Network Analysis
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Lecture – 75
Two Port Network – V

We are discussing about reciprocity condition of 2 port network and we have found out for a given 2 port network conditions which must be satisfied if it is Z represented if it is y represented in terms of admittance matrix or if it is h parameter weight is represented and of diagonal elements some conditions are there Z12 should be equal to Z21. Similarly, Y12 should be equal to Y21 but for h parameters there is a slight difference which should be once again h12 h21 but h12 should be equal to negative of h21.

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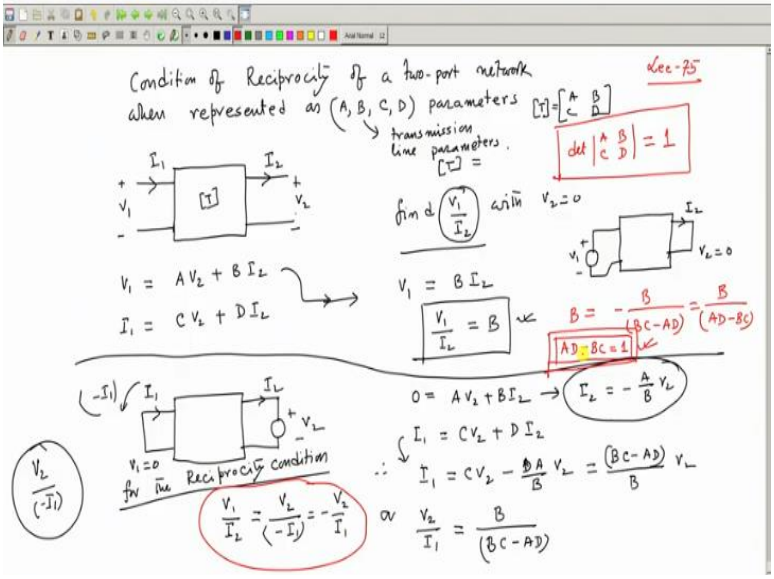

Now today I will discuss condition for reciprocity of a 2 port network when represented as A, B, C, D parameter or transmission line parameters as A, B, C, D parameters what relation A, B, C, D parameters looks like that is A, B, C, D parameters you know this is A, B, C, D from the other two you may be tempted to say maybe something to do with the off diagonal element B should be C we will see that what it is really.

In fact, it is not B = C let us see that so first of all draw the 2 port network and write down the general equations fundamental equations that is if you apply a voltage V1 here this current is I1

and here the voltage polarity is V2. But as I told you in case of A, B, C, D parameters which is also called T parameters transmission line parameters A, B, C, D parameters are also called transmission line parameter and usually written as T that is T is equal to this is T okay.

Anyway so this is the general thing I have drawn but while drawing this I should be careful about this I2 fellow it is assumed always positive when it is going out like that and then the input quantities basic equations are V1 and I1 sending a voltage and sending a current of a transmission line is represented in terms of receiving end voltage and receiving end current that is AV2 + BI2 and CV2 + DI2 this is the thing this is a fundamental equations good.

But once again I have to find out V1 / I2 relation therefore first what we do is this find V1 / I2 with V2 = 0 that is for this from which circuit this is to be found out that was the general circuit that is V2 must be made 0. So, I will short-circuit it V2 is 0 I2 mind you this way and this is you have applied the voltage here V1. So, from this network I have to find out V1 / I2 this ratio and how can I find it out so V1 / I2 I find out okay and this ratio if I want to find out V1 / I2 so from the first equation I will say that look here V1 / I2 okay it is right.

So this first equation will then become how much V1 is there from the first equation this V1 = AV2 that is 0 + BI2 that is very nice because I want to find out V1 / I2 and I have got it so I will keep this V1 / I2 that is the transfer impedance I have to calculate whatever it is V1 / I2 and this will become equal to B and keep this, this is an important result. Then what I will be doing is I will excite this output port with a voltage V2 and keep this side short it.

And I1 current direction of current as I have told you it is specified in this way; I should not play with this but anyway you excite this with V2 short circuit this V1 = 0 clear. Now I calculate V2 / I1, but do you think that V2 / I1 should be equated to V1 / I2 no because you have energized V1 and shown this current to be coming out I2. Therefore, you have applied V2 here current in the other side in this direction is -I1 I should take this ratio that is what important.

So anyway first calculate V2 / I1 that is V1 / I2 in this case because of this thing see the direction of I2 in this case and the direction of I1 should be consistent that is the ratio that is what

reciprocity theorem tells us. So V1 / I2 I have calculated I will keep it in which mode this is V1 + - and current is coming out from this output. Similarly, I have energized this with V2 and but I1, I2 is specified like this.

So I must look forward for V2 / -I1 and this should be equated with V1 / I2 anyway let us see what happens. So that is the thing with this thing if you come to the from the so V1 = 0. So first equation will give you 0 this is the second experiment we have put a horizontal line here first experiment is over. Second experiment but this equation is fundamental V1 = 0 should be equal to A into V2 is not 0 + BI2 this will be there.

And the second big equation is I1 is there which is equal to CV2 + DI2 nothing is 0 here but you want to find out V2 / I1. So first divide by I1 both sides any mistake point out. So I1 you divide so if you divide I1 it will be 1 = C into V2 / I1 + D I2, I2 goes is it correct DI2 / I1 because I have divided this to this or. Now who will give me this I2 / I1 business V2 / I1, I have to do so if you divide V2 this is 1 that is what I have to do.

So what I will do is this instead of dividing first this I2 I will represent it first step let us see, see in this equation V2 / I1 is necessary. So from this equation I will find out I2 as -A / B into V2. So that this equation I1 = CV2 was there plus D into I2 and for I2 I will put – DA / B into V2 is it not this is the thing, and this will be equal to BC- AD divided by B into V2 or I will say V2 / I1 = B / BC-AD that is what I will get.

But the question is here because this I2 I have a defined in a different way for one experiment you have found out you have excited this side got the current this way. So to have the networks satisfying reciprocity same polarity of voltage I have applied on this side, but I should take the current in the same way is it not. So I should not equate V1 / I2 = V2 / I1 what it will be equated to for the; this quantity for the reciprocity condition we must demand that V1 / I2 should be equal to V2 / -I1 that is what I require which is = - V2 / I1.

This is very important to note because what happens A, B, C, D parameters are defined like that all people are following this that is the convention because of this thing happens. Therefore, I

will say then V1 / I2 is B. B should be equal to -V2 / I1 that is - B / BC - AD which means that B / AD - BC B, B goes and I will say AD - BC should be equal to 1 that is the condition therefore if a network two port network is represented in terms of A, B, C, D parameters B and C alone does not decide whether it is a of diagonal elements same or no.

The condition is totally different it says that determinant of this matrix A, B, C, D determinant this must be equal to 1 that is the condition got the point therefore if it is represented in terms of impedance matrix Z12 is Z21 if it is in terms of admittance matrix Y12 should be equal to Y21 if it is h parameters then also h12 and h21 but h12 should be negative of h21 and if it is represented in terms of A, B, C, D parameters this is the condition final condition got the idea. So this is how it is to be found out see reciprocity property you must as I was telling if somebody says that one should not be have any doubt about the step one that is why I am telling.

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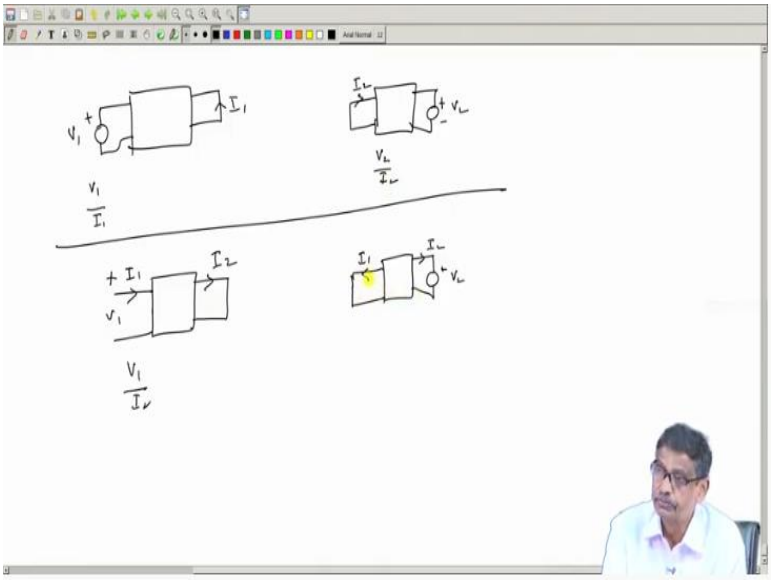

If somebody says here is a network and you apply a voltage here V1 forget about these Two port network V1 you have applied and you record this current in this direction I1 calculate this ratio V1 / I1 now what I am telling if the same network if you excite keeping the voltage polarity same with V2 and short it and tell this current to be also I2 like this then there is no conflict everything is then V1 / I2 should be equal to V2 / I2. Here current is coming in here also current is coming in case of A, B, C, D parameters what has happened if somebody says that this is V1 this is I1 and somebody says that I2 I will take like this calculate V1 / I2 of this.

But then in the second case this is V2 this remains I2, but this current is of interest transfer this voltage and divided by this current. If somebody says this current is I1 when you short it because deduction of current is my prerogative I will choose the way I like then this impedance V2 / I1 if you calculate and equate it to V1 / I2 that is not fair that is not correct that is what I want to tell what you should do V1 / I that is fine so V2 / I1 in this way if you assume that is okay. So i think you have got the point any doubt hopefully okay that is fine okay.

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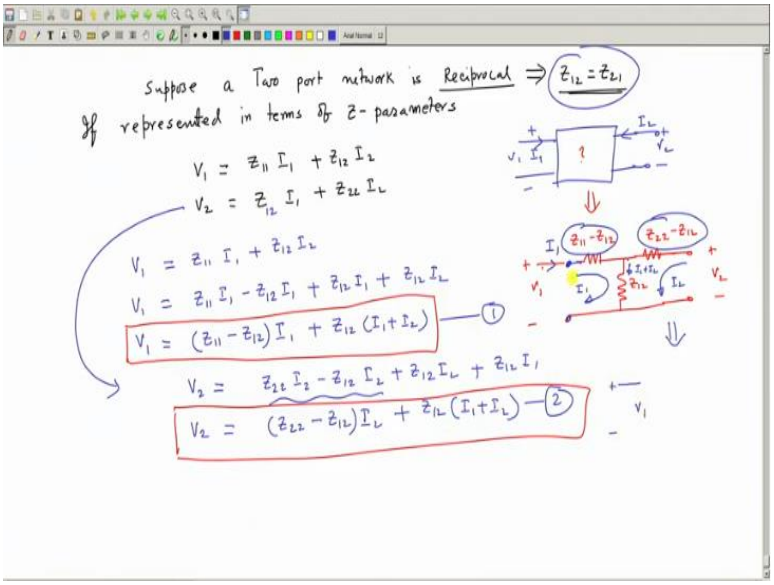

After I have done this now the question is you will be able to solve several problems okay today I will tell you an interesting thing that is suppose a Two port network is reciprocal okay we know what it means therefore if represented in terms of Z parameters I have represented in terms of Z parameters then these equations are true. So reciprocal so I know that then Z12 = Z21 that is what. So V1 = Z11 I1 + Z12 I2 and V2 = Z21 I1 + Z22 I2 is it not.

But in that case, Z12 = Z21 okay so this equation then by writing metrics way it is okay so I can replace it this one sorry by Z12 itself of diagonal elements will be same that is what we have found. Let us look at the first equation V1 can written as Z11 I1 + Z12 I2 that is fine then I will just manipulate this equation a bit this way I write Z11. I will go step by step so I will write it is equal to Z11 I1 was there then what I do I subtract this term Z12 I1 and add the same terms like this I can do that everything remains same.

Now this V1 then will become Z11 – Z12 into I1 and this one will be sorry this one will be Z12 into I1 + I2 ls it not like this so keep this the equation 1. Similarly, you play with the second equation what do you do that second equation V2 = Z22 I2. So, you write Z22 I2 then subtract Z12 I2 then add Z12 I2 so that it remains same as it was and finally the first term is always there Z12 I2.

Therefore, it can be written like this that Z22 – Z12 into I2 these two terms and this one will be +Z12 into I1 + I2 let this equation be 2. Now if it is like this then this 2 port network can be because see in these 2 port network what I told I do not know how the impedances are connected actual networks we have shown with some example earlier that + - V1 these are the terminal voltage current I am only measuring and trying to model in my own way whether Z parameters, h parameters or things like that that is fine V1 I1 V2 I2 now and I find out the parameters and I am told that it is a reciprocal networks.

So in that case Z12 must be equal to Z21 therefore this basic equation can be written like this that is highlight these two equations this equation and this equation looking at this equation I will say that inside this network I can presume that the network is a T connected network Z11 – Z12 and this one these are impedances Z21 Z22 and Z12 and here Z12 these are very interesting desserts and see earlier it was unknown how these impedances are there.

But what I am telling it was a black box I found out these and also concluded it is a reciprocal network during the course of experiment Z12 came out to be same as Z21 I concluded it is a reciprocal network if it is then I am telling manipulate these 2 equations in this way then you see the this is your V1 you are a applying plus minus this is your V2 you are applying this is also plus minus.

Now this current I1 is like this so what is this current I can think it is I1 goes in. Similarly, this current V2 I always assume in this way so this current is going like this that is this I1 I2 be thought up to be a look current of this state work of this T network it is called it looks like a T

where the elements impedance values of this one is Z11 – Z12 it is Z22 – Z12 and Z12 you see this what is the loop equation here the first equation look at V1 = Z12, Z12 into I1 plus.

What is the current flowing through this branch I1 +I2 into Z12? And these 2 voltages must balance V1 that is what exactly I am getting. Similarly, in the second loop applied voltage is V2 it will be = Z22 - Z12 into I2 but what is the current flowing through this Z12 it is I1 + I2 and I1 + I2 into Z12. So this then tells me that okay you may not be knowing the internal structure of the impedances how they are connected.

But if it is a reciprocal network you can always think this network is nothing but a T network with these parameter values are you with me that is I am now telling something more inside this I can now think of it is a network like this got the point which was hidden to not known to me and once you do this let me be in this page only once you do this see the interesting part is these are the ports these are available to me and inside.

Now I can think okay it is a T network or this one if it is like that these 3 impedances that connected in start I could also if I wish this star, I will represented by some delta impedance because I now know the parameter values here, there, there. So I can also represent it by an equivalent let me go to next page.

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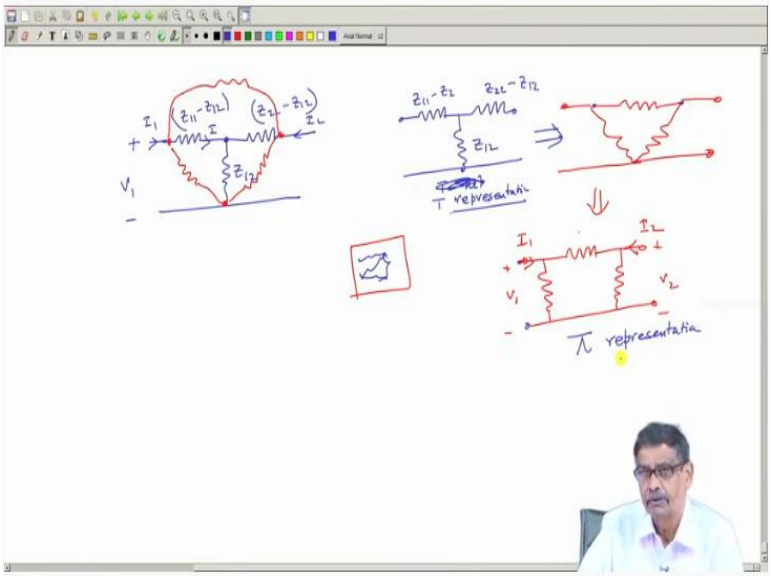

So I have got this thing I do not bother what or how complex the network was but ultimately I have shown that if this is a supply voltage this is the current I1 you can represent it as a this, this is Z12 know and this is Z22 – Z12 this is I2 this is I1 and this current is I1 + I2 and so on we have seen that let me now put too many things here and all the parameter values are known this, this and this now what I am telling these three impedances are connected in star.

Therefore, I can replace this by some equivalent delta these are the key points about which i can do the start delta transformation and I will put it like this, and it is the blue terms. So this this star connected networks that is this 3 that is a good thing Z11 – Z12 Z22 – Z12 and this is Z12 with respect to these 3 points this can be these 3 points are I cannot touch their 3 terminal 2 port network, this can be drawn as a circuit like this. Is it not, that is what I told you.

This is my input port, and this is my output port input port remains same or with respect to input and output port you can write like this. So I1 V1 this is V2 and this is I2 what will be the value of this impedance star connected rule I will apply this plus this plus this into this divided by this. So I can always find out these values also therefore I am telling that in a sense generally all the networks will be bilateral and linear networks.

And no matter how in the complex way the impedances are connected inside this black box. If you tell me the value I will always tell that okay this network can be represented as a T network if it is like this, this is the T representation T network T representation T this is not the way to A, B, C, D parameters T representation or you can say it is a pi representation do I know the values of these impedances yes i know because I know and vice versa.

Therefore, remember how or whatever complex way the impedances are connected inside eventually you can with respect to the input and output ports you can consider to be some lumped parameters connected in T fashion and delta fashion pi fashion and this is widely used in power system sort of representation and we will continue further with our discussion what do I mean by series parallel connection of 2 port networks in my next class. Thank you