Network Analysis
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Lecture 55
Star-Delta and Delta-Star Transformation

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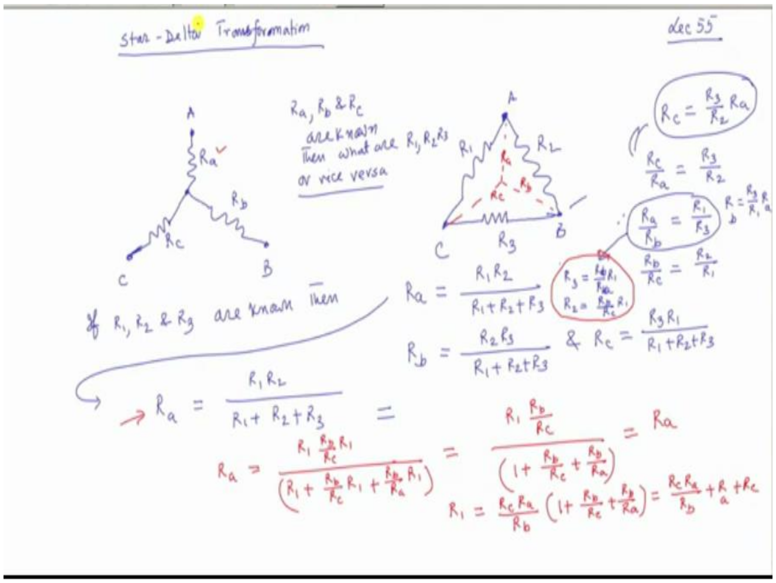

So let us start with star-delta formation in lecture 55 and this is the thing we have done for star delta transformation, that is if you know, this R1, R2, R3 in delta fashion at A, B and C, then you can replace R1, R2, R3 by some star connected loads RA, RB, RC and these RA, RB, RC if you know the values of R1, R2, R3, then RA, RB, and RC can be found out, okay. Also, now I want to find out the reverse formula that is if RA, RB, RC are known, then what should be R1, R2 and R3.

So to find this out, what we do here is this one. That is in lecture 55, you see if you take this ratio, RA by RB which are the 3 values is known, it comes to that. Now what we will be doing is we will express R2 and R1. Let me write these ratios very clearly that is we have got these ratio, RC is equal to R3 by R2 into RA, therefore and another important ratio is this one. Therefore, for example, from these, I express R3 in terms of RA by RB into R1, I can write it like this.

Similarly, I would express R2 in terms of R1, R2 by R1 is this. So it will be RB by RC into R1, R3 is equal to RB by RA, thank you, RB by RA. So these 2, I have expressed R3 and R2 in terms of R1, then I go to say, this equation which has already been established and I will say that RA is equal to from this; R1 into R2, so R1, I will express everything in terms of R1 here, R2 and R3 in terms of R1.

So that on the right hand side only R1 and RA, RB, RC will be there. So I will be able to express R1 in terms of RA, RB, RC. So R1 and for R2, I will write RB by RC into R1, this I will write, divided by this is R1 is there plus for R2, I will write RB by RC into R1 plus R3, but for R3, I will write RB by RA into R1, like this, this I can write, then you see that there are R1 square on the top and one R1 will come out.

So this will be equal to; if I clean this up, so this will be equal to R1 into RB by RC, one R1 will go and it will be 1 plus RB by RC plus RB by RA. This will be the thing and that I am telling it is equal to RA. Therefore, if I take all these things to the right hand side, keeping R1 on the left hand side, I will get R1 is equal to RC into RA by RB and whole into 1 plus RB by RC plus RB by RA, that is over.

But you push it this inside, then what you will be getting? You will be getting RC RA first term by RB plus 1 plus, this RB and RC will cancel out, you will get RA here and the third term will be RB and RA will cancel out plus RC and you are done. So R1, the value of R1 will be equal to RA plus RC plus product of these 2 divided by the third one.

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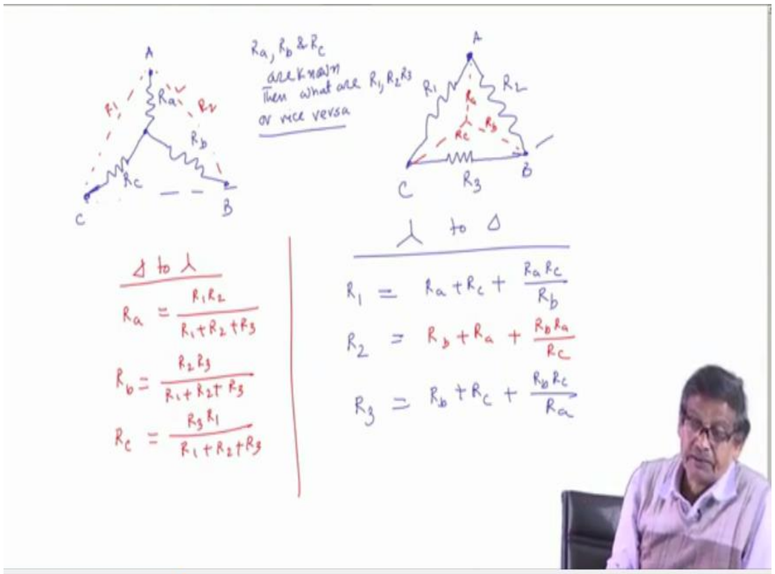

Therefore, it is still there. So finally, this is the thing. Therefore, if the equivalence between these 2 circuits we have already seen for delta to star transformation, delta to star, first we derived that is these that RA is equal to R1 R2 divided by R1 plus R2 plus R3. RB was equal to RB is this resistance, R2 R3 divided by R1 plus R2 plus R3. They could be also impedances, do not worry, RC equal to R3 R1 divided by R1 plus R2 plus R3.

So if the delta values of resistances are given, equivalence star can be made at those 3 points A, B, C and for star to delta transformation, that is you know the value of RA, RB, RC, then what should be this R1? This is here the R1 should come, so it is easy to remember. R1 will be equal to the sum of the adjacent resistance that is RA plus RC plus RA RC divided by RB, product of these 2 divided by the third one.

Similarly, for this I have done and similarly, for the other I will be, in the similar way, I will be able to do that. R2 that is this resistance, that is R2 will be how much? RA plus RB adjacent resistances sum of them, RB plus RA divided by RB RA by RC and finally, R3 will be, R3 is this one, so it will be RB plus RC plus RB RC by RA. So when it will be necessary? For example, if you have a circuit like this.

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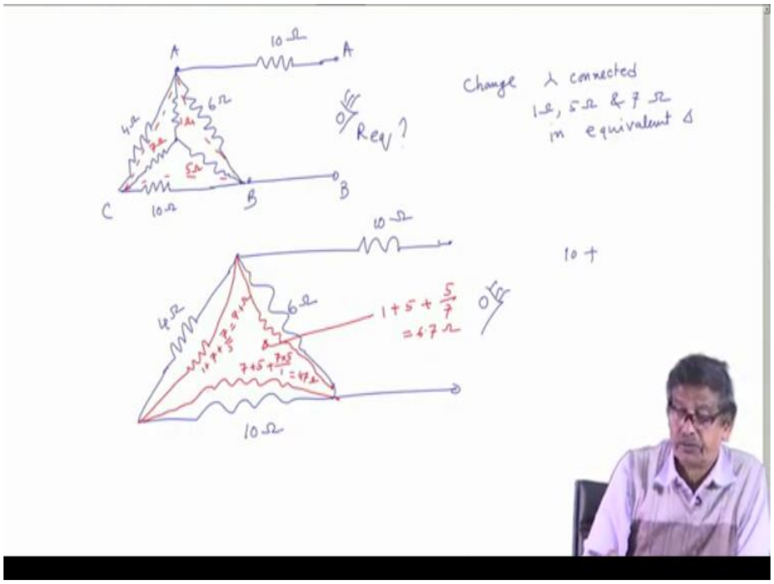

Suppose, I say that you have 3 resistances. Now I am coming to a problem, what I am asking to tell you and there are 3 resistances connected, just a simple example. Suppose, these values are known; 4 ohm, 6 ohm, 10 ohm, this is how 6 resistances are connected and this is a 1 ohm, 5 ohm and 7 ohm, it is there. Now, you have a circuit like this. Another resistance is a 10 ohm.

Suppose, I ask you what is the equivalent impedance looking from the points A and B which you are supposed to do after replacing the sources in any network to find out R Thevenin's or z Thevenin's. Now, here the knowledge of simple series-parallel equivalent resistance formula will not work because the resistance value RAB be will be 10 plus the equivalent resistance of this network between these 2 points and that is very difficult to find out.

Because I do not know this 1 ohm, whether it is in series with 6 ohm or in parallel, no, I am not sure, similarly 4 ohm, then what I will be doing is this, I will be changing, these 3 points are fixed, this is A, this is B, this is C. Question is what is R equivalent? So what I will do? I will replace this delta connected resistances, 4 ohm, 6 ohm and 10 ohm in points A, B and C by an equivalent star I can do or I can change this star to equivalent delta.

So change star connected 1 ohm, 5 ohm and 7 ohm in equivalent delta. So how it will look like? So there were this 4 ohm, 6 ohm and this 10 ohm is there, this I am not disturbing, 10 ohm is there. Here was some additional resistances, 10 ohm connected here. So let this remain, 4 ohm, 6 ohm and 10 ohm. Now this 1, 7, this I will replace it by the equivalent delta here. Another resistance will come there, another resistance will come there, another resistance will come there.

And then I will replace, forget about this star connected 1 ohm, 7 ohm and 5 ohm, so what will be this additional resistance across ABC? There will be this resistance and what will be the value of this? It will be 1 plus 7 star to delta; 1 plus 7 plus 7 by 5, is it not? RA this plus this, plus this into this, divided by the third one, 5 ohm, how much it is? Some value will come, is it coming 9.4, so let us put it. Similarly, this I will replace by a resistance here.

What will be this value? It will be 7 plus 5, let me write 7 plus 5 plus 7 into 5 divided by 1, 47, whatever it is and finally here there will be another resistance connected between these 2 points and this resistance value will be this one. If I write it here, it will be 1 plus 5, so much ohm, 1 ohm plus 5 ohm plus 5 by 7, 6.7 ohm or so. Now I will not carry on with this calculation, I just told you this is this portion.

Then I have been asked to calculate the equivalent resistance of these, then you see this 6 ohm and this red resistance are in parallel, formula can be applied. These 2 are in parallel. I can easily calculate, these 2 are in parallel. Then this and this will be in series and this equivalent resistance can be easily calculated, which will be 10 plus this equivalent resistance between these 2 points.

It is just to tell you that in some situations, when we cannot make out whether the resistances or impedances are really in series or in parallel the star delta transformations or delta star transformation are very useful. That is how it has been pointed out. I hope you have understood it. See problems to find out equivalent impedance, when the resistances are connected in an awkward fashion. Today now, what I am planning to do is to start another interesting theorem and that is called the Tellegen’s theorem okay.

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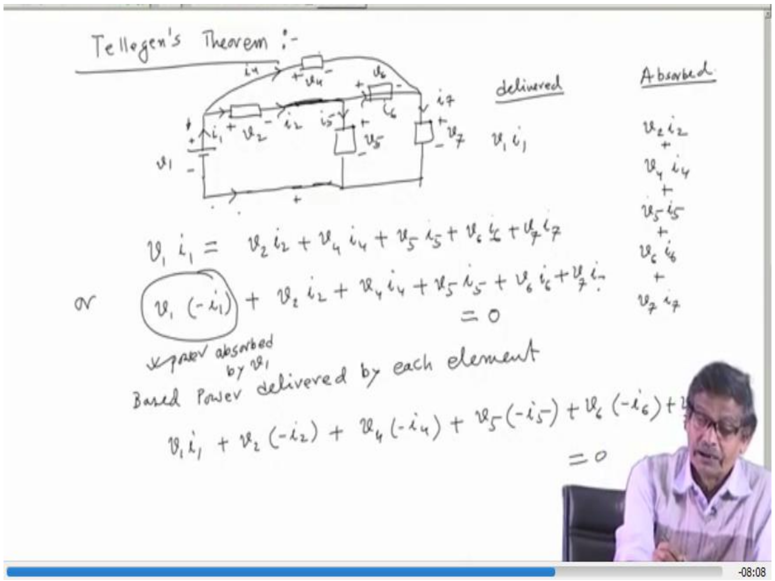

So let us start that, at least the idea, Tellegen's theorem. Tellegen's theorem is a very interesting theorem. It is only applicable, but before we start really Tellegen's theorem, you please try to recall that in any network, there may be sources, there may be several impedances connected like this. Suppose, you have a network like this, then in this network, there will be several elements connected and you can have another elements connected here in between these two.

Suppose, you are considering this network, these elements is could be also source, this is another element, another source here but whatever it is, in this network suppose, I say the voltage across each element, I will call it some value say, v1, voltage across this element v2, understood, this is suppose, v3 what not, v3 and so on and v4, v5, v6 and v7 and also arbitrarily, if I assume the currents; i1, i2, i3, and say, i4, say i6, i7, i5, this way I have done it.

This of course, you must understand in this diagram, this i3 and i1 are same. So let us consider only one element in this branch, they can be combined. So there is nothing like this one, but what I am trying to tell, given a network I will be able to calculate the voltage across each element. That element could be an impedance as well as an source and also I am able to calculate the voltage across each element.

Now in a network if you calculate the product of voltage and current and sum them up for all the elements, it will come out to be 0, okay. That is precisely whatever I am trying to tell. For example, I say that what is the power delivered by this element? This will be v1 i1, this element, after I have assumed, so delivered, I was writing like that delivered and here I will write absorbed and once you have assigned the voltage polarities and the current, do not play with that.

So this is the it is delivering power, what about element doing? Element 2 is absorbing power, because current is entering through plus, so it will be v2 into i2, element 2; it is absorbing power. Element 3 of course nothing, element 3, here it is element 4, it is also absorbing power, v4 i4 and element 5 is also absorbing power, so power absorbed is given by the v5 into i5 and element 6 is also absorbing power, v6 i6, element 7 also absorbing power, v7 i7, is it not.

So how many elements are there? 1, 2, 3, 4, 5, 6; 1, 2, 3, 4, 5, another is left out; v1 i1 is another element, therefore I will say that in this network, this equation is correct, v1 i1 delivered is equal to v2 i2, if I miss the terms, you please tell me, plus v4 i4 plus v5 i5 plus v6 i6 plus v7 i7. This we have learnt in any network it has to happen. It has to happen at all times even if there are time varying.

So volt-ampere supplied, power supplied instantaneous must be equal to some of all the powers of other elements. Now this equation also I could write it like this, that v1 minus i1, I will bring this term to write plus v2 i2 plus v4 i4 plus v5 i5 plus v6 i6 plus v7 i7. In that case, I will tell that I have written all the elements as if they are absorbing power. Total power absorbed by this network v1, how much power it is absorbing; v1 and minus i1 is entering through plus.

So v1 minus i1 is the power absorbed by this source, v2 i2 and why negative sign, because we know to absorb power through the plus current must enter. But since I have assigned current along the leaving the positive terminal, therefore, I have every right to say that minus i1 is going towards positive and calculate this and then you tell that power absorbed by v1, which will come out to be negative. That means, it is really delivering power.

Power absorbed by v1 and these are of course, all power absorbed. So this way I can write. Therefore, henceforth what I will do? In networks, I will assign the currents and voltages and always calculate power absorbed by different elements. I am sure that some of the terms will become negative. Let it will become, but mathematically that is also correct. So remember that in any network, this is one of the conventions, better follow.

Instead of separately keeping track of power absorbed, power delivered, always try to make a habit, okay how much power is absorbed by each of the elements. In that sense, I will write it and that sum has to become equal to 0 from this I get. One could also say that I will always calculate power delivered by the network, so power absorbed, same equation could be written based on power delivered by each element with that concept, each element as if delivered powers.

Then I will write v1 i1 it is really delivering, but v2 i2, how much it is delivering, v2 minus i2 plus v2 into, then I should not write plus i2, because in language I am written delivering, v2 minus i2 plus v4 minus i4 plus v5 minus i5 plus v6 minus i6 plus v7 minus i7 and this will be 0. In fact, this is the same equation I have written. I have just changed the side of each terms to the right. So you will be writing the correct equations.

No matter, whether you are calculating it considering all the elements to be absorbing power or delivering power. So I will assume all the elements are absorbing power. Now, this is not Tellegen’s theorem, this is known, in a circuit and with numbers I can very neatly write down and clearly write down, which are the elements, which are actually absorbing power, which are the elements which are delivering power. We have discussed it in length.

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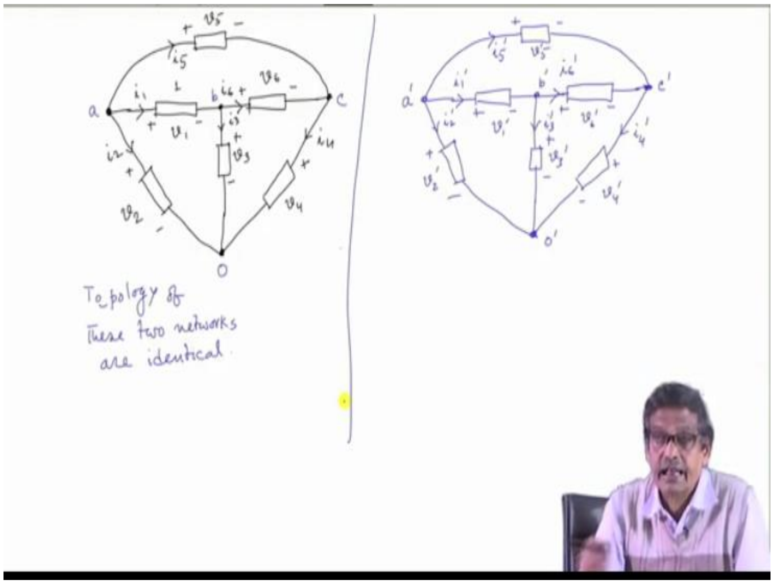

Now, what the Tellegen's theorem tells us that suppose, you have a network. Now let me be a bit methodical in telling the about the Tellegen's theorem. Suppose, consider a network like this and here also another element is there and there is also an element connected here, in general I am telling. So it is a network and these boxes are elements. Some of them could be sources as well, I do not mind, so source I am not drawing separately.

Now, what I will tell that, I will name the elements. For example, this element I will call element 1, voltage across it I will write v1 and i1, quite arbitrarily, this is my prerogative in solving network, you assume polarity and current in whichever fashion you like, but once you assign that write down correct KVL and KCL okay. So let this be element 1, let this be element 2, whose voltage at any time is assumed to be with this polarity and current is assumed to be i2.

Let this be element 3, v3, this is i3. Let this be element 4, voltage across it is v4 and current is suppose i4 and in this direction and then this is suppose element 5, v5 and current is flowing i5 and this is the voltage across the element with this polarity v6 and current is i6, I can do that, is it not. How many nodes are there in the circuit? 1, 2, 3, and 4 and let me name the nodes as a, b, c, and let this node be called o.

Say reference note with respect to that, I will calculate all the voltages. So, have you understood this thing? This is very important, a general network I have taken and in each branch there may be 2 series elements, I have combined them and I am calling one element is connected. One of these boxes must have been the source, otherwise how current will survive, okay. So this is a network and this is the topology of the network.

Now I draw a similar network; structurally similar network like this. I have drawn another network, now this network geometrically or topologically they are identical okay, same type, 3 loops, 4 nodes and so on, but only thing these elements could be totally different from these elements. Another network for example, here there was a resistance, here maybe there is an inductance, here there was a source; source is there.

So these 2 networks are totally different networks except for the fact that their topologies are identical. Topology of these 2 networks are identical, got the point and this network is seen separate. I will name their thing with dashed element, a dashed, this node is b dashed, this node is c dashed and this node I will call o dashed, like that I can write. Similarly the voltage and current assumed across each element in this network, I will follow the same rule as I have done here.

For example, I will say this is the voltage across the element, in this dashed network and I will say the voltage across it is v2 dashed and the current is i2 dashed. For this element, I will say the voltage across this network is v1 dashed and current is i1 dashed. This was plus minus v5. This is the voltage across this element current is i5 dashed. This is v5 dashed. This is also plus minus v6 dashed and the current in the same direction I will show, i6 dashed.

And this is plus minus v4 dashed and the current is i4 dashed and plus minus this is v3 dashed and the current is i3 dashed. So these 2 networks have otherwise totally different network. Elements may be different only they are structurally same, same topology and with different elements, but the voltage across each element, their polarities I have assumed in the same way as I have done here.

Obviously, the currents i1 dashed, i2 dashed, i3 dashed, i4 dashed will be decided by what are the sources present here, what are the impedances, they will decide the current, loop equations this, that you solve and that is supposed to be different from i1, i2, i3, i4, i5. Similar is the case with the voltage across each element that is v1, v2, v3, v4, v5. They are naturally expected to be different from v1 dashed, v2 dashed, v3 dashed, v4 dashed and so on, is it not. So Tellegen's theorem talks about 2 networks; 2 or more networks which have got same topology and we will continue with this in this next lecture. Thank you.