$, agree? So that you can say that the simplicity but what do we say that we will see that the number of measurements becomes within our positions, so we can have n number of and sufficient measurements to find the corresponding correct value of X. So in most of the cases, you will find that this A matrix make no more you can say that a square. So that becomes rectangular where this pseudo inverse matrix having these three terms are being used to obtain the corresponding unknown we can say that X parameter, we can say that having these two parameter *a* and *b*. Now let us you can say that how this corresponding least square estimation process that we define X equals to pseudo inverse of A into m that how we can apply you can say that for this power system application, particularly in relaying perspective we will see now.

Let us this signal V and that is n^{th} sample becomes equals to

$$v_n = Vsin(\omega t_n + \theta)$$

This kind of signal we have already used in the DFT application also, where v_n is this sample at t_n instant and V, θ to be found out. These are the two unknowns, V and θ which will reveal the corresponding phasors. V is the peak value so $\frac{V}{\sqrt{2}}$ provides the RMS and θ is the corresponding angle of this we can say that phasor at you can say that at the reference instant t_0 also. Now let us you can say that at one instant $t=t_0$ that is our starting time for the measurement, so in such condition

$$v_0 = Vsin(\omega t_0 + \theta)$$

If we expand this we can say that relation. So we have you can say the two terms,

$$v_0 = Vcos\theta \,\mathrm{sin}\omega t_0 + Vsin\theta cos\omega t_0$$

t_0 is the time that is under our control on measurements, so we know it and the $Vsin\theta, Vcos\theta$ are the two unknowns. So we define this $Vcos\theta$ and $Vsin\theta$ in terms of X_1 and X_2, the two unknowns, and the corresponding other two terms $\mathrm{sin}\omega t_0$ and $cos\omega t_0$ are defined by a_{01} and a_{02}. So this becomes equals to

$$v_0 = a_{01} X_1 + a_{02} X_2$$

Now for the corresponding measurement v_0, where v_0 is our measurement, the voltage sample at t_0. So we define

$a_{01} = \mathrm{sin}\omega t_0;\ a_{02} = \mathrm{cos}\omega t_0\ ; X_1 = Vcos\theta\ ;\ X_2 = Vsin\theta$

We relate you can say that the capital A in the least square formulation. Now similarly at next instant after the time Δt, $t = t_1$ we got another sample v_1 and that you can represent as

$$v_1 = V sin(\omega t_1 + \theta)$$

And then that becomes if we expand the like above, we can say that this becomes

$$v_1 = V cos\theta \ \sin\omega t_1 + V sin\theta cos\omega t_1$$

Two terms where the $V cos\theta, V sin\theta$ term again comes simultaneously $\sin\omega t_0$ and $cos\omega t_0$ are defined by a_{11} and a_{12}. Therefore, this becomes equals to

$$v_1 = a_{11} X_1 + a_{12} X_2$$

And the corresponding

$a_{11} = \sin\omega t_1; \ a_{12} = \cos\omega t_1 \ ; X_1 = V cos\theta \ ; \ X_2 = V sin\theta$

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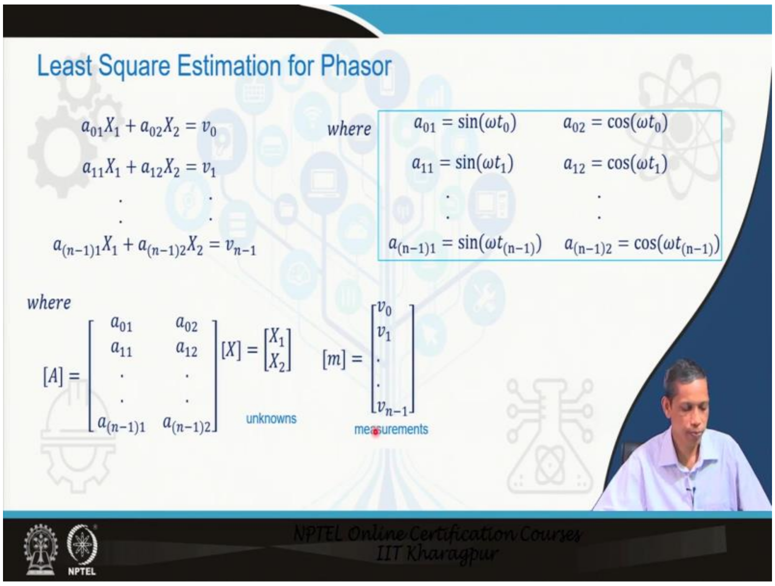

Similarly, we can say that we can have n number of samples from v_0 to v_{n-1} for the considered measurements and we can relate the corresponding equations as

$$a_{01} X_1 + a_{02} X_2 = v_0$$

$$a_{11} X_1 + a_{12} X_2 = v_1$$

.

$$a_{(n-1)1} X_1 + a_{(n-1)2} X_2 = v_{n-1}$$

So this is the set of equations for a set of measurements we can write as we formulated for this least square sense also. Now here for the particular sinusoidal signal of the voltage we can write,

$$a_{01} = \text{sin}\omega t_0;\ a_{02} = \text{cos}\omega t_0$$

$$a_{11} = \text{sin}\omega t_1;\ a_{12} = \text{cos}\omega t_1$$

$$.$$

$$a_{(n-1)1} = \text{sin}\omega t_{(n-1)};\ a_{(n-2)2} = \text{cos}\omega t_{(n-1)}$$

And like that we can say that these coefficients for the unknowns X_1 and X_2 are like this, where we will now formulate the matrix A as

$$A = \begin{bmatrix} a_{01} & a_{02} \\ a_{11} & a_{12} \\ \cdot & \cdot \\ \cdot & \cdot \\ a_{(n-1)1} & a_{(n-1)2} \end{bmatrix}$$

Note that these a's are nothing but in terms of the time index and the corresponding frequency, omega the fundamental frequency. Therefore, we can say that in the t_0 t_1 and t index in this index in the time index being known to us, so A matrix is assumed to be known to us. X is the unknown,

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

X_1, X_2 have already defined in terms of $Vcos\theta$ and $Vsin\theta$ term and the corresponding m are the set of measurements,

$$m = \begin{bmatrix} v_0 \\ v_2 \\ \cdot \\ \cdot \\ v_{n-1} \end{bmatrix}$$

So we have *n* number of measurements for this purpose.

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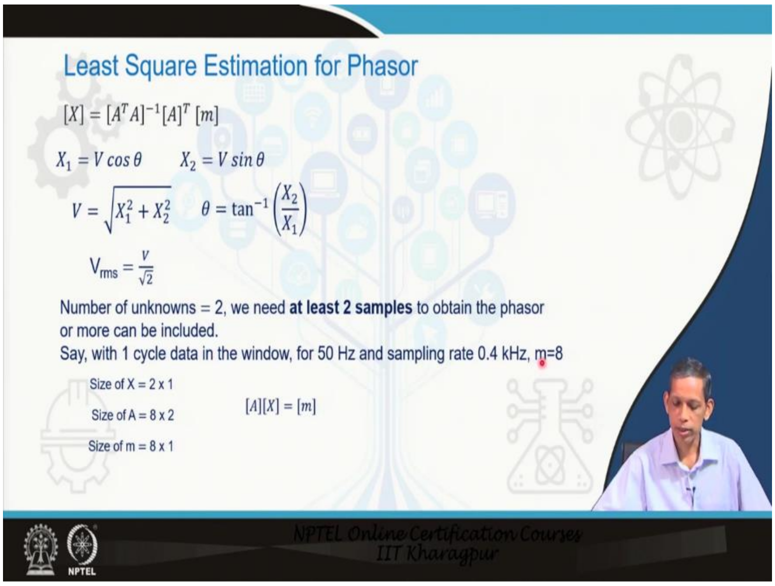

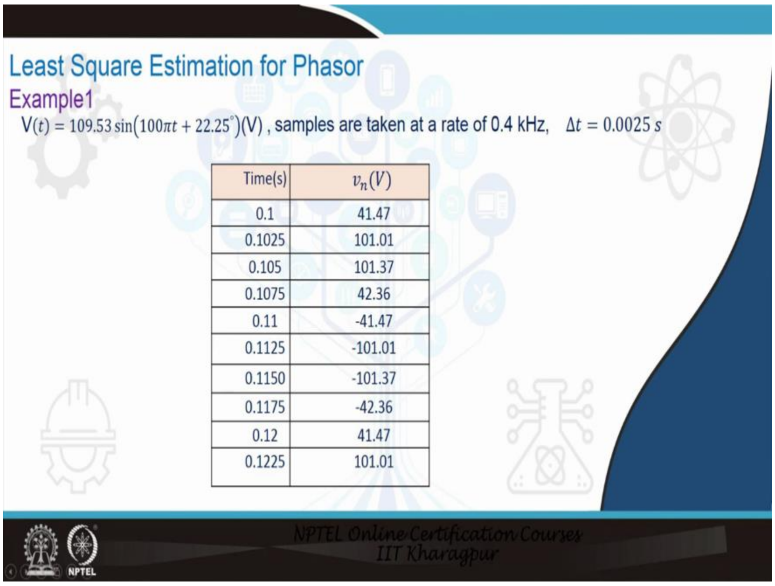

Now then we can say that we apply the corresponding least square technique which we have learnt.

$$X = [A^T A]^{-1}[A]^T[m]$$

Here we have considered $X_1 = V\cos\theta$, $X_2 = V\sin\theta$. Therefore, we can say that the X can be obtained from this pseudo inverse of A and m is nothing but v_0 through v_{n-1}. So the V can be computed from this X_1 and X_2 and that becomes

$$V = \sqrt{X_1^2 + X_2^2}$$

So this gives us the peak and then you do we can say, you can find the RMS value from this peak and θ you can say that

$$\theta = \tan^{-1}(\frac{X_2}{X_1})$$

From that X_2, X_1 we can find out the corresponding θ value. Therefore, from there we can say that we can find out these phasors to be $V_{rms}\angle\theta$ for the corresponding sinusoidal signal. So this is what you can say that how we can model the corresponding sinusoidal signal for the power system relaying applications with the least square estimation sense. Now in these formulations we saw that there are two unknowns, X_1 and X_2, so we need at least two measurements, two samples of v_1 and v_2 to obtain the phasors, obvious or if more measurements are available then also we can peek into the corresponding least square sense. Say if you have there are more measurements like one cycle of data for the 50 Hz system with same you can say that 400 Hz sampling, where number of measurements that *m* will have eight number against that samples. Then size of X becomes 2 ×1, size of *A* becomes 8 ×2 and then the size of *m* becomes 8 ×1.

We have eight samples for the one cycle, so eight measurements are available. Then the AX you can say that because AX equals to m that becomes valid as you test. Now let us you can see that we will go to some examples how we can compute the corresponding phasors using the least square estimation technique. So as usual you can say that in our earlier discussion also on phasor estimation using DFT, Cosine filter and so we consider a signal

$$v_t = 109.53\sin{(100\pi t + 22.5^0)}\text{(V)}$$

Samples are taken at 0.4 kHz and so therefore, the Δt becomes equals to 0.0025 s. We got the corresponding voltage samples which we have already seen and this is the corresponding time index for this one with an interval of Δt of 0.0025 second. So this leads to you can say that datasets available now and how we will apply the dataset for the phasor estimation technique that we will learn.

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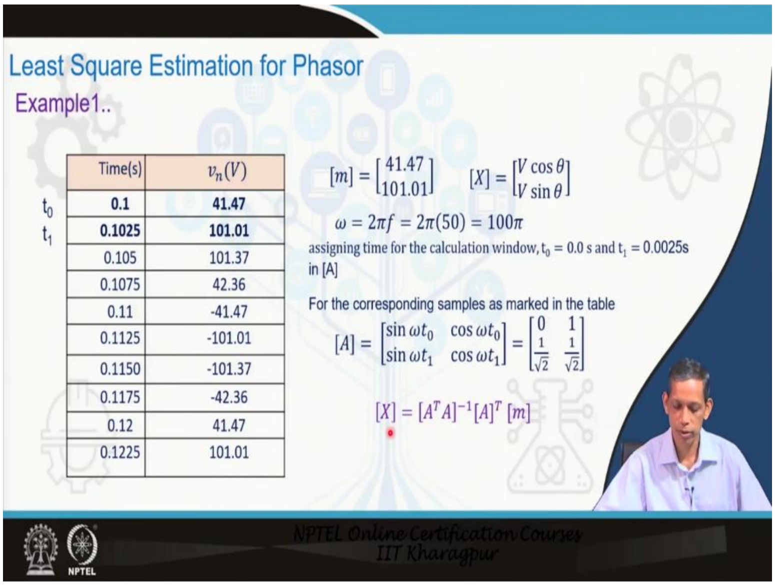

Now as already pointed out, we have two unknowns in this case, Vcosθ and Vsinθ which are already formulated. So for simplicity minimum two samples are required. These two samples corresponds to time index t_0 and t_1. So that corresponds to 41.47 at 0.1s and the next sample 101.01 is at 0.1025 s are the values of v_0 and v_1. Hence the measurement matrix m and the unknown matrix X can be represented as

$$[m] = \begin{bmatrix} 41.47 \\ 101.01 \end{bmatrix} \quad [X] = \begin{bmatrix} V cos\theta \\ V sin\theta \end{bmatrix}$$

In addition, considering $\omega = 2\pi f = 2\pi(50) = 100\pi$ corresponding A matrix for the respective time interval t_0 and t_1 can be written as

$$A = \begin{bmatrix} sin\omega t_0 & cos\omega t_0 \\ sin\omega t_1 & cos\omega t_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Now as per the least square estimation algorithm the unknown vector X can be represented as,

$$X = [A^T A]^{-1}[A]^T[m]$$

.

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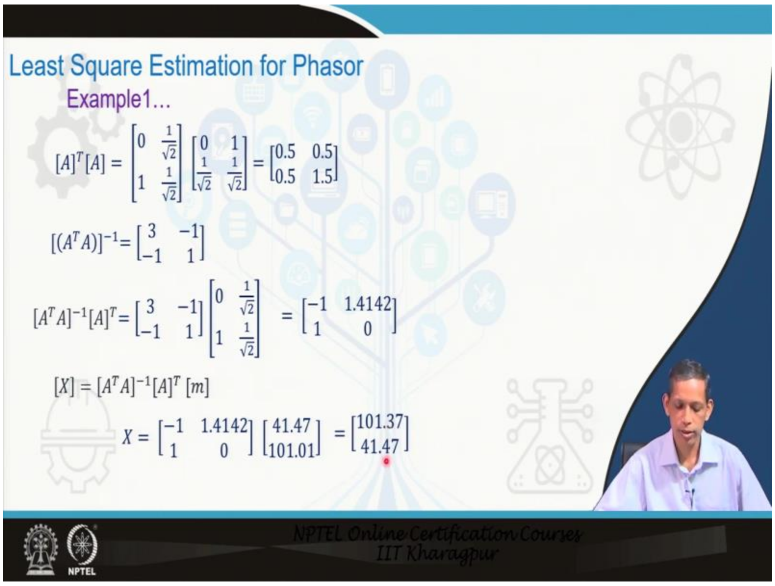

In continuation of that

$$[A]^T[A] = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ 1 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 1.5 \end{bmatrix}$$

Then you can say that

$$[A^T A]^{-1} = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$$

This pseudo inverse becomes

$$[A^T A]^{-1}[A]^T = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ 1 & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -1 & 1.4142 \\ 1 & 0 \end{bmatrix}$$

Note that because this is like a square matrix so this pseudo inverse is nothing but the inverse of *A* also. So we will get you can say that if we try, this inverse of *A* matrix becomes also same to this pseudo inverse agreed. This is to making a practice how to have the pseudo inverse computation. Now the unknown variable matrix X becomes

$$X = [A^T A]^{-1}[A]^T[m] = \begin{bmatrix} -1 & 1.4142 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 41.47 \\ 101.01 \end{bmatrix} = \begin{bmatrix} 101.37 \\ 41.47 \end{bmatrix}$$

So here, the first element X_1=101.37 and the second element X_2 = 41.47.

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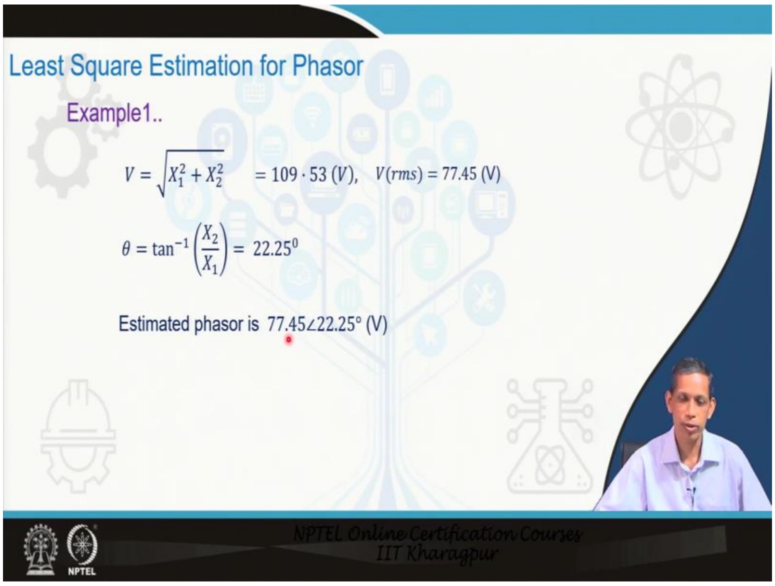

Then as per the already we have formulated the peak value of V becomes

$$V = \sqrt{X_1^2 + X_2^2} = 109.53\ (\mathrm{V})$$

The corresponding RMS value becomes

$$V_{rms} = \frac{109.53}{\sqrt{2}} = 77.45\ (\mathrm{V})$$

The angle theta is

$$\theta = \tan^{-1}\left(\frac{X_1}{X_2}\right) = 22.25^0$$

Therefore, the estimated phasor becomes $77.45\angle 22.25^0$. If you remember in DFT we got the same thing, also here in this example we are talking about that peak value divided by root 2 we are getting correct value for this part. This shows that the corresponding phasors being estimated by the least square estimation using the two samples become also pretty good.

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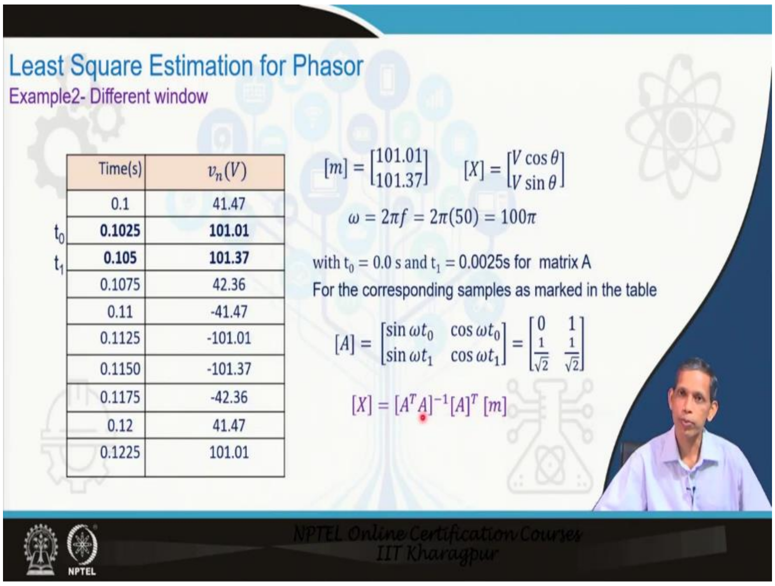

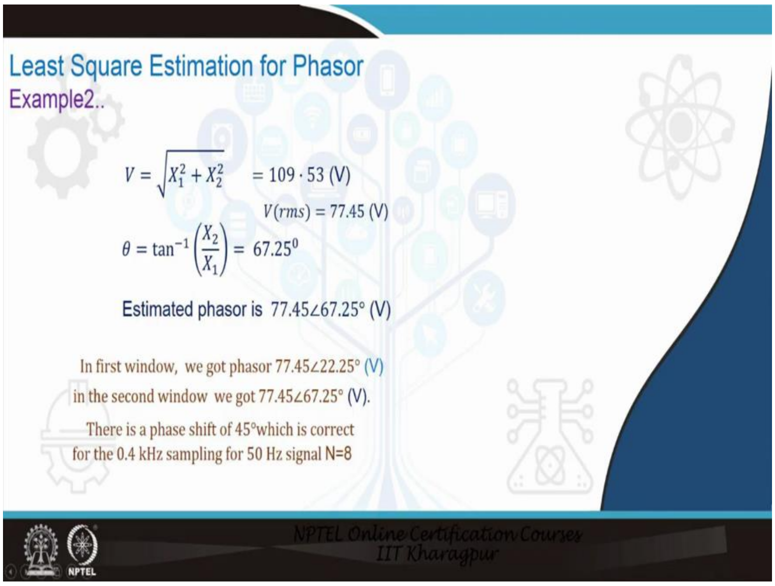

Now we will see how by shifting the window the corresponding algorithm works. For that we will leave this one and we have a fresh sample in replace. Now these are the corresponding two samples that constitute the window now. So in this window again, for simplistic for the least square perspective. We consider that this as t_0 and this as t_1, and t_0 again you can say that we are talking about 0 second and t_1 again 0.0025. This reduces the computation burden for this system also that is the advantage you can expect. Now the *m* matrix is given by

$$[m] = \begin{bmatrix} 101.01 \\ 101.37 \end{bmatrix}$$

The corresponding X again remains to be same given by

$$[X] = \begin{bmatrix} Vcos\theta \\ Vsin\theta \end{bmatrix}$$

Corresponding A matrix now becomes equals to

$$A = \begin{bmatrix} sin\omega t_0 & cos\omega t_0 \\ sin\omega t_1 & cos\omega t_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Same as what we see in the earlier example also. So what we are doing here also that is advantageous that the A matrix we are not changing that will lead to us to take a benefit of not computing the A matrix repeatedly. Not only A matrix, if the A matrix remains to be same in this process, even if we shift the corresponding window of the measurement then the corresponding pseudo inverse computation this become also fixed one. So the advantage you can say by doing we can say this approach is that A matrix can be computed a priory and then that so also the elapsed pseudo inverse can be also computed a priori and you fix it and then you only apply the corresponding m measurements whatever you do we have right now and then with that measurement if you apply that you can say that, you multiply this fixed matrix here then you can get the corresponding unknown vector X. Now this second window we will see you can see how the corresponding X can be obtained. So similarly, you can say that we got the corresponding A transpose is same you can say what you see here and the A matrix you can say that this pseudo inverse matrix becomes this which happens to be A inverse also and then you can say that it go, multiply the corresponding pseudo inverse matrix to this m and then we get the X. So the X becomes

$$X = [A^T A]^{-1} [A]^T [m] = \begin{bmatrix} 42.36 \\ 101.01 \end{bmatrix}$$

Then you can say that from this X_1 and X_2 we got the V to be 109.53 and the RMS value is 77.45, same what we got earlier also. And then the

$$\theta = \tan^{-1}\left(\frac{X_1}{X_2}\right) = 67.25^0$$

The phasors becomes $77.45\angle 67.25^0$. So what we see here that there are two windows we have observed now. In first window, we got the phasor to be $77.45\angle 22.25^0$. In the second window, we go the phasors to be 77.45, same magnitude with angle 67.25^0, a shift in angle of positive value of 45^0. Same thing we observe in the discrete Fourier transform also the, with the shift in you can say that the window with more and more observations.

The window is being shifted you can say that with the window progresses you can say that forward and then you can say that. Now what we say here that because this is you can say that the corresponding sampling rate is 4 kHz and then you can say we have 50 Hz signal, so the number of N becomes equals to 8 and then we are getting the corresponding 45^0 shifting by this one which is correct, that we have discussed in the DFT you can say that, DFT based phasor estimation.

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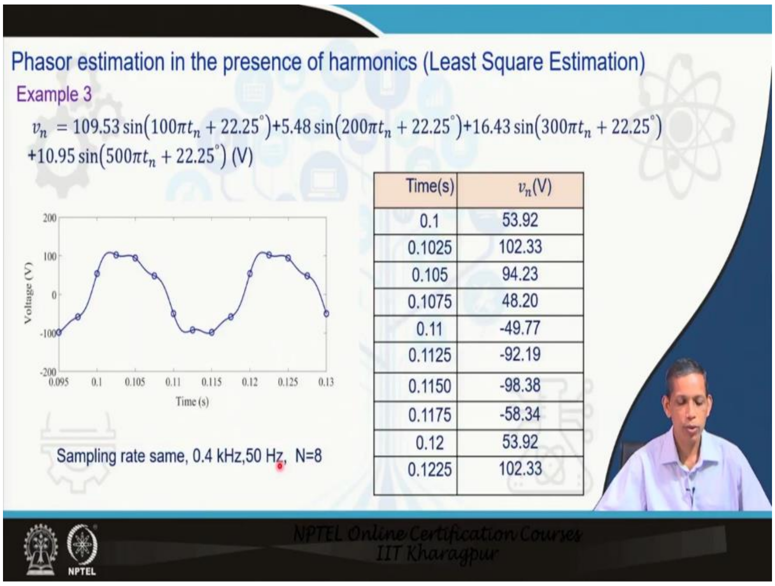

Now let us you can say that there is a chance that the signal may not be exactly fundamental and that this may be contaminated by different harmonics component, we call it noises because we are not, we are modeling in the least square sense only for the fundamental part. So we are not modeling any harmonics and so we can consider them as noise. So this is a signal which we have earlier also discussed you can say that with the fundamental we added you can say that now the different components to the second harmonic, the corresponding other harmonics component, third and fifth and so. So by considering the corresponding signal becomes distorted like this, no more pure fundamental and then you can say that the corresponding signal available, samples available are like this. With the same you can say sampling rate and this and with one cycle becomes n equals to 8. Now we will see how these at this situations, how the least square estimation technique will be good at.

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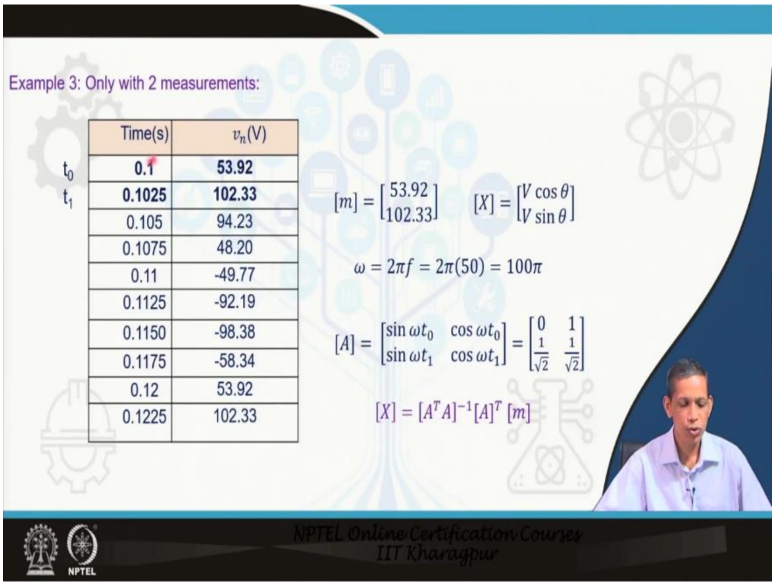

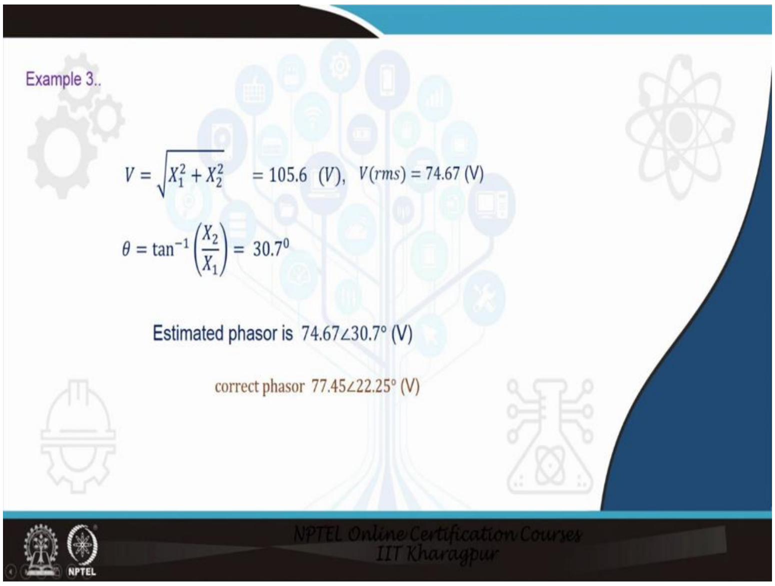

So again two samples, first two samples, distorted signal, again t_0 and t_1. So we progress like this *m* and X. So m becomes this,

$$[m] = \begin{bmatrix} 53.92 \\ 102.33 \end{bmatrix}$$

Only m is changing, X becomes

$$[X] = \begin{bmatrix} V cos\theta \\ V sin\theta \end{bmatrix}$$

Matrix A becomes same what we have considered earlier also

$$= \begin{bmatrix} sin\omega t_0 & cos\omega t_0 \\ sin\omega t_1 & cos\omega t_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Then we apply this one and we calculate the corresponding X and when we use the X, we get the corresponding V to be 105.6 and the rms value becomes now different one, 74.67 and the angle theta not twenty two point something, it is now 30.7^0. Therefore, you see that the estimated phasor becomes for the same fundamental component for the first example also. Now the current phasor you can say we got earlier was $77.4\angle 22.25^0$. Now we are getting you can say that phasor estimated by this process to be different. It means that only using two samples by the least square method we are not able to get the correct phasors as expected. So then what to do?

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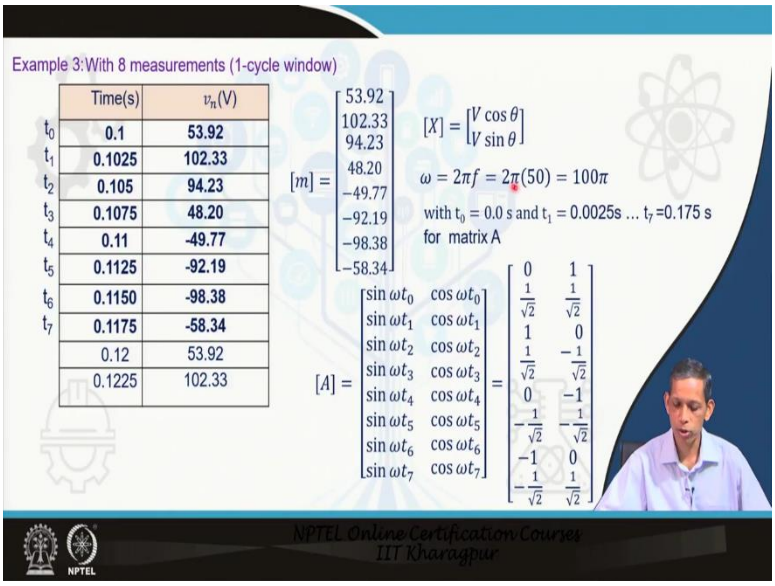

Now we will see what you can do also, we can have more measurements that is the flexibility in this least square approach. So let us you can say that time for simplicity you can take more than two, three, four, five, six, seven and then do. Let us say you can say that we are taking one cycle here like we did it for one DFT. So t_0 through we can say that t_7, we have eight samples here, one cycle. And these are we consider corresponding measurements, you can say that these eight measurements will be considered here. So our measurement m here you can see that are these eight measurements here. So X you can say that again happens to be same two unknowns, *a* and *b* and ω you can say as usual. t_0 we can say that we start from zero to t, you can say at 0175 that is the eighth sample we are talking about and for the A matrix we defined as you can say that having earlier because now we can say that eight points measurement, $\sin\omega t_0$, $\cos\omega t_0$

and like this you can say that $\sin\omega t_7$ to $\cos\omega t_7$. So by substituting the time index from t_0 =0s to t_7 = 0.175s and ω=100. Then we got the A matrix to be

$$A = \begin{bmatrix} sin\omega t_0 & cos\omega t_0 \\ sin\omega t_1 & cos\omega t_1 \\ sin\omega t_2 & cos\omega t_2 \\ sin\omega t_3 & cos\omega t_3 \\ sin\omega t_4 & cos\omega t_4 \\ sin\omega t_5 & cos\omega t_5 \\ sin\omega t_6 & cos\omega t_6 \\ sin\omega t_7 & cos\omega t_7 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & -1 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

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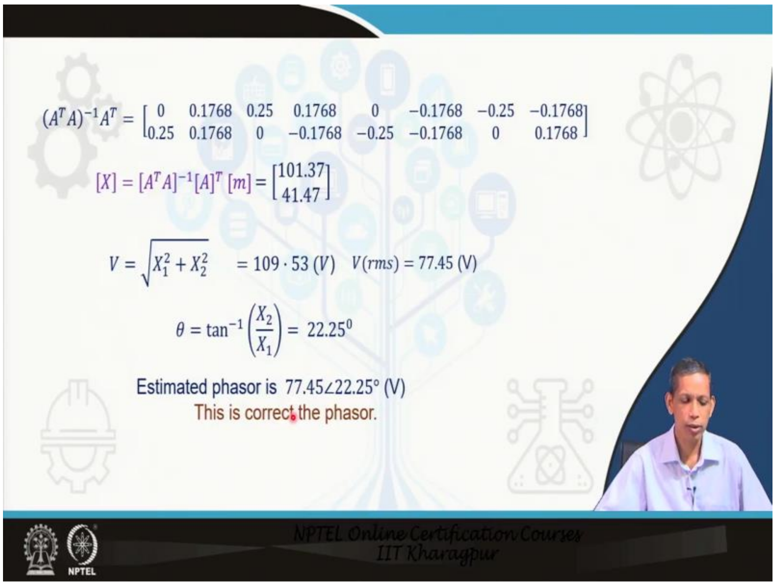

Now, if we apply this corresponding thing to that eight measurements for this estimation of the phasors then we get the corresponding A, the pseudo inverse matrix to be like

$$[A^TA]^{-1}[A]^T = \begin{bmatrix} 0 & 0.1768 & 0.25 & 0.1768 & 0 & -0.1768 & -0.25 & -0.1768 \\ 0.25 & 0.1768 & 0 & -0.1768 & -0.25 & -0.1768 & 0 & 0.1768 \end{bmatrix}$$

And the X matrix is written as

$$[X] = [A^TA]^{-1}[A]^T[m] = \begin{bmatrix} 101.37 \\ 41.47 \end{bmatrix}$$

Then the corresponding V to be like

$$V = \sqrt{X_1^2 + X_2^2} = 109.53(\text{V})\,;\, V_{rms} = 77.45\ (\text{V})$$

This is the correct one and

$$\theta = \tan^{-1}\left(\frac{X_2}{X_1}\right) = 22.25^0$$

The estimated phasor is $77.4\angle 22.25^0$ for this first window. So that is the correct phasors we are getting. So what we did here, that instead of two samples we took more samples which can be easily accommodated in the formulation and by that we can say that the corresponding eight measurements which we took from window. Thus by increasing the window size we are getting the correct phasor estimation even though the corresponding signal is being contaminated by the different harmonics. That is the beauty of the least square estimation technique what we see from this example. Now we will go something more beyond this.

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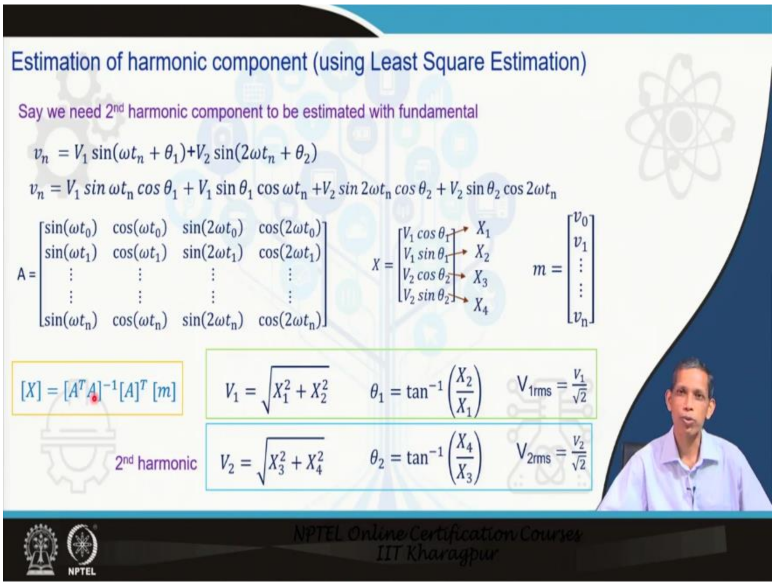

In many applications, what happens that the relay may require second harmonic component, third harmonic component, fifth harmonic component like this, like in one example in inrush current detection in transformer for that we require second harmonic component estimation, so different harmonic components may be required to be estimated correctly. Now in the DFT approach and other approaches now in the least square sense also what we have formulated till now only our target was to how to get the corresponding phasor value, fundamental component, we do not bother about other you can say harmonic components. They were being completely rejected in DFT one cycle and so. So if we did the corresponding particular harmonic component to be estimated properly for utility in the relaying perspective and so then what to do?

Now see here you can say that how we can do in the formulation in the least square perspective. So what we say that we need again second harmonic components to be estimated here. Let us say in addition to the fundamental, so we will formulate the corresponding signals v and whatever we can say the fault signal will be received by the relay in terms of two components, fundamental and the second harmonic component.

$$v_n = V_1 \sin(\omega t_n + \theta_1) + V_2 \sin\,(2\omega t_n + \theta_2)$$

So, V_1 , θ_1 and V_2 and θ_2 are the four unknowns for the corresponding fundamental and the second harmonic component. So we will expand the corresponding signal model to be like this as we have done,

$$v_n = V_1 \sin \omega t_n cos\theta_1 + V_1 \cos \omega t_n sin\theta_1 + V_2 \sin 2\omega t_n cos\theta_2 + V_2 \cos 2\omega t_n cos\theta_2$$

This for the fundamental component first two terms and the second harmonic component, two terms for the second harmonic component. Then the A matrix becomes for this case become

$$A = \begin{bmatrix} sin\omega t_0 & cos\omega t_0 & sin2\omega t_0 & cos2\omega t_0 \\ sin\omega t_1 & cos\omega t_1 & sin2\omega t_0 & cos2\omega t_1 \\ . & . & . & . \\ . & . & . & . \\ sin\omega t_n & cos\omega t_n & sin2\omega t_n & cos2\omega t_n \end{bmatrix}$$

So therefore, the corresponding unknowns becomes four. So therefore, this becomes the corresponding four cross something will be coming from the A matrix. So these results you can say that in terms of that you have n number of measurements you can say that, then you can put the corresponding matrix to be n times of this. So the X becomes

$$[X] = \begin{bmatrix} V_1 cos\theta_1 \\ V_1 sin\theta_1 \\ V_2 cos\theta_2 \\ V_2 sin\theta_2 \end{bmatrix}$$

So we have four unknowns, X_1, X_2, X_3, X_4 and we have *n* measurements

$$m = \begin{bmatrix} v_0 \\ v_1 \\ \cdot \\ \cdot \\ v_n \end{bmatrix}$$

Now this leads to we can say that same you can say that we will apply the corresponding least square sense and then we will find out we can say that fundamental

$$V_1 = \sqrt{X_1^2 + X_2^2}\ ;\ \theta_1 = \tan^{-1}\left(\frac{X_2}{X_1}\right)$$

And for the second harmonic case

$$V_2 = \sqrt{X_3^2 + X_4^2}\ ;\ \theta_2 = \tan^{-1}\left(\frac{X_4}{X_3}\right)$$

The RMS value you can say that can be obtained by from

$$V_{1rms} = \frac{V_1}{\sqrt{2}}\ ;\ V_{2rms} = \frac{V_2}{\sqrt{2}}$$

So this is what you can say that we say that if we like to get the corresponding second harmonic components then we have to formulate in the least square algorithm and then you can say that we can use the pseudo inverse perspective where the corresponding A matrix is being changed and so also the X matrix and then you can say that we can get this second harmonic component fundamental. We can include like this, you can say there are other harmonic components in the system also. Now note that you can say that we can accommodate more and more harmonics but that leads to more computation process because the A matrix size will also increase and more unknowns, more number of measurements also require for better estimation. Now also, if you have included more number of components in the process, then your modeling becomes more accurate and the estimation becomes better with more and more computational model. That leads to situation in terms of all these things, if you only require the corresponding fundamental part here V_1 for this part, you do not consider this required somewhere here even though you have modeled the systems, then we see that these matrices, these lines corresponding to first two part, this part you can say that this is nothing but corresponds to the fundamental part and then this block consider the difference for the second harmonic component. Therefore, you can say that in the A matrix also and in the corresponding pseudo inverse matrix those you can say the portions are being required for the computation perspective. So you can optimize your computation in accordance with the requirement for the relay application.

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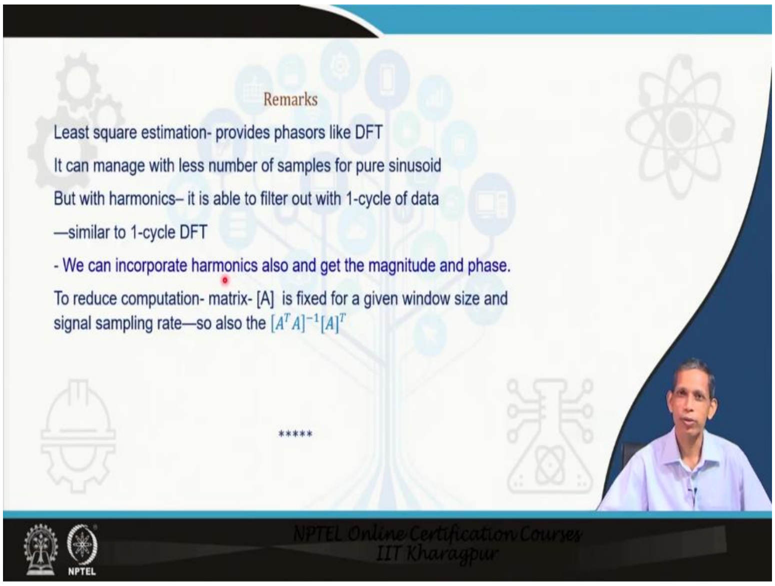

So what do we see? We can say that in overall that the least square estimation technique also provides accurate phasor estimations like in discrete Fourier transforms. If the signal is pure like the steady state or so, we can say that we can manage with the less sample also, but if signal having impurity, you can say that like in different situation we can have more and more sample accommodating systems. We can incorporate harmonic components in the estimation process also and you can say that to reduce the computation burden of *A* we can say fixed for that particular window size, also, we can say that this pseudo inverse is being fixed. Therefore, you can say that there is no need always to compute the corresponding inverse matrix so that reduces the computation burden in the process. In overall, we say that least square is another attractive technique for the phasor estimation perspective. In the subsequent lecture, we will see that how this technique can be used in different other applications also. Thank you.