

Network Analysis
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Lecture # 32
Numerical Example

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Numerical Problem:

Circuit diagram showing a voltage source $\bar{V}_s = 220\angle 0^\circ$ V connected in series with an impedance $\bar{Z}_1 = 3 + j4 \Omega$. The circuit then splits into two parallel branches. The first branch contains an impedance $\bar{Z}_2 = 6 - j8 \Omega$ with current $\bar{I}_2$. The second branch contains a series combination of $\bar{Z}_3 = 8 + j6 \Omega$ and $\bar{Z}_4 = 14 - j2 \Omega$ with current $\bar{I}_3$. The total current from the source is $\bar{I}_1$.

Calculations:

$$\bar{Z}_{eq} = \bar{Z}_1 + \frac{\bar{Z}_2 \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3}$$

$$\bar{Z}_{eq} = (3 + j4) + \frac{(6 - j8)(8 + j6)}{14 - j2}$$

$$= 10 + j3 \Omega$$

$$\bar{I}_1 = \frac{220\angle 0^\circ}{10 + j3} = 21.07\angle -16.69^\circ \text{ A}$$

$$\bar{I}_2 = \bar{I}_1 \times \frac{\bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3} = 21.07\angle -16.69^\circ \times \frac{(8 + j6)}{14 - j2}$$

$$= 13.116 + j7.06 = 14.89\angle 28.31^\circ \text{ A}$$

$$\bar{I}_3 = \bar{I}_1 - \bar{I}_2 = 21.07\angle -16.69^\circ - 14.89\angle 28.31^\circ = 14.89\angle -61.65^\circ \text{ A}$$

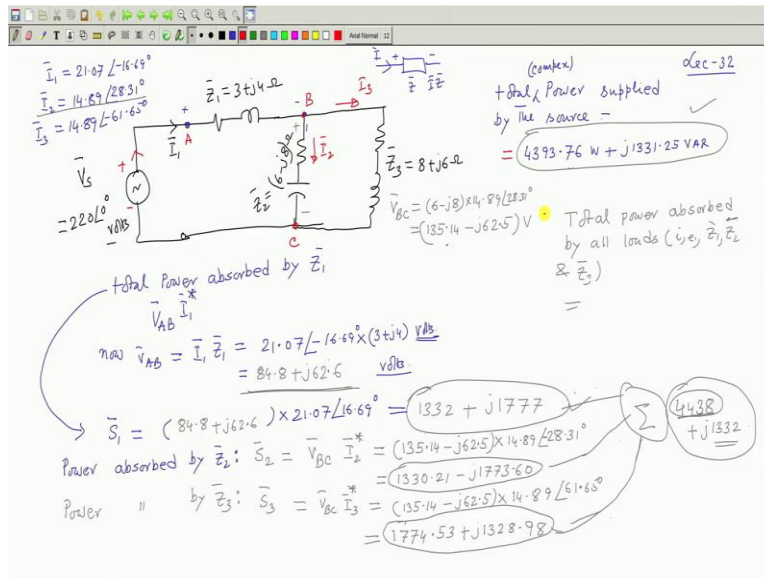
Complex Power Supplied by source:

$$\bar{P}_s = \bar{V}_s \times \bar{I}_1^* = 220\angle 0^\circ \times 21.07\angle 16.69^\circ$$

$$= 4393.76 \text{ W} + j1931.25 \text{ VAR}$$

So, we were doing some numerical problem started and we calculated up to this in the previous this was this circuit we calculate equivalent impedance just like resistance rule and calculated first calculate all the currents I_1 I_2 I_3 then we calculate it finally, what was the total power or complex power supplied by the source. I am talking about the power balance in AC circuit how to do it. So, these results I got. Now, what I will do is this I will calculate I will better copy this thing this circuit at least.

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So, this was the problem we are trying to solve. So, this is the thing and we have got this thing, power complex term I am not total power, whatever total power we have calculated up to this total power total means complex power supplied by source. Let me read at this result we already got supplied by the source = this number copy this is what we have got. So much watt so much volt now I will calculate what is the complex power. See this is 1 load 1 impedance z_1 .

So, I will write total power or complex power absorbed by the element z_1 , this is z_1 bar. So, these are each 2 terminals. And you see, current is entering through I_1 through A. And so if you calculate V bar AB into I_1 star, then in language I will tell this is the power absorbed by the source because through the + terminal of this voltage + - if you write it means I am talking about VAB, I told you many times. So I am writing nonetheless, very specifically, V bar AB into I_1 . But now the question is, what is V bar AB then I have to calculate. V bar A B can be easily calculated because I_1 , I_1 bar.

So in an impedance Like this, if this is z bar, this is the current then potential of this point with respect to this will be I bar into L so I will calculate so now V bar AB will be equal to I want do not take I_1 star here and that is that will be totally wrong I_1 into z_1 , this will be the thing and I won't already I have calculated in the previous page, which was 21.07 and angle is - 16.69 degree and z_1 value of z_1 is $3 + j4$.

So, much volt So, $\bar{V}_{AB}$ will be eventually coming as 135 in rectangular form I have calculated it can be $-j 62.5$ so much of volts this will be $\bar{V}_{BB}$ then at this calculation I carry on complex power absorbed by the load is a swan bar, I will say it is this voltage 135 this voltage is this $130.514 - j 62.5$ This is the voltage into I_1 star into I current absorb so I_1 star and I_1 was already known to be, let me write down I_1 bar was equal to we got it 21.07 into angle of -16.6 time

Let me write down all the currents we have already solved. I_2 bar I got it 14.89 and angle 28.31 degree and I_3 , we got it to be also 14.89 but angle is -61.65 this is what we got our last. So, here is I_1 star so it should be 21.07 and angle is 16.69 degree and if you calculate it the complex power absorbed by S_1 will be 1331 it was We will have a real and imaginary part $.83 + j 1775.64$ this will be the thing S_1 bar hopefully it is correct.

Similarly, I will calculate the power absorbed by get to and stressing upon this point am calculating power absorbed by S_2 . So S_2 once again this voltage should be known. So, let me write in longhand power absorbed by z_2 by z_2 calculating which will be equal to Say S_2 let us use a different color S_2 it will be $= \bar{V}_{BC}$ these voltage $\bar{V}_{BC}$ potential of B with a C, but to see and then I to start this is what I have to do. Now, the question is what is $V_1 V_2 \bar{V}_{BC}$ will be if you calculate, so, you calculate it here somewhere $\bar{V}_{BC}$ will be nothing but this current that is $6 - j$ into I_2 bar is 14.89 angle 28.31 degree.

And if you calculate at it comes out to be $135.14 - j 62.5$ volts this will be this thing $\bar{V}_{BC}$. So $\bar{V}_{BC}$ is this so I put that number $135.14 - j$ of 62.5 into I to star I_2 is this. So, I_2 star will be 14.89 into 28.31 degree, but I_2 is this so angle must be negative -20 and if you calculate it, will be equal to. So this will be 84 please correct that this real part calculation is it coming you tell me at $84.8 + j 62.6$ volts. So, this will be $\bar{V}_{AB}$ it looks like therefore, this I have to then correct it. We were a we so, I_1 has to do peaceful to calculate this one at $4.8 + j 62.6$ into this.

So, this will come then somewhat differently this how much it comes. So, I start into $V 1332 + j 1777$ this will be the thing, there are some calculation mistakes similarly power absorbed by J_2 will be divided V_c into this 1 and This calculation if you calculate as per my calculation it is

becoming $1330.21 - j 1773.60$ this will be the power absorbed by S_2 anyway will confirm this calculation.

Similarly power absorbed by z_3 will be V_{BC} same voltage V_{BC} into I_3 star now V_{BC} is already known that is 135 same voltage $135.14 - j 62.5$ into I_3 star I_3 is 14.89 and angle will be $+ 61.65^\circ$ So, this 1 will become $1774.53 + j 1328.98$ So, we have calculated the power supplied by the source and now putting some circle to them power absorbed by z_1 this 1 and power absorbed by S_1 is this 1 and power absorbed by z_3 .

Therefore, I will write total power absorbed by all the loads, total powers absorbed by all the loads all the loads, that is by, z_1 , z_2 and z_3 And there will be some of this + this 3 you add up the numbers how much it comes real part. Let me write whatever you are waiting 4438 reactive part 013289 then this will cancel out almost so how much - 1777 that is the 1332 this is the total power. So, real power it is coming close but there may be some little mistake I have made.

So, you see this some this number should be same as this number that is what I am telling total real power supplied by the sources 4393 it is likely I can use check once again this that calculation will confirm pretty active power it is matching So, calculate this V_{B1} . So, you see what we have done we have calculated the total power supplied by the source both the DL and reactive in 1 stroke voltage into a current start.

Similarly, the same steps I have repeated to calculate the power absorbed by z_1 z_2 z_3 and what I am telling is power supplied by the source must match the power delivered to the source or absorbed by the source and it will come same identical but anyway there is a slight mistake or whatever it is maybe 4.4 kilowatt, 4.3 kilowatt in terms of that we are getting, but do you just check 1 second your calculations it is close by but then maybe some bigger because of truncation I made some error but the point is clear in circuit analysis.

If it is a circuit, then supply voltage if you know all the impedances solve this I_t , get the currents then calculate individually if it is load how much power it is absorbing that you calculate and if it is source how much power it is telling many $P + A^2$ as the sum of all these things on the load

side and the source size must match. Therefore, in AC circuit source must be able to provide the reactive power. If you find the important point I want to make, for example, in this case previous simple problem that adults are the problem we have calculated.

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Apparent Power (S)
 VI

Active Power or real (P)
 $VI \cos \theta$

Reactive Power (Q)
 $VI \sin \theta$

Total complex power $\rightarrow S = P + jQ$

Circuit diagram: A voltage source $V = 200\angle 0^\circ$ V is connected to a load impedance $Z = 6 + j8 \Omega$. The current I flows from the source into the load.

Calculations:
 $\bar{V} = 200\angle 0^\circ$ V
 $\bar{I} = \frac{200\angle 0^\circ}{6 + j8} = 20\angle -53^\circ$ A
 $P = 200 \times 20 \cos(-53^\circ)$
 $Q = 200 \times 20 \sin(-53^\circ)$

Complex Power supplied by the source: $\bar{S} = \bar{V} \bar{I}^* = 200\angle 0^\circ \times 20\angle 53^\circ = 4000\angle 53^\circ = 2400 + j3200$ VA
 " " absorbed by the load: $\bar{S} = 2400 + j3200$ VA

Power triangle: $S = VI$, $P = VI \cos \theta$, $Q = VI \sin \theta$

Red box highlights: $\bar{I} = 20\angle -53^\circ$, $\bar{I}^* = 20\angle 53^\circ$, $200\angle 0^\circ \times 20\angle -53^\circ = 4000\angle -53^\circ = 2400 - j3200$ VAR

Power supplied by the source was $2400 + j3200$ p + j Q. So, active power supplied by the source was 2400 and reactive power supplied by the sources was 3200. Similarly, reactive power absorbed by the source has to be this 1. Now, what I am telling is the sign of the reactive power for lagging power factor load or inductive circuited is coming positive. If suppose the capacitor It was a capacitive sell it suppose $-j$.

Then you will see the reactive power are you getting me So, what I am telling in this note noted if Your 30s reactive only but $6 - j$ capacitance $-j2$ and if your supply voltage is same 200 euro degree this budget old then everything will remain same, only thing the currents will be now leading it will be this current will be leading 20 angle 53 degree is not current will be leading the supply voltage supply voltage is 230.

Now, if you calculate the power absorbed by the source absorbed by source by load it will be voltage 0 degree into current star $20 - 53$ legally if you calculate this, it will be real power will remain same that is 4000 real part, but this sine theta - sine theta so $-j3200$ this is so much part

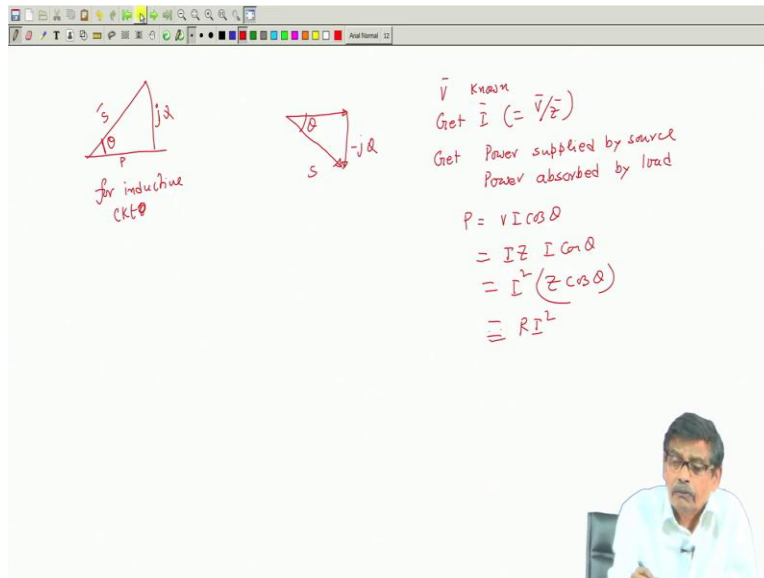
it will be and this will be so much cooler what that means the reactive power for capacity blowed absorbed by the load is - 3200 Had it been inactive would have said it is observing.

So, it is absorbing - 3200 what means it is delivered in 3200 bar to the system so inductive reactive power we just interpreted it is absorbing reactive power, but the same nature power and nothing doing that, but we make a distinction from this sign whatever it is, if I say is argued is observing $10 - j 50$ what and barred power I will come to know this is a capacitive circuit see if is a real part will be always positive.

Therefore, capacitor source can be considered to be generating reactive power. We could other way also say industry power generates, it does not matter, but we come from this side we will be able to come to know whether this circuit is capacity or inductive just looking at the sign of the reactive power got the point therefore, it is very easy to calculate the power balance of AC circuit also I will give you several problems.

But a kindly once again redo yourself this problem and if possible you correct the numbers here so that it is closely coming but it should come Equal if you do not truncate anything this that so that you can mail me and correct it, but the essential point is I have told you that in AC circuit we can calculate we can do also the power balance as we did in DC circuits, but here real and reactive power.

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And reactive power does not it is the oscillating power and if a circuit is inductive is reactive power will be positive that means absorbing power so, these volt ampere card that I have a triangle I have drawn, it can be drawn like this or inductive circuit this is p this is q Jq and this is this both real and inductive power will be positive load absorbs so much for capacitive circuit it will be like this P and it is reactive component will be - check you That is what I wanted s and this is the perfect angle of the load.

You just solved several problems and you will be able to get the idea you by this time you know all these things but calculations of the states will be then V is known get I using the relations supply voltage by z bar et cetera. Then get power supplied by the source and power means total power absorbed by load. This is what we did in this calculation once again you know the power is real power.

So, for real power is concerned it is like $VI \cos \theta$ I told you now, $V = IZ$ a totally magnitude of z and this is $I \cos \theta$. So, this is equal to $i^2 z \cos \theta$ but $z \cos \theta$ is R so, real part of the power absorbed by the load also can be calculated if you know a circuit is RL, but you know the current through it then you can be rest assured real power is $i^2 R$. R is current square magnitude into the value of the resistance got the point.

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Nodal & Mesh analysis can be adopted to solve network problems in the same way as in D.C. ckt.

$V = IR \equiv \bar{V} = \bar{I} \bar{Z}$

KVL in mesh 1 & 2

Solve for $\bar{I}_1$ & $\bar{I}_2$

Power delivered by sources ; $\bar{V}_1 \bar{I}_1 + \bar{V}_2 (-\bar{I}_2)$
 $= \bar{V}_1 \bar{I}_1 - \bar{V}_2 \bar{I}_2$

Power Absorbed by loads
 $\bar{V}_{Z1} \bar{I}_1 + \bar{V}_{Z2} (\bar{I}_1 - \bar{I}_2) + \bar{V}_{Z3} \bar{I}_2 = \bar{V}_1 \bar{I}_1 - \bar{V}_2 \bar{I}_2$

So, in case of AC circuit analysis we calculated current four simple circuit is series parallel circuits. You apply the same rules to calculate the currents once you know the current you can calculate the voltage across any of the impedances i into z and so on. Therefore, I will be now able to solve a rather complicated AC circuit once again by applying the nodal analysis as well as the mesh analysis. For example, you have a circuit like this nodal.

I will just indicate this nodal and Mesh Analysis can be adopted to solve network problems in the same way, in the same way as the same way as in DC circuit. Now, let me just give you one example. Suppose you have a circuit like this now you convert the impedances in terms of complex numbers there may be multiple sources present, I do not care. Suppose here is a voltage source whose phasor notation is V_1 , there is another supply whose phasor notation is V_2 polarities must be indicated.

And this is opposed z_1 bar this is z_2 bar, this is z_3 bar I want to get the currents. So, what I will do, I will now assume the current feathers as I_1 bar and why this can be done? Because we go to a year in this is not it here. Similarly, similar thing in essence there it is we were equal to it. So, no point in repeating to prove that it can be done because V and z follows this relationship. So, I will right by inspection the KVL equations in loop 1 and loop 2 what it will be KVL in mesh 1 and 2 will be like this coefficient of I_1 will be simply $z_1 + z_2$ into coefficient of I_2 will be this.

And this should be equal to be 1 by the sources, polarity of Plassey such as 1 same room. Similarly, the second equation will be coefficient of I_2 will be $j^2 + 3$ into it and coefficient of I_1 will be $-j^2$ into and this is + and this should be equal to the source voltage, but here it should be $-p^2$ then you have got 2 equations to solve it all is the complexity arises because these numbers you have to handle in complex numbers.

Then apply Kramer's rule is that whatever it is convenient to you get solved for I_1 I_2 (FL: 33:51) in this example tell me there are lead each elements Are there what is the voltage drop here I_1 what is the voltage drop here if you write this is + this is -, then you should write $I_1 - I_2$ phasors into debt to for this simple idea into geometry and polarity of the voltage it is known. Now here let us see what should a light power knowing the direction of the currents and polarity of the voltage Ohm.

I say that power delivered by the sources by sources this 1 really delivers. $V_1 I_1$ star this is what I will right. Now look at this source it is $V_2 I_2$ start but current is entering through +. So I should write powered absorbed by the source is V to it. But anyway I will then $-I_2$ is going away from class for the source, So, I can write it like this, we truly take care of this thing, but the point similarly, I will say net power delivered by the source will be given a monster and - it will be - 30 is wanting to leave. So, it will remain I_2 this sign this fellow delivers and this fellow absorbs - similarly, power absorbed by the source all the loads power absorbed.

This language you decide what you will right absorbed by the loads will be z_1 will be voltage equals z_1 , this voltage we had 1 and I will say I_1 star for $z_1 z_2$ what is the suppose and I am adding them now, we voltage across z_2 with this + this - this 1 I am lighting power absorbed. So, I will say it is $I_1 - I_2$ conjugate up these and powered absorbed by z_3 is we z_3 voltage across z_3 with this + this - into I_2 star and what I am telling is this must be equal to this $V_1 I_1$ start - $V_2 I_2$ star with numbers it will be much more easier to write you can easily write this.

So, total power delivered to the system or the circuit is this 1 and they are being observed by different loads. Now, whether actually a load is absorbing or delivering power that depends upon the side S the power deal for sign will be always + listeners, but the active part inductive, E will

E observes powers it means it is there is some inductance present it is delivering power a load reactive power, which will become - because the way I have calculated observed So, it becomes - means cities popping the active part to the system. Anyway, we will continue with this in the next class with nodal analysis. I will point out thank you.