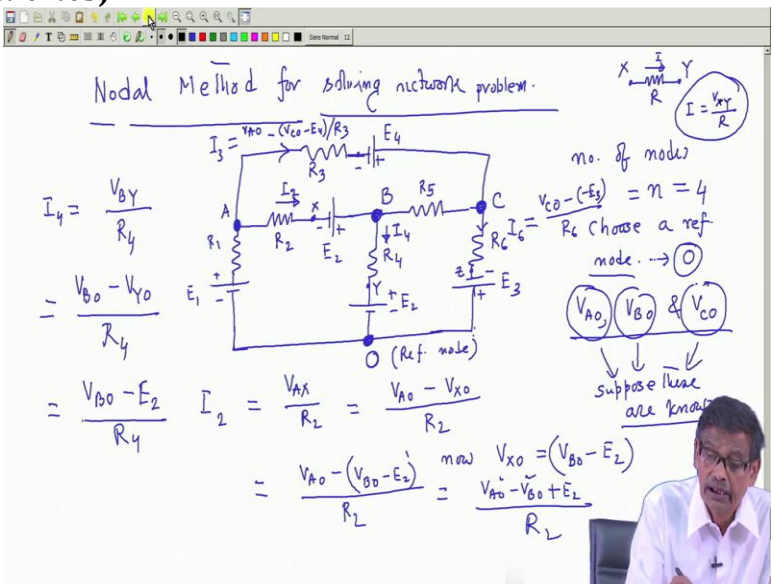
Network Analysis
Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology-Kharagpur

Lecture # 07
Nodal Analysis-II

Welcome to lecture number 7, and we have been discussing with 2 important methods of solving networks that is the mesh analysis and the nodal analysis in our last class. We went up to this, that nodal method for solving network. Just remember that current in any resistance, just reviewing that part very quickly. Current from X to Y will be given by VXY. We know what these 2 suffixes X and Y means. It means potential of X with respect to Y so, current direction since we are interested to know from X to Y. So, you have to take.

(Refer Slide Time: 01:03)



VXY and that you have to divide by the resistance that is the essence of this thing. Now, then I told you that in a network, there will be several nodes, each node is defined as the junction of at least 3 element ends, for example, A, B, C and O and then I told you that if by chance if you know one of these 3 nodes you take at reference O any one of them so, I have taken this to be as my reference node. So, with respect to that.

VAO, VBO and VCO are called the node voltages. And what I told last time, if by chance I know these three node voltages VAO VBO and VCO I am telling that you have solved the

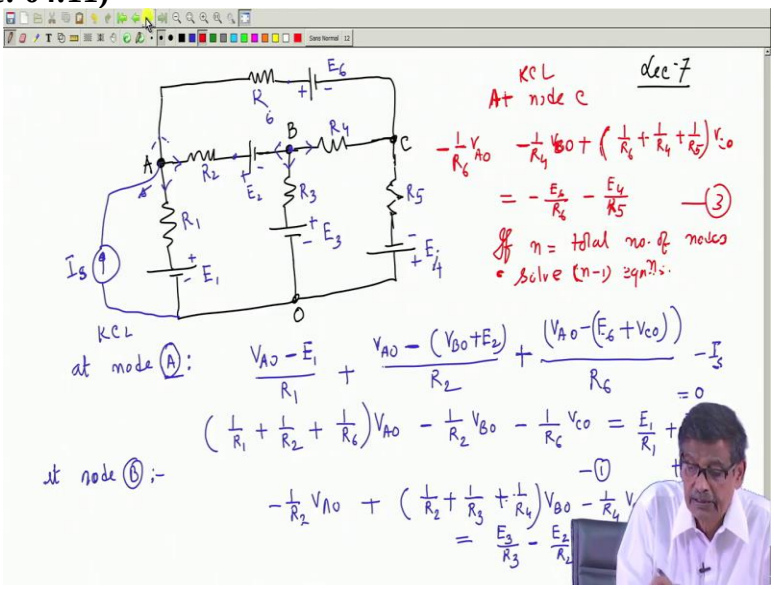
circuit, because if you are now, to find out the currency name any of the branches for example, in this branch, if you want to find out the current, that current to will be put in if grab the resistance of that particular path for example R3 and see, what is the voltage difference there.

This potential, the same potential is certainly VAO I know with respect to same point have to calculate the potential of this. Now, from how can you reach this point, this point you can reach VCO then you see there is a voltage drop here so, potential of this point with respect to same point O will be then VCO - E4. That is why the current in I3 in this direction is VAO - potential of this point which is VCO – E4/R3.

Similarly, the same thing I have applied to other branches as well, the point I want to make is that if the potential of these 3 nodes are known with respect to one of the nodes being defined as reference node, then you can find out the currency in any of the branches that is the essence of the thing. Now, I started my discussion telling that VAO VBO VCO unknown then only you can do this now.

The question is how can, I find out VAO VBO VCO that is our aim will be given a network. After identifying the node choosing one of them as a reference Then try to find out the node potentials of other nodes. So, how to do it that is the thing. So, let us see.

(Refer Slide Time: 04:11)



Suppose what I draw another circle which may not be exactly the same but similar type C as suppose we have a network like this and here is some resistance there is some source and there is another resistance source here is some resistance here and some source. So, and there is another elements here suppose this is a resistance and this is another source so, I identify the notes as usual.

This is 1 note this is another note and I choose this to be as a reference and say that this node name of this node is A this node is B this node is C our target is to node to node to solve for VAO VBO and VCO that is we are not targeting to calculate the currents in the branch straightaway we first calculate the node voltages and as I told you if these voltages are known calculation of current in any of the branches is taking the difference dividing by the resistance Et cetera will coming.

Now, how to solve this thing and let me also add another thing here suppose here is also a current source, just to a fixed current source ideal current source is connected here. Now, what is done is this to solve for VAO VBO VCO right the KCL add nodes A, B and C. That is what you have to do so, at node A KCL is the common thing at no day what it will be. And suppose these values are known this is E1 + - R1, this is + - E2, this is R2, this is + - E3 and this is R3.

And this is suppose some R4 this is - + if I E4 E2 E3 R4, say this is E4, how does it matter, and this is R5 this is E4 and this is R5 this is R6 and this is + - E6. Suppose, so, this is the circuit and there is also a current source connected now at node A KCL when I will calculate I will always calculate currents flowing away from node A in this branch this way and some of all the currents then will become 0 that is the whole idea. Of course, all currents I will calculate away from the circuits you read a stat assured that at the end.

When you get the numerical values some of them that is current going away from the node will become negative indicating they are really coming out. So, we because direction of current is decided by me. So, I just say that I will calculate current emanating from this node, in this direction, and this direction, and in this direction and some of all the currents will be 0. So first calculate this current, this current will be, in terms of node voltages.

VAO potential of this point divided by put - potential of this point that is to be noted that is VAO - what is the potential of this point with respect to O very simple + C1 divided by R1 This will be the current If somebody had assumed current flowing from below to talk, then you would have defined current that current to E1 – VAO/R1 I think you have understood the point. So, this is this current.

Similarly, what is this current these will be + this current + this current potential of grab the resistance try to find out the potential difference between these 2 points so, this is VAO - potential of this point with respect to O and that in terms of VBO it is VBO + C2 - VBO +E2 + this will be the and that divided by R2 will give me current in this direction that is all together 4 currents are branching off from A. So, 2 have taken.

Third one will be similarly + of VAO – E6 + VCO is not VCO + E6 is the potential of this point this resistance potential difference, this divided by R6. So, 1, 2, 3 this 3 current are remained then this is the current source S. I am writing, then in this direction the current source I have shown to be like this. So, this must be - IS which is coming out from A this way and this must be 0. So, what is my job my job is to generate 3 equations.

Involve being VAO VBO VCO and solve them to get VAO VBO VCO. So, this is 1 of those 3 equations mind it. Now, this equation if you collect if you try to find out the coefficients of VAO it can be written as 1 by R1, collect these terms involving 0 + 1 by R2 this will get + 1/R6 into VAO this will be the coefficients of VAO what is the coefficient of VBO. You can see that is this term - 1 by R2 into VBO.

Then what will be the coefficient of VCO this is - and this - goes in + 1/R6 is not + or - VAO - VCO R6 so, this I wrote wrongly + all some of the all the currents coming out these 0. So, this is +. So this once again will be – 1/R6 into VCO and then on the right hand side, I will write down the sources. For example, it will be E1/R1 on the right hand side and then this E2/R2 - so, it will go there.

It is also + E2/R2 and then + once again E6/R6 and then there is another current source that will also I will write it here because these things will be known sources are unknown, I want to find out the currents so, this will be equation one for this network got the point. Therefore, see dimensionally on the left hand side voltage by some impedance current ampere, this is also ampere. So right hand side also matches.

Each 1 of them is in ampere. So, this is 1 of the independent equations involving VA0 VB0 VC0. Now after writing down this equations, we see there is a pattern. If you are writing down the KCL at node A the coefficient of VAO will be some of reciprocal of all the resistances connected that node A that is 1/R1 + 1/R2 + 1/R6. And in these brands, there is no resistance nothing is coming out.

Coefficient of VBO will be any resistance connected between A and B because I am writing at node A. So, A is at the central point, whether there is any common resistance between A and B yes R2. So, reciprocal of that resistance but preceded by a negative sign VBO similarly, coefficient of VCO it will be – 1/R6 because there is a common resistance between A and C which is R6.

This is what I will only remember. Therefore, this will be the first equation. Similarly, following this logic of course, this I have written by rigorously right each term from the fundamental things, then after that writing I discovered there is a pattern that you must understand. So, similarly, I can write from very basics, the KCL here, this current some of this current is 0, then keeping VAO VBO VCO on the left hand side, bringing all the sources on the right hand side, I can get that.

Now, coming to this first equation, what about the right hand side right hand side as I told you this will be all sources. So, for example, E1 is a source and it has to be current on the right hand side So, E1 divided by the resistance in series with E1 that will be the term here E1/R1 and whether today right + or - it will be + If the current natural direction of the current that E1 is supposed to deliver with regard to this polarity is E1/R1 that is if the positive of the battery is connected to node A then it will be + E1/R1.

Similarly, there is a current source a voltage source a R6 which is by R6 connected to A. So, this term will also come and this will be E6/ R6 because after all it is has to be ampere E6/R6 and +E is connected towards A so + this is this term. Similarly, about E3 it will be E3/R2 because +E is connected towards it why E3 are not another term will be there now + because there is another source what is the E3 R3 is not there it is not involved in at node A this E3 R3 are not involved.

So, this is that solved that is correct. So, this is first equation now let us write down KCL at known at node B at node B in the same way I will write so if because that node be at the center stage now, so, coefficient of VBO will be Some of the reciprocal of all the resistance connected here 1/R2 + 1/R3 + 1/R4 and these should be VBO. If you simplify that writing from basics coefficient of VAO will it be present there yes.

Because between the end be there is R2 so it should be - reciprocal up that resistance into VA0 plus and coefficient of VCO, S-1/R4 into VC0, left hand side is over and right hand side will be all these sources will appear here in the form of current Therefore, you see, there is a battery connected here to be with + connected towards B. So, there will be 3/R3 and that will be positive.

So, right hand side E3/R3 this is correct, there is another battery to connected here to be so, the C2 by R2 but it negative is connected to be so, - E6/R6 will not appear at B because they are not connected to not there between B and C there is no source so only these 2 sources but node down the and node current source is also not coming so that is all this is no equation to hear.

And I will write also the equation at no C on the same page it will be congested but let it let me write it here with a different color. So, everything is there on the same page at note see what it will be. So, I let me rub this up now, at the third at note C what will be the equation let us write and add node C KCL at node C. Again once again start from the basics, but I will not do that. Because I know there is a certain pattern in which the coefficients of VAO VBO and VCO at decided.

So at node C coefficient of VC0 will be some of reciprocal of all the resistance is connected to node C that is 1 over R6 + 1 over R4 + 1 over R5. This will be the left hand side and coefficient of VC 0 is this coefficient of VB0 will be - 1 by R4 into VB0 will be the thing. This is the VB0 and coefficient of A0 will be – 1/R6 into VA0 and this will be equal to this is VC0 this will be equal to sources right hand side sources so at C 0, there is a E6/R6 will appear.

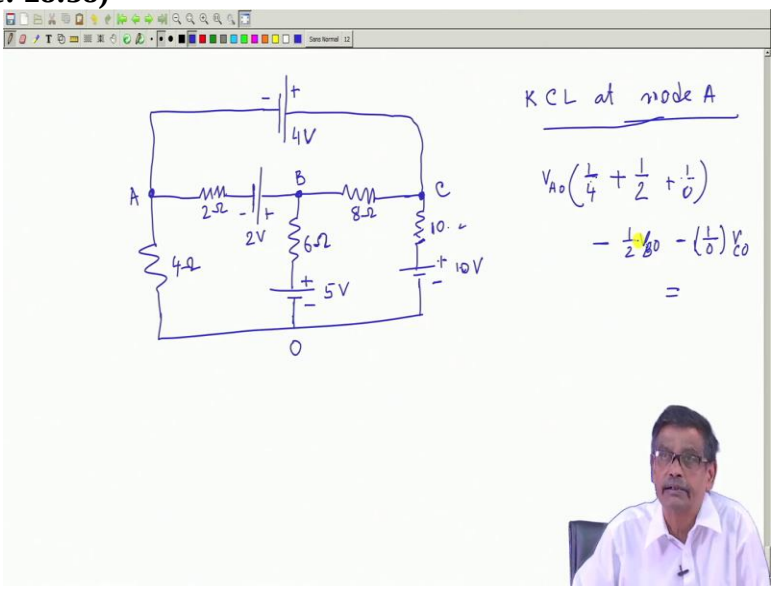
But negative is connected to C so it should be - E6/R6 this will be the current. In this branch no resistance no term appears sources is not present here. In this case, it will be once again minus is connected to C so – E4/R5 so this is the third equation. In essence what I am trying to tell in nodal analysis, you identify the nodes. If total number of nodes if in equal to total number of nodes.

Then you have to generate n - equations solve you have to solve n - 1 equations. For example here n = 4, so, 3 equations and 3 node voltages you have to solve. Now, in conclusion, I will say that to write down the KCL at the different nodes I will not start from scratch after knowing this I will straight away write down this equations by inspection. I will not spend any further time to once again scratch my head and see this way current is flowing that way No I will not do that.

Because that have done for the first time and then I soon discovered that there is a pattern existing and then I will encase that E To write down all the three equations in no time practically, I mean very quickly, we have to be very fast and write down this equation and get it. So, number of equations to be solved here is 3 is that point. Now, so, in the mesh analysis number of equations to be solved in general is equal to the number of meshes present in the network. What is the mesh we, know number of equations to be solved.

If you want to adopt nodal method is n - 1 where n is the total number of nodes. We have seen that in mesh analysis, if E will appeared that in one of the outermost meshes, there is a current source. Then number of equations to be solved will be even less than the number of meshes one less is not that we have learned. Now, in this case, I have not discussed that aspect whether it is always in - 1 equations to be solved or also there may be situations where number of equations to be solved maybe less. For example, let me.

(Refer Slide Time: 28:38)



Consider just draw a circuit like this and it will bring out another thing that suppose you have a network now I will write some numbers so that you understand what I am doing suppose you have a network like this and I will bring out another important concept as well. Suppose this is the thing you will do. Now let me put some numbers say this is 4 ohm this is 2 ohm this is 6 ohm. This is 8 ohm and this is 10 ohm.

Let me write you can also make your own circuit for understanding this thing and this voltage is supposed 10 volt this is suppose 5 volt and this is supposed 2 volt. Now, this network has got suppose another branch this point to listen carefully what I meant to say and here is an ideal battery no internal resistance + - and this is suppose 4 volt this network problem you have to solve.

Now, the point it that if you assume mesh analysis method there are 3 meshes very. So, assume I1, I2, I3 and you can generate those 3 equations by inspection coefficient of I1 will be some of all the resistances et cetera like that and writing sources on the right hand side. Now, suppose I insist that solve this network by nodal method. Now, in the nodal method, the first thing is you identify the nodes and name them.

For example, you name it A, B, C, and suppose O and then your target will be to solve. For VA0 VB0 and VC0 if you have chosen O as your reference note and we know how to write down the

KCL said the different nodes. Now suppose, I have chosen O as my reference node. Then coefficient of VAO that is KCL at node A suppose I want to write down node A what is the rule I told you coefficient of VA0 will be some of reciprocal of all the resistance is connected 2A that is 1 by 4 + 1/2 and in this branch there is an ideal battery.

So, resistance here is 0. That is it should be 1 by 0 it looks like I mean, the way I told you and this should be coefficient of VA0 then coefficient of VA0 will be 1/2 into VB0. And coefficient of VC0 once again is a dreaded term 1/0 into VC0. Because common resistance between MC0 resistance and this should be equal to the sources whatever it will it there will be also some problems.

So, it looks like you can proceed what you will do with coefficient VA0 as I, I becoming infinity, you cannot solve that equation is not. So, that is it looks like there is some inherent problem and in fact, it is an inherent problem and we will discuss this very interesting aspects in my next class. Thank you.