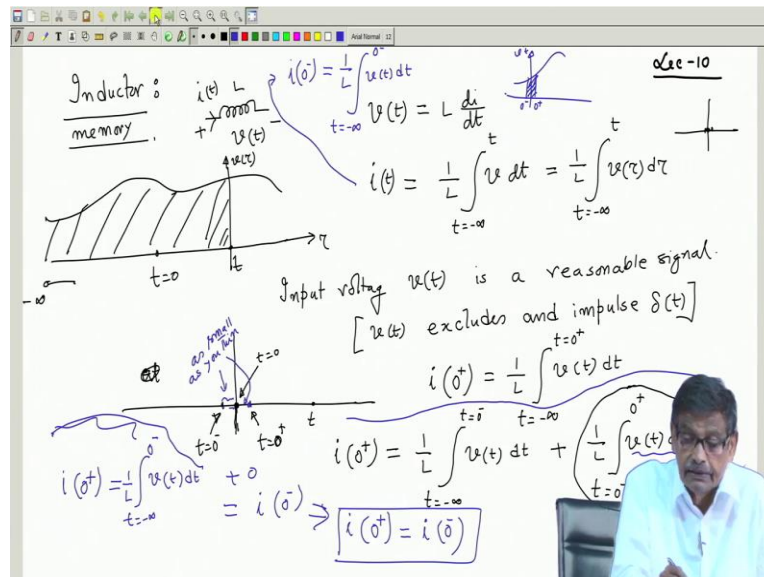


Network Analysis
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Lecture # 10
Initial Condition for Inductor

Welcome to lecture number 10 and we have started discussing about inductor its terminal relations between voltage and current.

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And I told you that if there is an inductor ideal inductor having no resistance, if you show the voltage applied across see to be $v(t)$, this means potential of this point with respect to this is to be $v(t)$. And if you show this current to be i , which is expected to be a function of time also and then the relationship between voltage and current to will be $L di/dt$ you should be consistent in noting the polarities of the voltage as well as the direction of the current.

Now, from this we showed that the current i will be nothing but one over L and integral the $v dt$. Now, what should be the limits of the integration that is the current at the present time t depends upon all the voltages that is t is equal to minus infinity to t . Sometimes this creates confusion equal to here also dt . So, this integration this definite integral will not change if you write give it

some other variable v and dt and w is equal to minus infinity to t that is, this is one and the same thing, because it is after all a definite integral.

So, the change of this variable, you can do anything, so I have need to be t inside and I am interested to know the current at time t . So, this will be the thing. Therefore, we note that this inductor present current to integrate this he will required the value of v that is v_t for all the time. If this is your time axis here is your $t = 0$ this we discussed but I am just telling that all the information and this is supposed to be your t and this is your time axes.

So, you require all the values of the voltages till time t this $v dt$ this is v this voltage if you integrate from these it means this area under this curve that is what I told last time and this you have to take till $t =$ time t . So, this is $t =$ infinity - infinity. Therefore, inductor unlike resistance the current present value of the current in the resistor is dependent solely on the present value of the voltage no previous voltages are required but in case of inductor present value of the current depends upon the past history of the voltage that we applied across it.

So, this is the thing so, therefore inductor is said to be a device or an element with memory. It remembers how you treated the inductor previously with what kind of voltage that it makes sense. Anyway so, this is the thing so, inductor is a device with memory. Now, this input voltage across the inductor, this v_t is the applied voltage current is the consequence current and these are the relationship in differential form and this is the relationship in integral part.

Now, naturally the question is not it to find out the current at time t you require all the previous voltages, things like that we will assume that this input voltage v_t is a reasonable signal means it is not an impulse function or any function it may be exponential it may be triangular wave it may be a stable form it may be sinusoidal what not those are all reasonable signals.

Input voltage is a reasonable signal that is a v_t I will exclude v_t for the time being v_t excludes and impulse signal there input signal could be also impulse signal which is generally denoted by δt except for that we are discussing now at present, we will see the implication of this

impulse excitation of a circuit later in great detail that will do that, but for the time being, we will assume it is not an impulse exclude impulse.

Now, if that be the case then you are i t is like this (FL: 06:37) now, suppose an inductor across an inductor at $t = 0$ let us try to understand $t = 0$ business. This is $t = 0$ got it now, at $t = 0$, I am going to apply some voltage across the inductor at $t = 0$ and that voltage is known to me whatever voltage I will apply now, as I told you and I will be interested to know the current at time anytime T that is what my goal is, but as I told you, the current at time t will depend upon all the voltage integrated up to time t including the non-voltage a and for t less than zero.

I am not sure what voltage I applied things like that, unless it is known, it cannot be done, but I will tell you how this problem can be tackled. See, for example, the value of the current suppose I take I am showing in a larger scale suppose you take a time here this is a this vertical line is it called it this is this point is crucial this is $t = 0$. Now you take 2 times here 1 left $t = 0$ which is lesser than $t = 0$ by a very small amount maybe 10 to through -32 whatever way you think so, that time I will tell is that t equal to 0- that is very close to 0 but left to $t = 0$ this point is $t = 0$.

Similarly, on the right hand side immediately after $t = 0$ I will denote the time by this notation t equal to zero plus means it is almost zero, but on the right hand side, Now, why it is necessary and this option in a larger scale it is like these what I am trying to tell this is $t = 0$ and $t - 0$ is here t_0 plus is just here immediately after that. So, zero minus zero plus indicates those 2 points (FL: 09:33) Now, you see, suppose, I want to find out current through the inductor at t equal to zero plus this I want to find out what should I do? I should then right they should be equal to $1/L$.

$t = -\infty$, this upper limit becomes zero plus $t = 0+$ is that and v_t this is what have to do to find out the current at $t = 0+$ plus at $t = 0$, something happens to this I have applied voltage some changes occurred. So this must be true. Now this integration can be this step is very crucial. i_0+ will be equal to $1/L$ disintegration I will break it up into 2 parts. t is equal to minus infinity to $t = 0$ minus $v dt$ v_t , there is v_t here.

You are doing integration this one $\int_{-\infty}^{0-} \frac{1}{L} \int_{-\infty}^t v dt dt$ is not. So the same thing is now broken up into 2 pieces, 1 is from $-\infty$ to $0-$ integrator then from $0-$ to $0+$ to get the current at $i(0+)$ now, as I told you, our voltage signal is a reasonable signal, not an impulse, what impulse is we will discuss later. Therefore, what will be the result of this integration let us ask yourself what will be the result of this integration.

The result of this integration has to be 0. Why, because of the fact suppose, dt like this, then I am trying to integrate this is supposed to be dt in the interval $0-$ to $0+$ and these are as small as possible in the calculus sense. Therefore, any on that cart will be practically 0 because you have gone a little here a little decide so, area enclosed by this is nothing but the area enclosed between $0+$ to $0-$.

That area will upload 0 because of the fact in $0+$ is only you have crossed 0 immediately after that, if you say I have crossed by an amount 10^{-32} . I will say no it is 10^{-64} that is the sense of but nonetheless it is some positive number by which you have crossed to zero. Similarly, this is as small as possible these interval as small as possible as you think.

But as this distinction it is not equal to 0, that is what I am trying to kill similarly, this fellow, this is also as small as possible. This fellow, this interval so, $0+$ is a number which is practically 0 for all practical proper purposes, but it is it is to the right of the equal to 0. There were these 2 points collapses. Therefore, you will get a very theme sheet. They are worried under this card has to be 0 for all practical purposes, if that be the case then so, this is the thing.

So, if that be the case, then let us see what is it $i(0+)$ will be then equal to this whole integral is equal to 0 this part it will be $\frac{1}{L} \int_{-\infty}^{0-} v dt$ because and then $+0$, this term is 0. But what is this thing by definition, $\frac{1}{L} \int_{-\infty}^{0-} v dt$ is equal to minus infinity to $0-$, it is nothing $i(0-)$ because I have written $i(0+)$ like this is the general time t , how much will be the current so, I am rewriting using this relation from this I get this $i(0-)$ of course, you have understood but I am still writing what you want to know current at equal to $0-$, then integrate the voltage applied across the inductor from t is equal to infinity to t that is $0-$.

So, this integral is nothing but the current at 0- therefore, this is the crucial thing we know that i_{0+} plus has to be equal to i_{0-} got the point. Therefore from this we conclude that an inductor excited by signals which is not an impulse signal the current existing just prior to your switching in this arcade that is i_{0-} whatever it was. And we were doing something here. Then at $t = 0+$ immediately at $t = 0+$ it is equal to these 2 currents possibly same that is current through an inductor. We will continuous it cannot have a sudden jump is that point clear? That is 1 point, and 2 point is see our that previous history.

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$$i(0^-) = \frac{1}{L} \int_{t=-\infty}^{0^-} v(t) dt$$

$$t > 0$$

$$i(t) = \frac{1}{L} \int_{t=-\infty}^t v(t) dt = \frac{1}{L} \int_{t=-\infty}^{0^-} v(t) dt + \frac{1}{L} \int_{t=0^-}^t v(t) dt$$

$$i(t) = i(0^-) + \frac{1}{L} \int_{t=0^+}^t v(t) dt$$

$$i(0^+) = i(0^-)$$

$t=0^- \text{ or } 0 \text{ or } 0^+ \equiv 0$

This 1 I will write it in a special way that i_{0-} what we have seen it is equal to $1/L \int_{-\infty}^{0-} v(t) dt$ this is the thing. Therefore, to solve this arcade from $t = 0$ onwards as I told you one may demand that okay I must then know what are the voltages you have applied before $t = 0-$ that is one thing, but see this previous history all this previous history is nothing but to say that inducted when you are doing some switching had an initial current i_{0-} it captures the previous history this current in the inductor at equal to 0- is this whole thing.

Therefore, our anxiety that I must know that is the 1 thing if all the voltages are given I will be able to solve it that is you have to integrate. But another way of telling this is this current i_{0-} captures the previous history of the inductor that is how you treated the inductor will be written

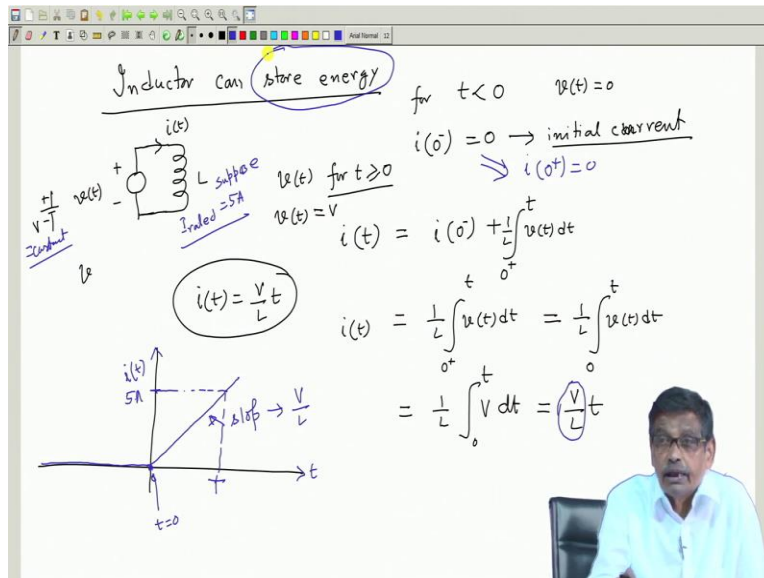
in this 0^- itself. In other words, what I will then write that current through the inductor at any time t will be an, t is greater than 0 suppose at equal to 0.

I am doing something, it will be equal to this way, then I will read $1 - i$, t is equal to minus infinity to let me write that 1 also, t v dt , this is what I have to do this I will now break up into 3 zones, 3 pieces $1/L$, t is equal to minus infinity to 0^- v dt + $1/L$ integration t is equal to minus infinity to 0^+ t equal to 0^- liabilities. So, 0^- to 0^+ v dt , this I will do and then + $1/L$, $t = 0 + 2t$. This is what it looks like.

Then I will say current at time t , i t will be this part is nothing but current that i_{0^-} and this part, as I told you, v dt is not an impulse this will be equal to 0, plus this third part only have to integrate where do you know the book what kind of voltage you are applying, you play with that $t = 0 + 2t$. So this is the story. How inductor current at any time t can be found out if you are done some switching at $t = 0$. Now, for us, those who are using the circle for them, $t = 0^-$ or 0, or 0^+ , this is as good as 0 is not what is the difference? We cannot make any difference, but to your inductor we will make the difference.

So, that is why we could say that this is the thing and what is i_{0^-} this 1. And with this condition that i_{0^+} will be equal to i_{0^-} this is has to be inductor current has to be continuous. I think you have got the point will solve some small problems to emphasize this point further, but this is absolutely essential that you must be very clear about what you were doing now, this inductor.

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So, sometime suppose I say that an inductor. Now, inductor is an energy storing element inductor this very important inductor can store energy stored and now suppose you have an inductor L inductor and here 40 less than 0 no voltage was applied try to understand all the points I am telling for $t < 0$. The voltage applied across the inductor was $v_L = 0$ no voltage was applied and at $t = 0$ we applied voltage v_L and v_L could be any reasonable function that is all. So, game starts here from $t = 0$.

Now, the question is, I must say that inductor current was 0, $t = 0^-$ was 0. Because you did not apply any voltage if the inductor was kept open circuited anyway that current past be 0 before $t < 0$ and I say that $i(0^-)$ is 0. So, that is there now I will implace a voltage v_L appears for t less than or equal to 0. I have applied a voltage across the inductor which is like this and I want to solve for the current.

So, I will say current through the inductor at any time t or t greater than equal to 0 will be equal to $i(0^-)$ plus 0- to 0+ that integration will be 0, then 0+ $v_L dt$ by $1/L$ this will be the applied voltage across the inductor. And since $i(0^-)$ is 0, at any time t since $i(0^+) = 0^-$ is 0. So, this I will write it as this 1 and which is for all practical purposes. So, for our time thing is concerned it is equal to 0 to $v_L dt$ this will be the case.

For example, somebody says that v_t for t greater than 0 is the battery constant voltage, this v_t is battery v with this polarity you have applied, this is your $i(t)$ mind do so, suppose if you applied a constant voltage v they will say the current through the inductor, which had no initial current these are this is called initial current so that so, here is the practical example, previously you did something and whether you did not something $i(0^-)$ was 0 it is given to me.

Therefore, current at any time t is $i(0^-) + 1/L \int_0^t v dt$ etc and then I say that this v_t is constant and then what will happen $1/L \int_0^t 0 dt$ $V \int_0^t dt$ I will right. So, this will be equal to $V/L \int_0^t dt$. So, current through the inductor if you apply a constant voltage will be equal to $V/L \int_0^t dt$ this will be the solution and it is always better you skate the current. So, the current skating will be 40 less than 0 you did not apply any voltage v_t was 0.

Therefore, your current was 0 this is $t = 0$. So, what was the current at $t = 0^-$ it was 0 what will be the current at $t = 0^+$ once again 0 it cannot start abruptly from some other value, which also which means this $i(0^-)$ is 0 means I will immediately conclude $i(0^+)$ is 0. So, it has to start here for me, it does not make any difference 0^- 0 and 0 this is equal $t = 0$. And then I notice this being constant $y = mx$.

So, the current will increase linearly, this is time and this is the current through the inductor it will evolve and increasing linearly. So, apply a constant voltage across a pure inductor current will rise linearly this slope of this slow of this cart depends upon the magnitude of the voltage constant voltage by L . This is constant battery we have connected to we were simple example. Of course, 1 you can easily see, I should not apply a constant voltage across an inductor for long.

If you do what happens as time passes current limitlessly go on increasing after all, you have made the inductor with a wire which has got a definite cross section, it has got a definite current carrying capacity. Therefore, if you just keep a battery connected across a pure inductor after some time you will see smoke will come out from the sky because your coil cannot bear this current you are not there will be fire.

Each wire with which you have made the inductor has got a current carrying capacity. Suppose, 5 ampere is the current carrying capacity of the inductor. Therefore, I will say in that case suppose I rated up this coil is equal to 5 ampere suppose and here is this 5 ampere. Then I will say if you apply a constant battery voltage across an inductor and this is the time axis you can apply safely that voltage up to a definite time 5 ampere you know V/L you will be able to calculate what is the value of t up to this capital zero to t if you connect the constant voltage, current will rise from 0 it will reach 5 ampere.

Then integrity productive action no I should not allow this voltage to act upon the inductor further because it will then go beyond its rated current and your inductor will be spoiled, it is dangerous. So, constant voltage can be applied across and inducted, but for how long that you decide, depending upon the current rating of the inductor anyway will address this problem. Now, as I told you, I will concentrate upon energy storage in an inductor unlike resistor whatever is the power $I^2 R$ or voltage across resistor into current they are 1 and the same thing.

That power in the resistor will always receive power and that power whatever it receives, it immediately dissipates into the atmosphere. The power that you have injected in a resistor will be lost and lost forever you cannot recover that power back. We will see in my next lecture, how inductor can store energy. Thank you.