Network Analysis
Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 17
Linearity of R, L, C - Inductor with Initial Current and Capacitor with Initial Voltage

So we were discussing about linear elements which elements are linear. Obviously resistance is a linear element V= RI and when input voltage is 0 current will be 0 always no problem.

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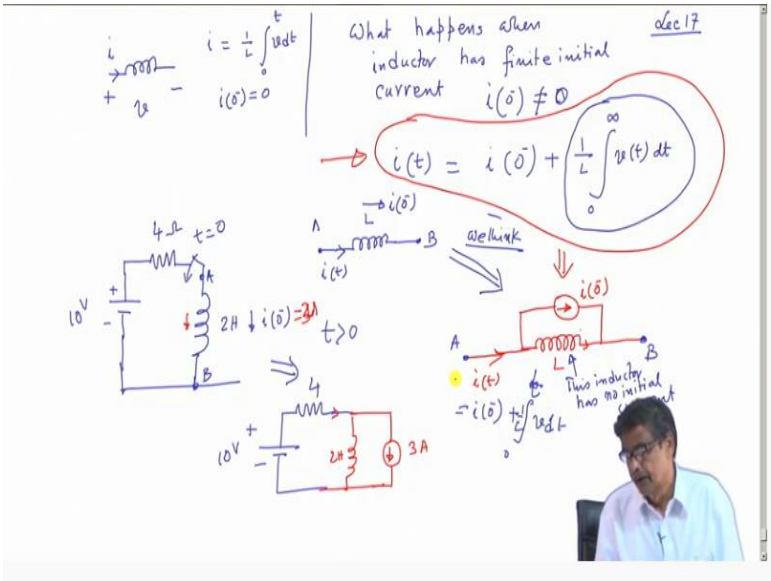

But in case of an inductor it is not going to be if current is your response you are applying some voltage v then we have seen i=1/L Vdt. If your game starts at t=0 it will be 0 to t at any time and the only thing that inductor will be linear. That is what I showed in my last class provided there is i0- must be 0 then only inductor will be like an linear element that is super position will hold good if you apply V1 voltage if the current is i1 then if you apply V2 voltage current will be i2.

Then I am saying if you apply V1 + V2 then current will be i1+i2 it is a very powerful tool in circuit analysis. That is why linearity is always preferred. Now the question is there will be situations as we have seen inductor current, initial current may not be 0 you have connected it to some other circuit inductor got some initial current i0- then you are connecting it to another circuit and current through the inductor will start from i0-.

Therefore, at that time super position theorem application will become definitely a trouble something and super position is such a powerful tool. Now what happens when inductor has a finite initial current that is there is some definite i0- is present non-zero i0- is non-zero and by this time you must have understood what do I mean by this? This inductor was connected in some others circuit.

At some time you are connecting this inductor to some other circuit and subsequently want to know what will be the current then current variation in the inductors should start from i0- and I am telling if the inductor has got initial current you it cannot be treated as a super position it cannot be treated as a linear element. Hence position cannot be applied things like that I am telling is there a way out to avoid because that will be the situation in most of the cases.

So what happens when inductor has finite initial current i0- then the current at any time t in an inductor we have seen it will be equal to i0- +1/L 0 to infinity Vdt this several times I wrote like this where v is a reasonable voltage and not an impulse. This is the situation so initial current plus this extra current in the inductor. Okay and this expression is the current run inductor which has got no initial current.

Therefore this point you please be with me and try to understand what I am telling a crucial point an inductor and it is current at any time t is it and it has got some initial current i0- and these two points are suppose A and B terminal of this inductor with initial current. Now from this equation I can say this inductor L without no initial current and this is it. And here is a current source i0- and these two points are your A and B got the point.

That is an inductor with an initial current i0- current at any time t you see i0- plus this term. But this term is the current in an inductor without any initial current. Therefore, what I what we are doing is this inductor this red coloured coil L having same value L this part, this inductor has no initial current. Therefore an inductor with initial current i0- can be thought of as the same inductance cell without any initial current.

But across it another source is connected whose values is i0- you see these it is indeed a i0-+ this term 0 to infinity 1/L 0 to t 1/L Vdt. So that is why it is connected in parallel got the point. This is a crucial point an inductor if it has got no initial current fine, this inductor will remain inductor AB but an inductor having an initial current i0- you can think that we think without any mistake you can replace this arrangement with an inductor same value without no initial current need and that initial current you treat it as a current source across it and everything will be fine.

Now why people do like that? People use this concept replacing a charged inductor in this fashion simply because that an inductor if you treat it with i0- show in this circuit. This L is not a linear element and you will struggle while applying this super position theorem. In fact, you will not be able to apply super position theorem. Therefore, translate these to an inductor having no initial current therefore super position theorem can be applied to this element.

And this is just another source in superposition theorem when many sources are present, consider one source at a time and find out the current in this circuit and get the solution. We will take up several problems but this concept please try to understand appreciate what is done from here to there. Therefore, in a circuit when such an inductor is present with initial current I will simply say for example let me draw a circuit so that everything is in place.

Suppose you have a circuit like this. I say it is a 10 volt battery here is a plus minus 10 volt source. And I say that here is some resistance 4 ohm and I say that I will connect an inductor L which is suppose 2 Henry but it has got some initial current. I will indicate it by this i0- got some other circuit and connect it to this circuit at t=0. Okay I can write down the differential equation and initial current is 0 that is one way of solving.

Now I am telling from the point of view of super position theorem I want to do there are now. So this circuit call this as A and B you say that circuit for t > 0 will be like this 4 ohm your battery is there 10 volt and I will draw it with the red colour to indicate that it is an inductor 2 Henry with no initial current and this initial current is here show it by say suppose i0- Ampere is 1 Ampere say 3 Ampere.

So I will connect it like this so in this circuit there are two sources one is the voltage source, current source and both these elements are linear. This 2 Henry and 4 Henry therefore I will consider one source at a time and find out the currents in the circuit got the point. So this example you try first then I will solve it. But only thing is we are looking for current in this inductor after you find current in this inductor 2 Henry who each has got no initial current, do not forget to add this + 3.

Because actual circuit this inductor current is this current in the 4 ohm resistance and that current is here. You should be careful that is all. Otherwise you can apply super position theorem as well it is not very interesting that is okay. One way of handling this situation I know I will write down the differential equation, solve it initial condition and i0- = 3 Ampere one way of solving but keeping in mind a principle of superposition will be so often used in later stage of this course.

Then we ask ourselves we are investigating whether an inductor really is a linear element or not. Then only you can apply super position theorem. Then we discovered that okay inductor indeed will be linear provided there is no initial current in it because principle of homogeneity must be satisfied. If applied voltage = 0 current must be 0. So i0- = 0 then there is no problem fine. But as we have seen earlier an inductor is connected to this circuit for some time, then it is switched over to another circuit.

Then building off of current in the inductor when it is connected to this new circuit will start from this i0- current is continuous in an inductor and this will spoil the game. So far as principle of a super position is considered. What is the way out? Way out is this equation. This equation tells you so many things okay it = i0-1 by this one in an inductor with initial current which is non-zero. Suppose AB is the terminal of the inductor then what I am telling, this is the actual thing. You can treat it as an inductor same value L without any initial current.

And in parallel there is a current source i0-. But in your actual circuit the inductor current is it. So this current in this circuit there will be some current when you solve it, this current is not the current you are looking for. Do not forget to add this current with i0- to get it. I hope you have got the point so this is the thing. So an inductor with initial current can be modelled between two

terminals A and B as the inductor without an initial current and across it there is a current source. Okay hope you have understood.

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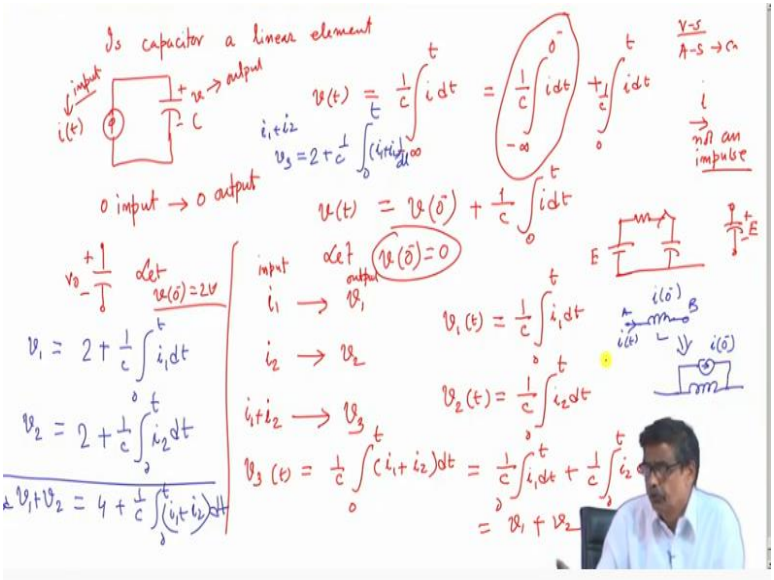

Similar is the case with capacitor is capacitor a linear element. Okay this is your c and here you are connecting some say current source it and we know that the voltage across the plate of the capacitor Vt is the output v is output. This current which you are injecting feeding to the capacitor plate is the input. Suppose we think in that way this is my input, so voltage across the plate of the capacitor, we have seen it is equal to - infinity at any time t = idt is it not.

By the by remember that to build up voltage across the plate of the capacitor you have to spend coulomb ampere second. Recall that in inductor to establish a current we had to spend volts second. Similarly to build up voltage across the plate of the capacitor you have to spend ampere second ampere into second is nothing but coulomb that is you are dumping it has a physical significance. Similarly volt second can be shown to be equal to the flux linkage. Those things we will see but idt is the one you have to spend to establish voltage across the capacitor plate.

Anyway, this is the thing now this one =1/c into - infinity to 0-idt and if i is reasonable, this will be 0- or 0+ whatever you write it does not matter t idt. These are the two parts i not a delta impulse not an impulse for this only this is valid, not an impulse signal. Forget about this so general functions it so this will be the thing.

Now this thing voltage across the plate of the capacitor at any time t this quantity is your V0- at t=0 you are doing some switching and this one is a voltage at any time t is this current, this one and then 1/c idt 0 to t. Now what is the what we demand for this element to be linear. What is the input here it when current is 0 no current is applied I will demand there should be no output 0 input 0 output?

This is the first thing 0 input must cause 0 output 0, 0 ampere input must cause 0 output. This is the condition of homogeneity for super position theorem. If this is not true, then it is a not going to behave like a linear element. That is what I am trying to tell. Okay? But we know that a capacitor, if you charge it from some other circuit and leave it like this with some initial voltage V0, what is the current now applied to the capacitor? 0. But it will be able to retain this voltage. It has been charged from some circuit and disconnected.

For example R this capacitor like that here is a voltage you charge it. Then after it is fully charged open it this capacitor, I mean in ideal condition it will retain this voltage. If this voltage is E it will have charge to this voltage and it will retain this voltage forever. I mean which is theoretically not possible through the atmosphere there will be leakage capacitor will slowly discharge that is one thing, but in ideal conditions it will retain that voltage.

Similarly, a capacitor now no current is injected to it and it has got V0 that means V01. Let us see let V01 initial voltage across the plate of the capacitor is 0. Is it then linear let us see? I will give i1 current and see what is the voltage V1. So I will write V1t with V01 = 0 will be simply 1/c 0 to t i1dt fine. Input i1 voltage output V1 and they are related by this. Then what I will do, I will give another input i2 with still maintaining V0-=0 initial voltage across the capacitor is 0 and I note down what is the voltage output.

So V2t will be equal to 1/c 0 to t i2dt this will be the thing i1, i2 at the input it is not that this will be the thing. Suppose now you exceed the capacitor we take current source which is i1+i2 and suppose it gives rise to a voltage V3. I am not sure whether it will be V1+V2. Suppose it gives

rise to this v3 when the input is i1+i2 what will be the voltage developed across the plates of the capacitor.

Then that I will write V3t = 1/c 0 to t at any time t and this will be i1+i2 dt and this can be broken up into these two parts 1/c 0 to t i1dt + 1/c 0 to t i2dt is it not. But this one is nothing but V1 and this is nothing but V2. Therefore a capacitor with V0- = 0 must be a like a linear element and super position is holding good. Nothing is better than that. However, if there is V0-1 you can easily show a super position theorem will not hold good is it not?

Suppose let although I have not done in case of inductor, suppose V0 - = 2 volt then let me use separate colour. Then I will say when you apply V1 what will be the output voltage? This, 2 volt + 1/c 0 to t i1dt. You apply i1 get this V1 is it not. Suppose I say V0 -1 is not 0 which is 2 volt. And then you are applying V1 voltage 2 +1/c and so on.

Similarly when you apply separately V2 voltage, what will be the output to the with any same initial voltage to volt it will be 2 + 1/c 0 to t idt i2dt is it not? What will be the response of it that is V1 + V2 if you write these two equations it becomes then 4 + 1/c 0 to t i1+i2 dt. If you just add these two equations I have not applied separately V1+V2 from these two equations. I am telling you this is going to be this.

Now indeed when you apply V1 + V2 to this circuit. Then the voltage when you apply V1 + V2. Let the voltage in this circuit be V3 oh sorry. When you were apply i1 + i2 are you following me? I1 + i2 you apply I do not know I am not assuming anything I am telling that okay let the output voltage be v3.Then v3 voltage should be equal to the same condition 2+1/c +i1+i2 dt. This will be the thing 0 to t and which is not same as V1+V2 it is 4.

So this term will spoil the linearity property of a capacitor. If it has got an non-zero initial voltage got the point. Therefore, what is the way out as I am telling you principle of super position is a very powerful tool I would like to have in analysing circuits involving R, L ,C. R is a linear element inductor with no initial current is a linear element.

Capacitor also will be like a linear element without any initial voltage fine, but the fact remains that there may be situations when this capacitor and inductor will have their initial currents voltages and currents. In that case is there a way out? I will exploit the principle of super position while analysing this circuit. I would like to do that and at the same time I know that okay, initial conditions then it is not linear looks like that in case of inductor resolve that problem.

We told that okay if an inductor has got an initial current i0- L and this is the current at any time t across these terminals A and B this is you treat it as an inductor without any initial current, but do not forget to connect a current source. Another source will be introduced into this circuit. So super position you apply this inductor no current. Therefore in case of capacitor whether similar things can be done. Yes it can be done.

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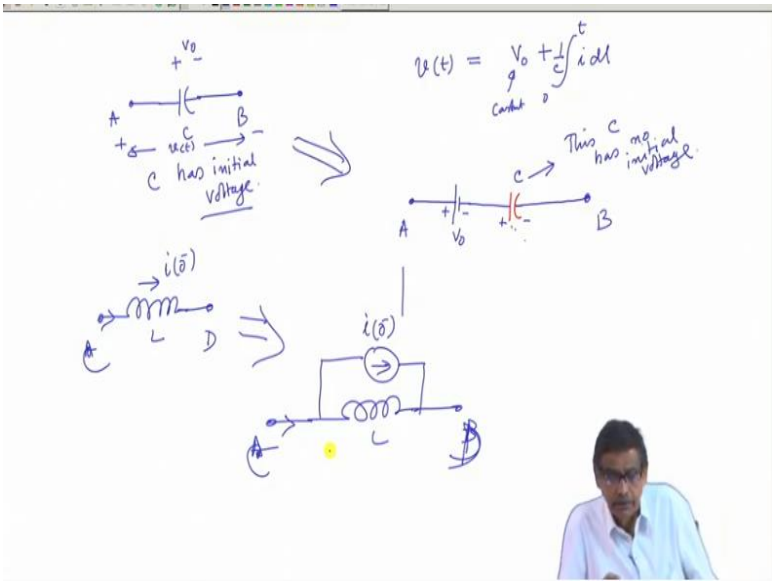

Because of the fact look at a capacitor c and let it has got an initial voltage V0 with this polarity and let this point be A and B. So this c has initial voltage and voltage across the plate of the capacitor at any time t will be V0 + 0 to t idt/c q/c this is the equation and this is the voltage applied across the capacitor Vt. So this Vt is a constant number this is constant a number constant and I soon discovered that, okay this Vt is nothing but a constant voltage V0.

Therefore, between A and B there is, I can consider as if there is a battery present V0 and plus a capacitor c which has got no initial voltage? This c has no initial voltage then it is very nice. So

only thing you have to alter this circuit bringing in another source. Okay in this circuit, there will be sources present. Okay another source comes but to solve this circuit, perhaps I will apply super position then for V0 also I will solve this circuit.

But only thing to be remembered this voltage which is which has got no uncharged voltage is not this Vt. Ultimately in this circuit VAB is the terminal of the capacitor. So while calculating the voltage across the capacitor, you solve what these voltage then do not forget to add this V0 get the actual capacitor voltage. What are they? So this is this c is our thought process told me that okay a charge capacitor can be thought of a capacitor. Why in series? Because voltage, some of two voltages. That is why it is in series in case of inductor the current was sum of 2 currents.

That is why it was parallel I hope you have got the idea and we will use this substitution that is a charged capacitor can be substituted by an uncharged capacitor with the V0. What is the motive for doing that? Because a uncharged capacitor will behave like a linear element. Similarly an uncharged inductor will be so to summarize this so this is the thing for capacitor and for an inductor L with initial current i0- and these two terminals are suppose CD.

This is equivalent to an inductor with no initial current and in parallel. This initial current is shown to be as and source and you can solve this is your AB, mind you this current is this current and not AB CD. I hope you have got the point. Anyway we will tell more about these things when we will be trying to solve circuits with initial conditions. One way of solving this forgettable disease, you do not know how to change these to these or that to that; write down differential equation apply boundary condition, get the solution. It has nothing to do with this transformation. But now we will look ahead and we will try to apply several network theorems.

Formally, I have not told you anything about network theorems, like super vision theorem but this is the time I slowly start telling about those things. I hope you have understood this point. This lecture is very important go through it. Thank you.