Analog Electronic Circuits
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Lecture - 05
Analysis of Simple Non-Linear Circuit

So, dear students welcome to this course of Analog Electronic Circuits and today we are going to discuss some of our early topics namely how do we analyze a simple non-linear circuit. So, to start with we will be covering the diode circuits, but then whatever the concepts it will be discussed here, it is equally applicable in other non-linear circuits as well. So, let us see what are the plan we do have today and we will see that how it is consistent with our weekly plan.

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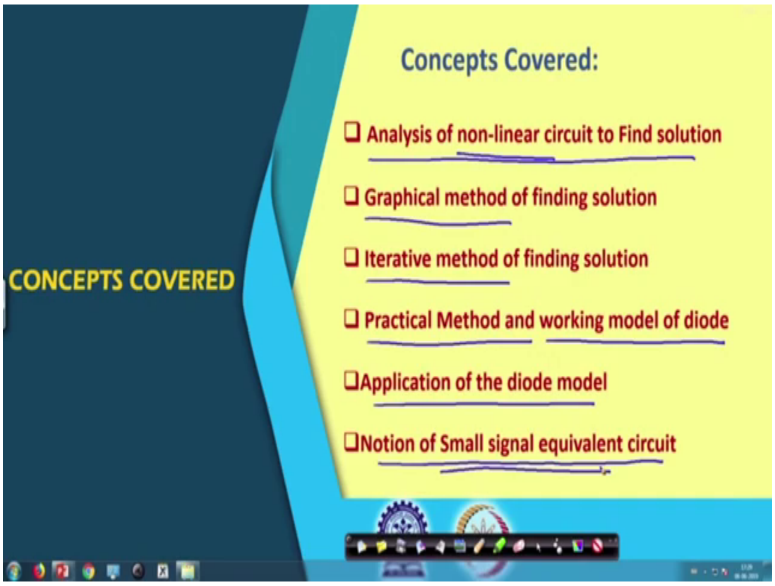

So, what we are planning today, it is that we will start with non-linear circuit, we will try to seek how to find the circuit solution, namely the circuit voltage and circuit branch currents consistent with the KCL KVL of the circuit and also we will be seeing that the device characteristic need to be respected. So, then we will start with generalized methods namely graphical method or graphical interpretation of the method to find solution, then we will be covering iterative method which is finding numerical solution of a given circuit with known parameters and then we will be moving to practical methods.

So, typically this non-linear iterative method to find the numerical values may be used for a computer, using computer programs or circuit simulator, but for hand analysis that maybe bit clumsy particularly when the circuit it grows and then we need some practical method and we need some working model of diode which can be used to solve the circuit equations to find a reasonably accurate solution.

And that diode model the working model it today we will see that how it can be deployed for different examples and finally, will be giving a notion something called small signal equivalent circuit. So, small signal equivalent circuit not only it is applicable for diode or a simple non-linear circuit, it is also applicable conceptually for any other non-linear circuit.

Essentially, in small signal equivalent circuit what we do is we do linearize non-linear circuit, so whenever we do have non-linear circuit it is very essential to translate into a simpler form for possibly to manage the situation. So, let we also see what is our weekly plan and we will see how we are consistent with that.

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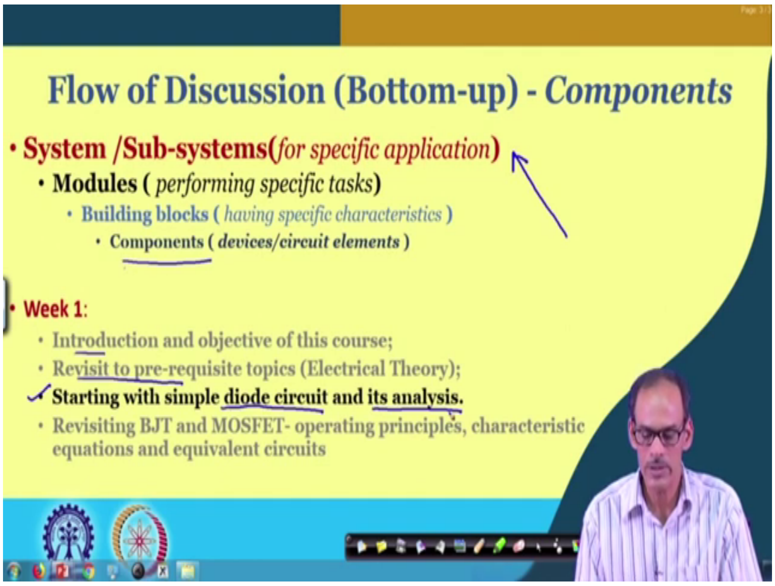

So, this is our overall plan of the this course, namely will be going for analog circuits in the top down approach, namely the components, then building blocks and so and so. And in the first week, so we are essentially some here, here and here and in the previous two classes what we have covered is apart from introduction, we also have revisited some of the prerequisite, and little bit we already have started about simple diode circuit.

Today, will be going little deeper into the analysis of diode circuits and as I said that the analysis method it will be discussed here it is not only valid for diode circuit, it is also valid for any other non-linear circuit particularly in the context of analog circuit.

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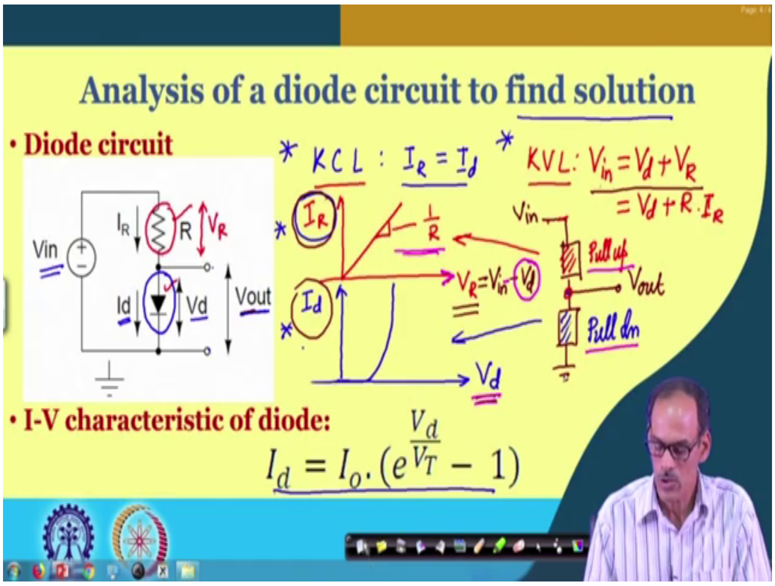

So, let us look into one simple example. So, to start with in fact, we will be having this example throughout, what we have in this circuit is we do have input voltage which is applied across series connection of the register and diode. And our objective here is to find what may be the voltage at this output port eventually this is same as diode voltage V d.

Now, while we will be solving this circuit what to find the circuit solution essentially we need to find V out the voltage here and the branch currents here so that they are consistent with other requirements. So, what are the requirements? We do have KCL. In fact, if you see at this node the KCL implies that the I R and the I d should be equal. So, this is the KCL.

So, likewise we also need to satisfy KVL. So, in this circuit what we have? We do have one loop here. So, we can say that KVL is giving us equation V in equals to V d plus V the drop across this register, so you may rearrange this equation in this in different form. So, you may say that this is V d plus R into the current flow through this register.

So, we do have KVL and also we do have KCL and also we have to give a respect of the behavior of the two elements, this element and this element. So, if you see here this element it is having its own characteristic namely it is having some characteristic getting represented this I R and V R relationship. What is V R? Voltage across this resistance. So, as you know this is linear device, so based on the value of the resistance it will be going through the origin and the slope of this line it is 1 by R.

On the other hand, when you see the other element we do have diode element and the relationship of its current and voltage it is given here, the exponential equation. And what is this equation? We do have for this element the voltage drop across this element is V d and current flow through this element is I d and it is you can say it is exponential kind of behavior, ok.

Now, while we will be solving this circuit to get the final value of V out and the currents and all we need to as I said we need to respect KCL, KVL and then register characteristic and then diode characteristic. Now, before I go into the solution, so let me introduce some term. In general if you see here we do have one element series with the other element connected to ground, this is connected to V in, incidentally this is register and this is diode and V R to find what will be the voltage at this point.

Note that based on the strength of the this element, this element if it is strong then it will take this voltage towards this input voltage. So, it may pull up this output node towards this input voltage. On the other hand, if you see the other element the diode in this case it will try to based on again its strength it will try to take this node voltage towards the ground. So, we may call this element as say pull down element.

So, likewise the upper element we may call it is pull up element. So, that is how will be naming. So, we will say that pull up, in this case pull up characteristic is given here and then pull down characteristic it is given here. Now, as I said that this is a generalized method, so whatever the discussion we will be having it can be deployed, any other circuit having these two paths pull up and pull down need not be this is linear or this is non-linear, in general it can be anything.

And end of the solution or end of the analysis whatever the solution will be finding we must as I said we must respect the KCL, KVL, pull up characteristic and then pull down characteristic. Now, if we see that this KCL suggests that this current and this current, these two currents are equal. So, we can say these two axis they are equal.

And on the other hand if you see the KVL the V R if you see here, V R it is essentially, V R I can rewrite by following this KVL. So, if I use this KVL I can write V R equals to V in minus V d. So, we can say this is V in minus V d. Now, this V d and this V d eventually they are same as V out, but unfortunately in this characteristic curve and this characteristic curve they are different.

Luckily, in this x axis equation we do have though this V d present, but unfortunately it is having a minus sign here and also it is having V in. So, as is if I try to overlay these two characteristic equation or characteristic graphs it may not be giving us a meaningful information. So, before we really try to compare these two characteristic curve on single one let we try to rearrange this pull up characteristic.

So, we like to rearrange this characteristic so that after rearrangement the x axis is consistent with V d. So, that is the next thing we will be doing. So, please keep in mind that these terminologies pull down, pull up, basically referring the pull up characteristic and so and so.

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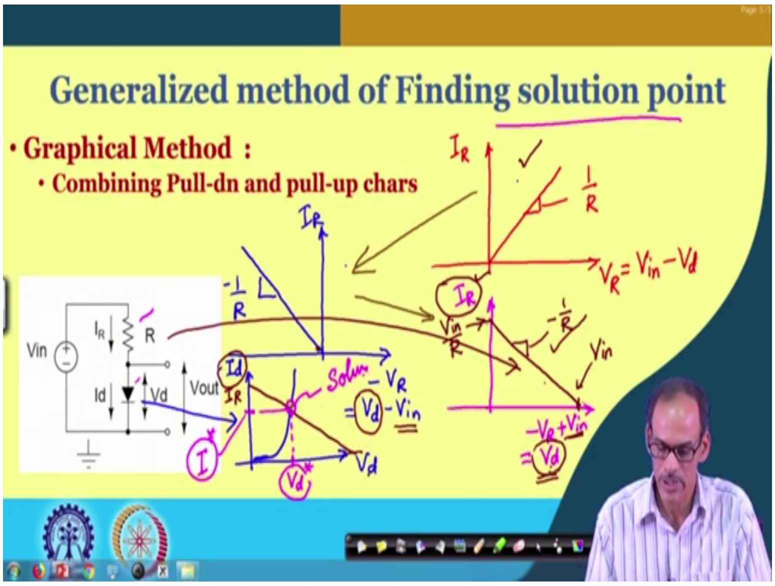

So, let us move to how do we rearrange the pull up equation. So, to start with we do have pull up characteristic, it is originally we do have pull up characteristic which is going through the origin as I said and y axis is I R, x axis it is V R, incidentally V R equals to V in minus V d and as I said that our main task is to translate this x axis in different form and of course, the slope it is 1 by R. So, the first translation what we will be doing is to get rid of this minus sign.

So, let us see how we do, to avoid this minus sign let we flip the characteristic curve. Namely, let we plot this I R with respect to minus V R. So, what is minus V R? This is V d minus V in and once you flip this, the axis and the corresponding characteristic curve of the pull up element it is going to the second coordinate where the slope remains 1 by R, but of course with a minus sign because the axis got changed.

So, we can say that this characteristic and this characteristic essentially representing the same thing. So, here now we can see that we do have V d, but still we do have minus V in. So, still we need to have some more rearrangement. So, let us see how do we further rearrange. So, with this minus V R if we add V in; that means, along this x axis if we say that minus V R plus V in that eventually comes to V d and let we retain this y axis as I R.

So, yes we obtain this V d and of course, by the virtue of adding this V in which means that the whole characteristic curve it is getting shifted, right by an amount of V in. So, we can say that this point of the characteristic curve it is getting shifted here and its value of this expression or the V d equals to V in. So, at this point it is V in, slope of this line; however, remains same minus 1 by R.

Now, this straight line having the slope information and this point information you can easily find yourself that this point where this part it is equal to 0; that means, V d is equal to 0 then we can say that the V R at this point equals to V in. So, if V R at this point is equal to V in which means that the corresponding current it will be this V R divided by R.

So, we can say that this is nothing, but V R by R or V in by R, because at this point when V d equals to 0 V R and V in they are becoming equal. So, that is how we define or rearrange this pull up characteristic in this form. So, originally we are having this characteristic curve. Now, we are happy to have this characteristic curve where the x axis it is V d and y axis we are happily retaining to the I R characteristic. So, if you recall then we do have the diode characteristic which is V d versus I d that is remaining to be exponential.

Now, we do have the pull down element characteristic here and we do have the rearranged pull up characteristic near. And both of them the y axis and x axis they are exactly matching. So, if I overlay this two characteristic incidentally we are making this I R equals to I d which means that we are satisfying KCL. So, likewise this two characteristic, this two axis since they are same we are actually satisfying KVL.

So, if I superimpose this characteristic curve here, what we mean is basically the pull up and pull down characteristic curves are falling on the same graph and this two axis matching of this two axis is ensuring the KCL and the KVL. So, along the y axis we do have not only I d, but also I R.

And if you see at this point at the intersection point of this two graph what we are getting is both the curves are consistent to have the same current and the to support the same voltage. So, whether you call V d or V out. So, we can say that pictorially this is our solution point. So, corresponding to this point whatever the voltage you are getting either you may call V d star and you may call this is you know I star, either you call I d star or I R star. So, this current and this voltage they are nothing, but whatever the solution we are looking for.

So, pictorially what we have done for non-linear circuit is that we are overlaying the pull up and pull down characteristic curve, but definitely only after this nice rearrangement. So, first of all we made this rearrangement first step and then we have rearranged further, so that the x axis of this characteristic curve is consistent with this one and that is how it is by combining we can say that we are getting the solution. Now, this is referred as graphical method of finding the solution of this simple circuit.

Now, in case if you want to find the numerical value of this point what may be the procedure? That is also again it is generalized method and this circuit is an example.

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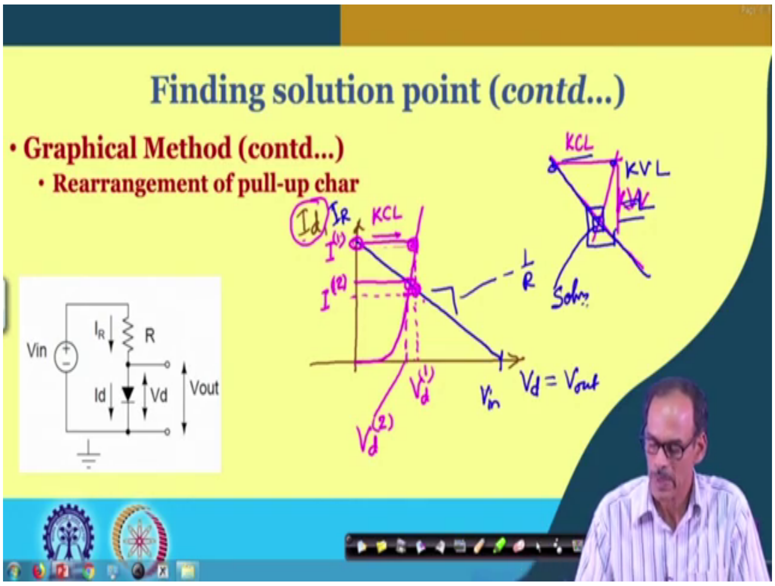

So, let us see what is that method called, ok. So, this is what we already have just now we have discussed. So, I will just probably redraw the whatever the characteristic we obtained. So, along the y axis we do have I d and also we do have I R and along the x axis we do have V d which is eventually V out.

So, I R with respect to V d characteristic is like this, slope is minus 1 by R, this point it is V in, and on the other hand the diode characteristic curve, let me use different color. So, this is the diode characteristic curve. Now, let us move to how we proceed to find this solution. Maybe we can start from this point and then we can move to this point and then we can come to may be this point and so and so on.

So, if we do not know what is the solution here somewhere you have to start and typically we do start from this point. So, that is how pictorially you may say that from here we maintain this current which means that at this point we do have I R and if we are moving horizontally representing that we are equating this I R with I d because this curve is basically representing I d. So, that ensures our KCL.

And then we come down here to, so this point of course, this point it is not consistent with this line. So, we do come down here saying that, we are getting some V d maybe V d 1, but again this is not the solution, but it is we are coming here to make the diode and pull up characteristic rearrange pull up characteristic consistent.

So, we do move from this point to this point and then we come to this point again and then pull up character pull up characteristic may be consistent at this point, but then pull down characteristic is not consistent. So, again we move along this direction to find the updated current. So, that is how it is you know iteratively it is done.

So, let me again elaborate using the equation and here you may say that we call this is I 1, I subscript 1 the voltage you obtain this is V d 1 and after moving to this point the d whatever you say that the updated current in the second iteration and then wherever this current is intersecting the diode characteristic namely the pull down characteristic curve we call this is V d 2 and so and so. And they can then again from here you can go up here, so to find the updated current.

So, you may say that we are moving horizontally satisfying KCL, we are coming down satisfying KVL and in between you see that this is diode characteristic curve and this is sorry this is the register or pull up characteristic curve. So, while you are moving from this line to this line we are trying to respect both the elements pull up and pull down element and at the same time we are also satisfying KCL and KVL, ok. And in a process we are moving like this and finally, in a almost like a spiral way we are converging to this point called solution point, ok.

So, let us move to what are the equations involve into this movement to have a better understanding about the or different understanding rather.

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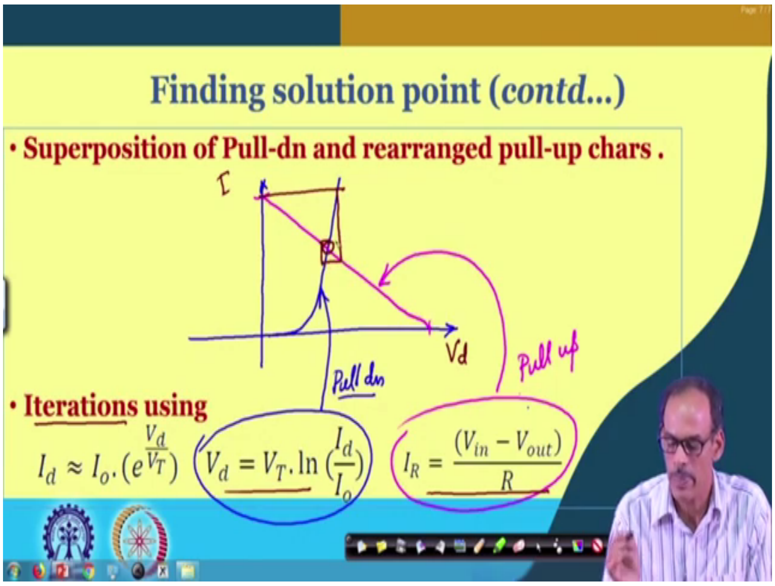

So, we already say that we are superimposing the two characteristic curve and we are respecting this two characteristic curve and iteratively we are moving to the final solution. It is kind of reputation what just now we said, but this is what we are saying that we do have this characteristic curve is basically the diode characteristic curve this is the rearranged pull up characteristic curve and the other one it is the diode characteristic curve or you can say pull down characteristic curve.

So, in this superimposed I-V characteristic curve of this either you can say I d versus V d or I R versus V d we are trying to find this solution. So, while will be going through the numerical procedure called iterative procedure we will be using this equation and also we will be using this equation and let us see what are the steps involved into that. And as I said that our movement is going like this coming here and then moving horizontally and then vertically, and then horizontally and so and so on.

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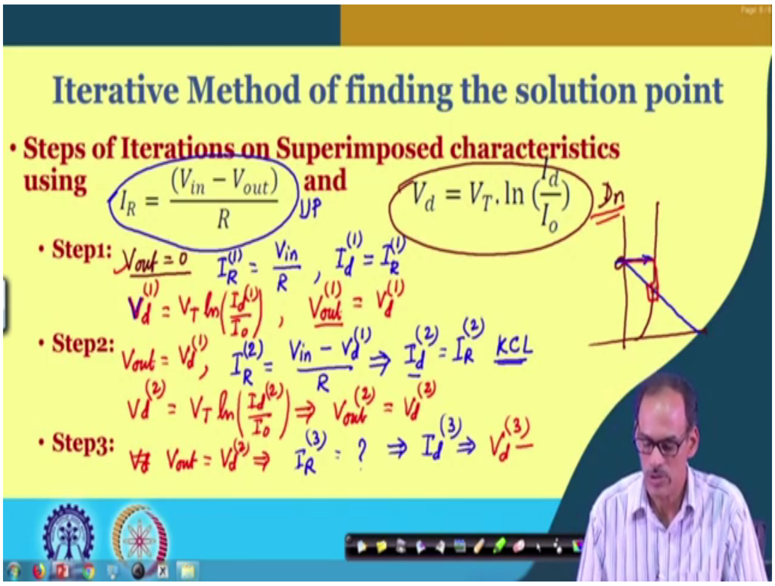

So, let us see how we are what are the equations involved. In step 1, in step 1 using this, so this is pull up characteristic as I said. So, this is pull up characteristic, for simplicity I will say up characteristic and we will see that this is down characteristic. And using this pull up characteristic first we start with V out equals to 0. So, you may recall that we do have pull up characteristic like this and then we do have the pull down characteristic like this.

So, we are starting from say this point where V d equals to 0 and V d is nothing but V out. So, we can say that if I assume the V out equals to 0, then by using this pull up characteristic we can say that I R equals to V in by R and you can say this is end of the first iteration, so this is this level call I R 1. And then while you are moving horizontally what we are doing is we are saying that diode characteristic curve after this iteration is nothing, but this I R 1.

And then once you get this I d then you can find the corresponding V d. So, let me use different color here. So, we are getting this V d using this down characteristic curve it is equal to V T ln I d 1 by I naught, I naught is basically the reverse saturation current. So, this is again end of the first iteration, so we call V d 1 and that gives us new V out call updated V out which is equal to V d 1 because we know that V d 1 and V out 1 they essentially same. So, we started with this initial guess, we are landing to some updated value.

In the next step we will use this V out. So, we will be using V out equals to V d 1 or V out 1 and using this pull up characteristic what you can get is, we can get I R this is updated 1 and now we will be using nonzero value of V out call V d 1 and whatever the value we are getting it is we called it is I R at the end of second iteration. So, again this will be giving us I d 2 equals to I R 2; that means, we are satisfying this KCL, right.

And then once you are obtaining this I d 2, you can use the pull down characteristic to find V d 2 as equal to V T ln I d 2 divided by I naught and then that gives us updated V out. So, now V out is V d 2, right. And subsequently you can go to the third step.

So, we can say that V out sorry this V out, new V out it is V d 2 and then you can find this I R third iteration and then that gives you I d third iteration, that gives you V d 3 and so and so on. So, that is how you can as I said that you are progressing towards the solution and it looks like it is converging. So, but still we do not know pictorially it looks like converging. So, we need to check whether actually it is converging. So, how will you do that? Probably, we can use one numerical example.

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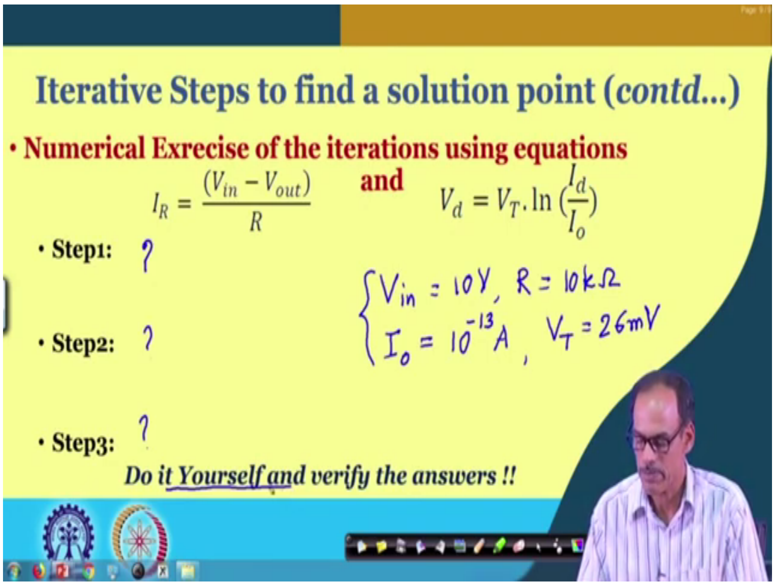

So, what you can do? You can take say as an example you can take V in equals to 10 volt R equals to 10 kilo ohm. Say for the diode reverse saturation current let you consider 10 to the power minus 13 ampere and thermal equivalent voltage maybe we can take 26 millivolt, typically that is what we do get at room temperature.

Using this values of this parameter you can find the corresponding current here in the first step, second step and in the third step. So, my suggestion would be please do it yourself and verify it with the answer given on the next page. So, what we have it is on the next page? We do have the solution.

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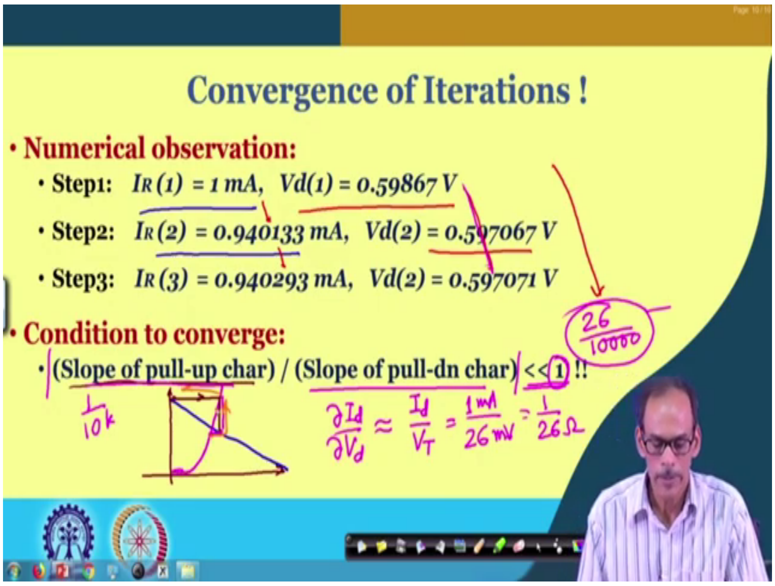

So, I guess you already have done it and you will be or maybe you will be doing it offline and then verify whatever the solution we obtain. End of the first iteration we obtain say current is given here and the voltage the updated voltage it is given here starting from 0, now we obtained this voltage.

And then in the second iteration we are you know improving the accuracy of the voltage the current as well as voltage and if you see here with progress of iteration you can see that the difference relative difference of the previous iteration to the next iteration and so and so they are gradually decreasing and even for the voltage also.

So, it looks like the numerically at least for this example it is converging and but the natural question is that will it always converge? Yes. There may we cannot give a guarantee, but of course, probably we can say something about special characteristic of this circuit and we may say that under certain condition the way we have moved interestingly the way we have moved from the pull up characteristic to the pull down characteristic and their nature is helping us.

So, let me illustrate what does it mean is that while you are moving in this direction and then coming back here we are converging to this point. In fact, if you see this shift and this shift if you take the ratio that is nothing, but the slope of the pull up characteristic. On the other hand, while you are moving say downwards like this and then moving horizontally that is nothing, but the slope of the pull down characteristic.

So, the while you are coming from this point to this point essentially that is nothing, but the ratio of the two slopes of the two characteristic curve and you can see that this is resistive element and this is exponential curve. Numerically, if you calculate for whatever the solution you have done the slope of this line which is 1 by R of course, having a minus sign and we are concentrating on the magnitude.

So, and on the other hand if you see the slope of this line. What may be the slope of this line? It is change in I d divided by change in V d, and if you see the diode characteristic curve you may say that it is well approximated to I d divided by V T. And this current it is in the order of milliampere.

So, if I say that this is 1 milliampere and this is 26 millivolt, so this is 1 by 26 ohms and this slope of course, 1 by 10 k, ok. So, what does it mean is that if you take this ratio it is nothing, but 26 divided by 10 k and interestingly for this example it is I should say much much lower than 1. So, as long as it is less than 1 there is a guarantee that this will converge. And in this case it is very small and hence it is rapidly converging. So, that is why this practical, you know method it is working fine.

Now, the natural question is that if you are not moving in the right direction, on the other hand if you are moving in the wrong direction maybe in this way and then this way and then this way then of course, the same situation with wrong movement you will be diverging out.

Whatever the initial guess you do have it will be diverging out. So, movement of this characteristic curve is very very important. I think, let me see what I do have next.

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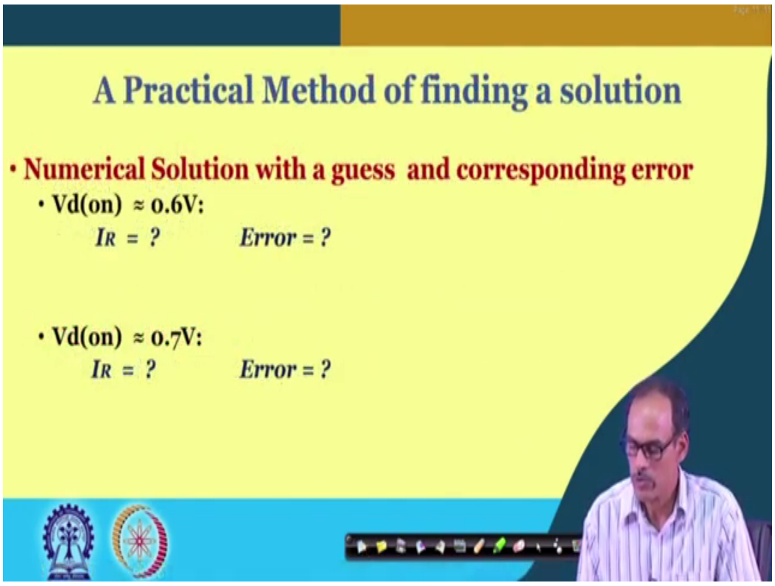

So, before we go to the next slide probably let you take a break and then will be again restarting from here. So, we have discussed about generalized method, but as I say that it is a iterative method we may be looking for some practical solution. So, in the next part after may be short break we will be coming, we will be resuming to look into some practical method to solve this kind of circuit.

Thank you.