VLSI Signal Processing

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Lecture 38

Baugh Woolley Multiplier

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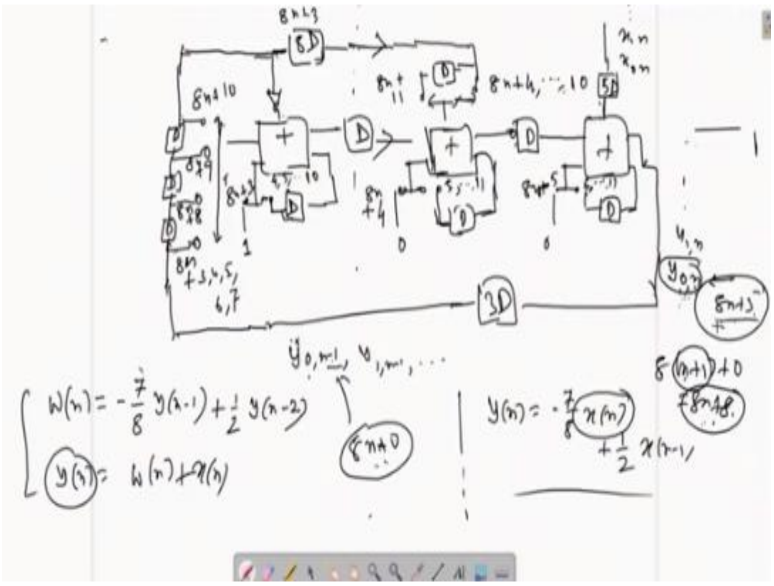

Okay, consider this IIR equation actually you have cleverly chosen so that it has got some kind of overlap with the previous example, so yn depends on yn minus 1, yn minus 2 and of course xn. Since, it depends on its past it is an IIR filter all of us know okay. In fact if you take z transform takes transform function this will give as to z inverse into 7 by 8 it will give rise to z inverse 2 into half.

So, transform function we will have if you do this numerator by denominator kind of form this denominator will give polynomial up to z inverse 2 so there will be 2 poles and all those things then there part of DSP, but this is an IIR filter. So, the IIR filter consider this part this let me call Wn which is depending on yn past values of y. So, the equation can now be written as Wn is equal to there is Wn and what is yn, yn is this wn with that you add xn.

Now, remember this previous FIR filter that time yn was minus 7 by 8 xn plus 1 by 2 xn minus 1. So, this 2 equations are very identical here input was xn at n th 8n plus 0 plus 0 when I am computing the nth cycle output of yn that time nth cycle input of xn that was coming xn0, xn1, xn2 like that and then I have developed the inter circuit. This was in (()) (02:18). Here instead of yn I have got Wn and instead of xn I have got yn minus 1.

But filter coefficients are same so I can use the same circuit, but instead of giving here earlier I was giving xn, 0 at 8n plus 0 clear because that was the input now this is the input, nth cycle output was found that time yn now same nth cycle output only to be found. Input was xn so xn0 came in at 8n plus 0 etcetera here yn minus 1 is coming. So, yn minus 0, y0 n minus 1 with the LSB of why this fellow.

This would come at 8 n plus 0 at 8n plus 0 by input that time was xn0 or x0 n now it is y0 n minus 1 because it is xn it is yn minus 1, yn minus 1 you can call it as some single zn, so z0 n which is nothing but y n minus 1 this comes, but this comes at 8n plus 0 then we would go y0 n minus 2 dot, dot, dot, dot. Sorry instead of writing in that direction let me write in this direction this followed by y 1 n minus 1 dot, dot, dot, dot okay this is to be given.

And at 8 n plus 4 that time LSB of yn was coming out and then subsequent bits at what cycle 8n plus 4 okay. Now, at 8n plus 4 the LSB of Wn will come LSB of Wn and then next one LSB, next to LSB and all that, but equation is Wn plus xn. So, LSB of Wn that time I should have LSB of xn, xn is coming as (())(04:32) 8n plus 0, 8n plus 1 etcetera LSB of xn x0 n coming at 8 n plus 0, but this fellow the LSB of Wn that is coming at 8 n plus 4, 8 n plus 4 which means input xn also they has to be delayed by 4 cycles.

So, LSB of xn comes not at 8n plus 0, but 4 cycle later 8n plus 4 so 8n plus 4 here, 8n plus 4 here you can add. So, you can have adder and this is how this things come 0n, 1n dot, dot, dot, dot and it has to be delayed since 8n plus 4 at that time only LSB of Wn you can call W0 n comes to be added with x0 n that is you need (())(05:20) delay, but then you have to give it directly here with 2 (())(05:25) again come one after another in the same cycle.

So, critical path goes up so I do not want that to happen so I will put a delay here which means this also has to be delayed further by one more so 5D okay and this is output and carry this is your carry so 8n plus 4 here means 8n plus 5 you shall carry 0 so it will be 8 plus 5 then I have got 6 to 11 here 6 dot, dot, dot 11 and output coming here. What is the output? Output is nothing, but this output at what cycle y0n followed by y1n dot dot dot.

So, this is coming at 8 n plus 5 so y0n followed by y1n dot, dot, dot so this is coming at 8n plus 5. So, y0 n this is coming at 8n plus 5. Now, you see I presumed that at 8n plus 0 this fellow is available that is a previous word LSB was available at 8n plus 0 previous word LSB and so on and so forth subsequent bits and then I generate it current word nth word LSB at 8 n plus 5.

So, by the same logic this will be required here in the next cycle, next cycle means at when I have got n plus 1th word, n minus nth word LSB I am giving at 8n plus 0 at 8n plus 0 all right for calculating yn for which 8n plus 0 that time I am giving the LSB of the previous word and then going through this at 8n plus 5 I am rewriting the LSB of the current word, but LSB of the current word then will be required for the next word.

Where I will have to give the next word at 8 into n plus 1 plus 0 at 8n plus 0 I give y0n minus 1 I am assuming it to be available okay previous word LSB available at 8n plus 0 I am giving him. So, at 8n plus 5 instead of y0n minus 1 y0 n comes. So, this I want to make available here for the next word. So, for that the starting cycle will be 8 into n plus 1 plus 0 that is 8n plus 8 it is available here at 8n plus 5.

Which means I have to delay this by 3 cycles this is 8 n plus 8 and this is 8n plus 5 so this fellow I have to delay by 3 cycles and they give here and the loop is completed. This is the BIT serial relation of the IIR filter. Now, remember one thing sometimes it may so happen that word length is such and other parameters are such that this may be less than this. So, difference of this cycle minus this cycle that will be negative which means it is a negative number of delay which is not possible.

So, in those cases BIT serial relation is not possible you have to change the word length because design cannot be changed the word length W has to changed appropriately and all that. So, that this starting cycle here which is 8n plus 8 for the next word that is n plus 1th word that is more than the time when this LSB current cycle LSB, current word LSB comes which is 8n plus 5 in this case.

Here there is no problem 8n plus 5, but I require it here at 8n plus 8 there is a starting cycle of the n plus 1th word 8 into n plus 1 plus 0 which is 8n plus 8. So, 5 here 8 here so 3 cycle delay, but if this is less than that then there is a problem then this is not possible you have to change the word length okay. So, this gives you BIT serial relation of IIR filter you can try your hand with some other examples given in PARI's book.

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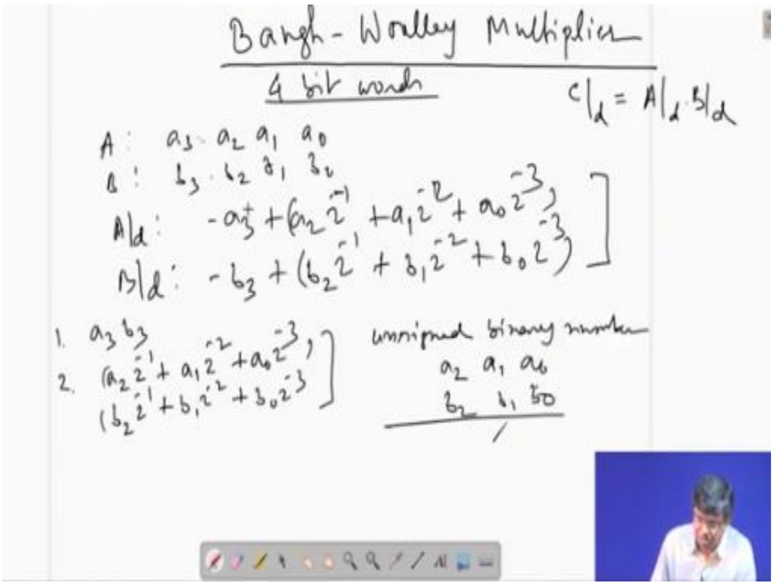

Okay my next topic is a very important thing you should know this Baugh Wooley I do not remember whether there is Woolley or single ley okay you have to check it I am not sure of the spelling Baugh Woolley Multiplier okay. Earlier we have seen this multiplication by (()) (10:08) okay there is a multiplication table, there is a row containing the PP10, PP11 everything that we sign extend it add with another row.

If this addition leads to overflow bit we are not checking the overflow bit and so we move with error. So, there is a scope of generating error there, but Baugh Woolley Multiplier is a very powerful multiplier where this error has been taken care of I mean it is free from overflow and there are 2 types of Baugh Woolley Multiplier so I will just consider one, but I will also take 4 bit as an example I will consider 4 bit words.

Suppose, A is given in two scope form. Now, I want to do the multiplication so that I get a number c whose decimal value under to scope them in formula is same as the decimal of A times decimal of B. Okay whatever you do in terms of hardware I do not care, but finally you should get some w bit 4 bit word or maybe larger bit word if we do not truck it anything whose decimal value under two scope system formula is nothing but decimal of A into decimal of B.

Then I will be happy with that c I will take it to be the product word okay multiplication word multiplication output of A and B all right. Now, what is A by d. Now, you see decimal expression minus a3 plus a2 into 2 inverse a1 into 2 inverse sorry 2 inverse 2 and a0 into 2 inverse 3. B by d minus b3 now a3 b3 a2 a1 they are all decimal digits if it is binary 1 here it is decimal 1 if it is binary 0 decimal 0 so on and so forth okay.

So, I can happily put minus sign multiply by 2 or power of 2 and all those. Multiply the two there will be what I will do I will put this part here this part here under parenthesis and I will multiply like this I will have minus a3 into minus b3 there is one term a3 b3 decimal term another term product of these two. You can see this product is nothing like multiplying two unsigned number unsigned there is ordinary binary numbers a2, a1, a0 b2, b1, b0.

It will be like you know a0 dot b0 which is decimal will be a0 b0 I will assign power 2 to the power minus 6 there then a1 dot b0 which is nothing but decimal a1 into b0 I will assign power 2 to the power minus 6 minus 5 related to be 0 2 to the power minus 4 then again a0 b1 a0 dot b1 which is nothing a0 b1 here 2 to the power minus 5 so it will come below this and like that.

No question of you know minus and all those things okay sign extension minus I think just a binary polynomial multiply by polynomial, polynomial in terms of powers of 2 negative powers of 2 again negative powers of 2. So, if I arrange this bits like this just multiply like an unsigned number okay I will get the result so this is later, but cross terms are important. What is a cross term minus a3 times this and minus b3 times this okay the cross terms.

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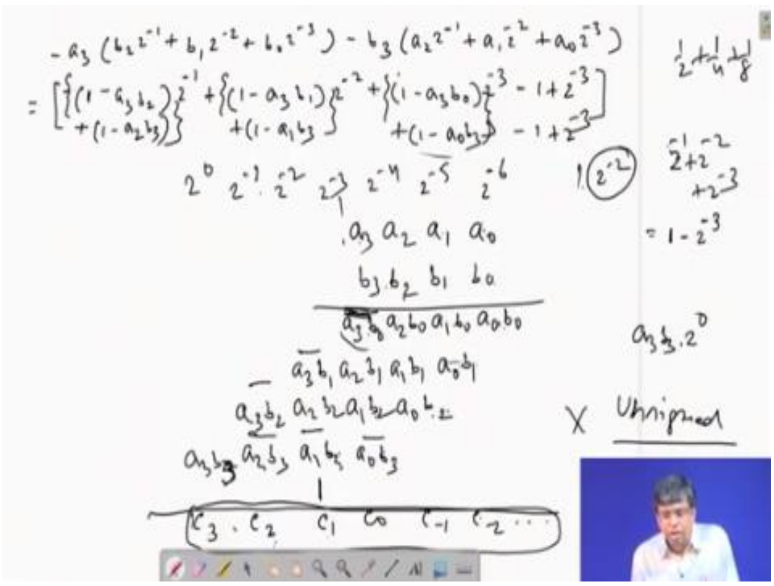

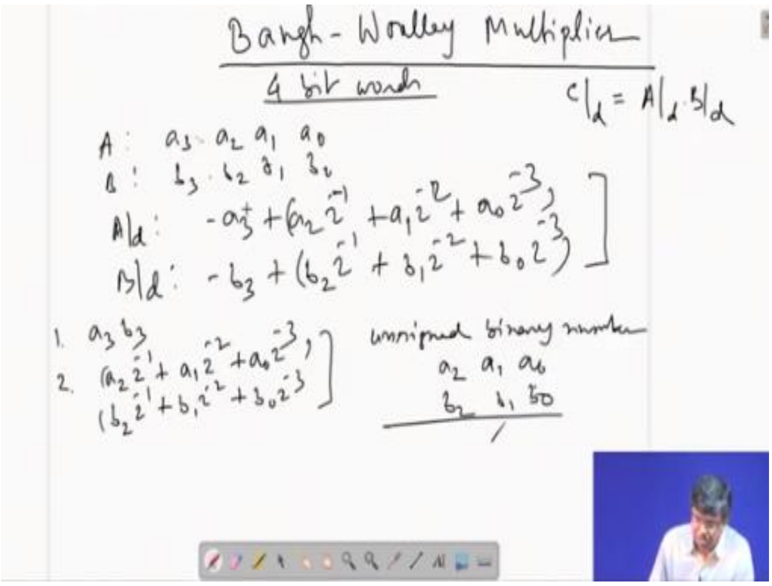

The cross terms are minus a3 times b2 2 inverse I will write it again minus a3 so minus a3 into this and minus b3 into (())(14:15) a2 2 inverse a1 2 inverse 2 a0 2 inverse 3 this is what I have. What I will do we will do some manipulation okay. I will write it like this. I will bring out 1 here minus a3 into b2 let that be as it is minus a3 into b2, but I am bringing 1 with 2 inverse this is basically I am adding a 1 into 2 inverse so I will cancel it out later.

Similarly minus a3 into b1 so I will write it as 1 minus a3 times b1 2 inverse 2 and minus a3 into b0 so I will write it as 1 minus a3 b0 2 inverse 3. So, I added 1 into 2 inverse 1 into 2 inverse 2 I added 2 inverse here 1 into 2 inverse 1 into 2 inverse 2 so 2 inverse 2 and 2 inverse 3. This kind of summation (())(15:28) series we had worked out earlier it is like 2 inverse 1 2 inverse 2 dot, dot, dot 2 inverse W minus 1 it was what 1 minus 2 inverse W minus 1 that is 3.

You can easily check also half plus 1 by 4 plus 1 by 8 this is what 8, 2, 4 and 2 7 by 8 and this is what you have here 1 minus 2 to the power minus 3. So, this I have to cancel so I will write minus 1 plus 2 inverse 3. So, those additional ones I have brought in they get cancelled same thing in the case of B same thing I will do in the case of B, so if you do that in the case of B I will have another term here minus b3 a2.

So, it will be 1 minus b3 a2 or a2 b3 that is also power 2 to the power minus 1 this also power 2 to the minus 1 that is both I am putting under same bracket. Similarly, 2 to the power minus 2 power I had here minus a3 b1 and 1 minus a3 b1. When I do the same thing in the case of b3 minus b3 1 it will be 1 minus b3 b1 1 minus b3 a1 or 1 minus a1 b3 so I put in bracket 1 minus a1 b3.

And similarly here I have b0 minus a3 b0 into 2 to the power minus 3 it became 1 minus a3 b0 2 to the power minus 3. If I do the similar thing in the case of b3 it will be 1 minus a0 b3 power is same 2 to the power minus 3. So, out of x this I had minus 1 plus 2 to the power minus 3 out of this side also we will have similar another minus 1 plus 2 to the power minus 3.

Now, you remember a3 times b2 in decimal it will be a3 dot b2 in binary. Firstly, 1 minus a3 b2 a3 b2 is a decimal digit. It can be either 1 or 0 is not it a3 can be decimal 1 or decimal 0 b2 can be decimal 1 or decimal 0 so it is a decimal digit which can take either 1 or 0. You remember sometimes if I told that a is in binary in decimal if I write 1 minus a that is if a is binary 1 it is decimal 1 if a is binary 0 it is decimal 0.

So, it is corresponding binary equivalent will be a bar if a is binary 0 in decimal it is 0 so 1 minus 0 is decimal 1 and a bar is binary 1 binary 1 decimal value is decimal 1 so I am getting 1. Similarly, if a is decimal 1 sorry binary 1 it is here is decimal 1 so 1 minus 1 0 decimal 0 if it is binary 1, 1 bar is 0 so binary 0 decimal 0 so they are matching right. So, that gives 1 minus a3 b2 means it is not that of a3 b2.

And what is a3 into b2 it is nothing but a3 dot b2 all right. In product of 2 bits here in the decimal means in binary it will be end of the two binary bits product of the 2 digits here decimal digits okay all this we have done earlier which means you can say this is you should give rise to a3 power b2 2 to the power minus 1. So, let me not write anything here. You understand one thing this we give rise to binary.

In binary when I go to binary forward this entire thing will give rise to a3 b2 bar. This will give rise to a2 b3 bar this will give rise to a3 b1 bar this will give rise to this 1 minus a1 b3 this will give rise to a1 b3 bar. This entire thing will give rise to a3 b0 bar this entire thing will give rise to a0 b3 bar this is minus 1, minus 1, minus 2 and 2 to the power minus 3 twice means 2 to the power minus 1.

If we do all that if you do all that just let us go the previous page. This is unsigned number multiplication a2 a1 a0 b2 b1 b0 this one. I have got a3 b3 also and with them I have to add that part. So, I carry out this a2 a1 a0 sorry when you multiply (())(20:36) power 2 to the power minus 3 this side minus 2 to the power minus 3 so it will be 2 to the power minus 6 line a0 binary domain I will write like a0 b0, b0 dot b0.

But when you take a decimal value it will be a0 into b0 times 2 to the power minus 6 this is 2 to the power minus 5 line. This will be a1 dot b0 in binary domain in decimal domain it will be a1 times b0 and then into 2 to the power minus 5 these are unsigned number not two is compliment business just this into this in decimal times this power this into this in decimal into this power a2 b0 they had power 2 to the power minus I had 2 to the power minus 3 2 to the power minus 3 so we will multiply 2 to the power minus 6.

I had 2 to the power minus 3 sorry 2 to the power minus this was 2 to the power minus 1 minus 2 minus 3 so 3 and 3 6 this is 2 to the power minus 2 this is 2 to the power 3 that is equal to 2 to the power minus 5 this is 2 to the power minus 1, 2 to the power minus 3 that is equal to 2 0 minus 4 a2 b0. Then again b1 had power 2 to the power minus 2 this I had power 2 to the power minus 3 that is why it is 2 to the power minus 5 that is coming under this line.

So, 2 to the power minus is common for this fellow also and this fellow also with a0 b1 then a1 b1 a1 had 2 to the power minus 2 this side 2 to the power minus 2, so 2 to the power minus 4 that is the line for 2 to the power minus 4 so a1 b1 and a2 b1 a2 had 2 to the power minus 1 b1 had 2 to the power minus 2 so 2 to the power minus 3 I have got 2 to the power minus 3 line so it will be a2 b1.

And then a0 b2, b2 had 2 to the power minus 1 power a0 had 2 to the power minus 3 power so 2 to the power minus 4 2 to the power minus 4 is here so a0 b2 will come here a1 b2 a1 had 2 to the power minus 2 this had 2 to the power minus 1 so 2 to the power minus 3 a1 b2 will come here b2 to the power minus 2 power a2 b2 a2 had 2 to the power minus 1 b2 had 2 to the power minus 1 multiplication 2 to the power minus 2.

So, a2 b2 will come under this power line this is what you have. Now, start counting here 2 to the power minus 3 then 2 to the power minus 2, 2 to the power minus 1 2 to the power minus 3 had a3 b0 bar and also a0 b3 bar so this are 2 to the power minus 3 line these are 2 to the power minus 3 line okay. So, a3 b0 bar so if I have a3 now included here a3 b0 like a0 b0 a1 b0 a2 b0 a3 b0 that will come here.

And I will write it under this a3 b0 bar all right a3 coming under 2 to the power minus 3 b0 coming under 2 to the power minus 3 sorry a3, a3 and it power b0 a3 power b0 yeah that is fine. So, a3 power b0 should have 2 to the power minus 3 that is why this fellow is coming under this line 2 to the power minus 3. Similarly what about this one it is a0 b3 bar a0 b3 bar same power 2 to the power minus 3.

So, under this a0 b3 bar so a0 b0 the next line a0 b1 next line a0 b2 so if I put a b3 here a0 b3 bar but under the same power 2 to the power minus 3 so 2 to the power minus 3 has this line so a0 b3 bar all right a0 b3 bar then comes this a3 b1 bar 2 to the power minus 2 so under this line a3 b1 bar a3 b1 bar okay. So, a0 b1 a1 b1 into be in this line give rise to this line so a3 b1 here you write a3 b1 bar.

It should come with power 2 to the minus 2 that is why it is coming. Similarly a1 b3 bar also should under 2 to the power minus 2 a1 b3 bar so a1 b3 bar okay a1 b3 bar. So, a0 b3 here a1 b3 a0 b3 bar here a1 b3 bar here okay then here a3 then I have got 2 to the power minus 1 line a3 b2 bar so a3 b2 bar b2 is here, so a0 b2 a1 b2 a2 b2 in this line so a3 b2 here b2, b2, b2 so a0 b2, a1 b2, a2 b2, a0 b2, a1 b2 a2 b2 so a3 b2 bar.

Okay that is for this side and this side a2 b3 bar a2 b3 means a0 b3 a1 b3 a2 b3 these are to be added then 2 to the power minus 3 into 2 means 2 to the power minus 2 so under this 2 to the power minus 2 line there will be 1 1 into 2 to the power minus 2 that will give 1 this is to be added and there is a a3 b3 means a3 b3 into 2 to the power 0.

So, here I have got 2 to the power 0 a3 b3 so a0 b3 bar a1 b3 bar a2 b3 bar a3 b3. Only thing that I have not added is the minus 2 minus 1, minus 1 so whatever I get here if you call the result and then c minus 1 so on and so forth this is the result I get is decimal value as I know this plus this times to inverse next time to inverse to all that I calculate with that I have to add a minus 2 with minus 1 minus 2 okay that is a job it has to be done.

But we will now show that if you what we are doing now we are taking it as a plain binary one not 2’s compliment just this into 2 to the power 0 this into 2 to the power minus 1 we are not putting any minus sign here unlike 2’s complementary system this into 2 to the power 0 this into 2 to the power minus 1 this into 2 to the power minus 2 and dot, dot, dot adding that is called unsigned number system.

No question of plus or minus sign minus sign all are positive, all are positive numbers okay unsigned there is no question of signed number just check it and multiply the various powers of 2. We will start with c3 (())(28:03) 2 to the power 0 c2 2 to the power minus 1 c1 2 to the power minus 2 and dot, dot, dot, dot value with that I have to add minus 2 we will show that you can bypass that minus 2 you cannot be require to add that minus 2.

Provided you create this binary word not as an unsigned word, but as a 2’s complement word that is when you find out its decimal value treat it as a decimal number this decimal value will be minus c3 plus c2 into 2 inverse and c2 1 into 2 inverse and all those things then you will get the same decimal value which otherwise you are getting when you are taking it as a unsigned number.

Just c3 into 2 to the power 0 c2 into 2 to the power minus 1 c1 into 2 to the power minus 2 and dot, dot, dot and plus this minus 2. You will get the same result if you forget minus 2 but treat this word as a 2’s complement number find its decimal value that way that is minus of c3 plus c2 into 2 inverse plus c1 into 2 inverse 2 plus c0 into 2 inverse 3 and dot, dot, dot this is what I will show.

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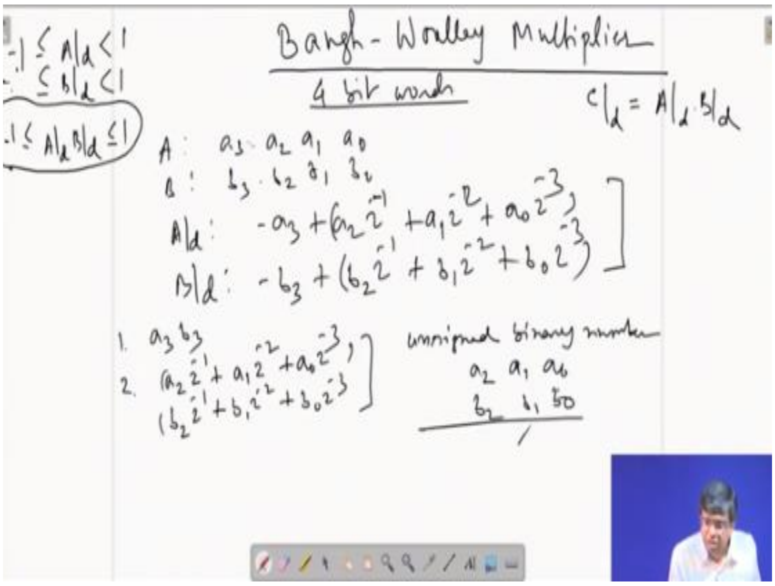

Okay how to show, now you see what we are doing after all we are multiplying a by b this is a decimal value of a this is a decimal value of b we know that A by d we have done the 2’s complement number system properties it is less than 1 greater than equal to minus 1 okay. Same for B by d when you multiply these 2 numbers A by d and B by d okay obviously the product cannot exceed this range.

Suppose both are positive but less than 1 if you multiply positive but lesser than 1 they both are negative multiply you still get positive, but value will be lesser than 1 okay less than equal to 1 so this product will be here I am putting less than equal to because here it can be minus 1 it can be minus 1 when you multiply product is plus 1 so product can still be here this is a range okay.

So, whatever business you do whatever business you do its total decimal value if you multiply the 2 decimal values here there is no 2's complement, no word sign business nothing this is decimal. Decimal of 1 into decimal the other that will not exceed the range from minus 1 to 1 it will not go above 1 it will not go below minus 1 that is guaranteed because of this.

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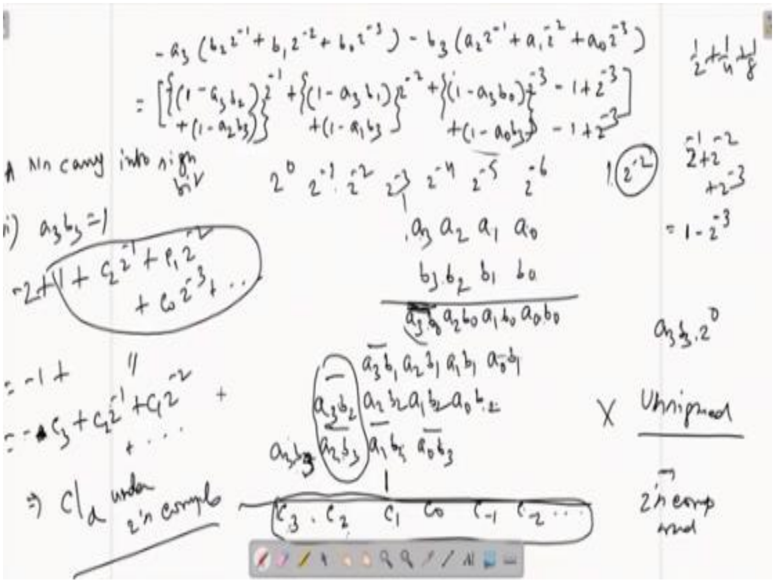

Keeping that in mind let us now see one thing we are doing this addition okay let me so we will do this addition everything when you add this two a carry may come in here, carry may not come in here both are possible we will consider both the possibilities. Okay no carry here and then carry here okay. Suppose case A no carry into MSB means from this addition when you add there is no carry here that is case A.

Under bit case I can have two possibility if there is no carry c3 a3 b3 only this will be the result here a2 b3 suppose is 0 that is c3 is 0 if c3 is 0 then what is the decimal value decimal value will be minus 2 of course plus this is 0 so c2 into 2 inverse c3 into sorry c1 into 2 inverse 2 and dot, dot, dot, dot all right. Now, this is minus 2 what is a even if all are positive all are plus 1 obviously this will be maximally positive.

So, we will add the maximum positive number to minus 2 so overall will be minimally negative what is the minimal negative value. Now, if you have 1, 1 into 2 inverse 1 into 2 inverse 2 and all those things okay whatever be the power up to it will go up to the power 2 inverse minus 6. So, it will be 1 minus 2 to the power minus 6 if you add then this is 1, this is 1 all are 1 then there this part is maximally positive so overall is minimally negative because this is minus 2.

So, even if it is minimally negative it will be 1 minus 2 to the power minus 6 here and minus 2 here so result will be minus 1 minus 2 to the power minus 6 which is less than minus 1, but this is not possible because the product when you multiply two numbers as I told you the decimal value when you multiply it can neither go below minus 1 not go above minus 1 so that means this causes possibility cannot arise okay this possibility cannot arise.

So, 1 and then this part c2 into 2 inverse c1 into 2 inverse 2 c0 into 2 inverse 3 and dot, dot, dot, dot, but minus 2 and plus 1 means it is minus 1 plus this part which is same as a3 b3 is minus 1 so minus a3 b3 there is minus c3 plus c2 2 inverse 1 c1 2 inverse 2 dot, dot, dot, dot which is nothing but if you take it as a 2's compliment number this decimal value will be minus c3 which is here then c2 into 2 inverse, c1 into 2 inverse 2 and all that.

That means it is C by d under 2's complement. So, originally it was unsigned numbers we added minus 2 under the assumption this is 1 plus 1 because it cannot be 0 so other person is 1 in that case whatever you get here just take the 2s complement just take the decimal value of that under 2s compliment formula you get the current result no need to add minus 2 you either take it as an unsigned number so that was the value with a3 b3 there is 1 2 that you add minus 2 you get the overall value.

But overall value is same as this which is nothing but the decimal value of this 1 as 2s compliment 1 c3, c2, c1 just forget about any minus everything just take it as 2's compliment 1 and find its decimal value which is minus c3 and c2 2 inverse and all that. Okay so in this case you do not have to bother about minus 2 take it as it is and take it as a 2's compliment number finds its decimal value under the formula.

This is when there is no carry into sign width from this previous addition then a3 b3 0 not possible that is c3 0 not possible a3 b3 1 possible and when that happens you do not have to bother about minus 2 just take this final word as a 2's compliment word and take its decimal value as this formula so far so good.

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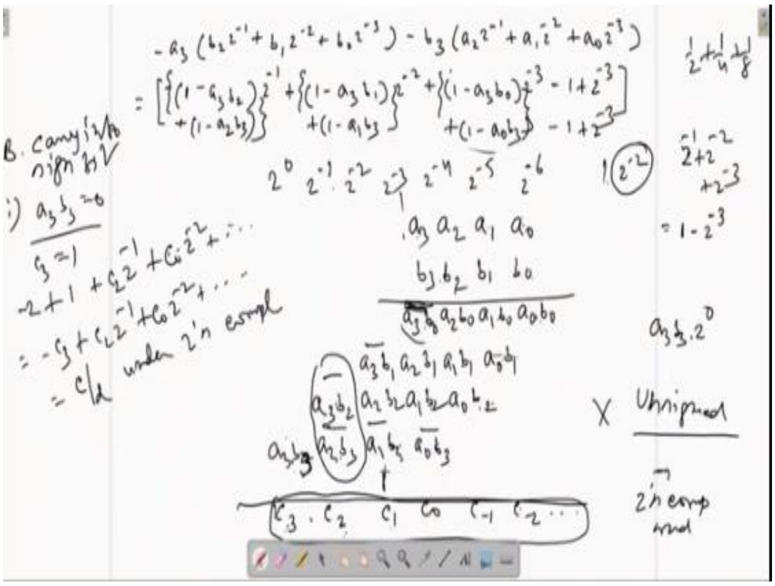

And when there is carry then again let us see 2 possibility 2 case B case B is carry into sign bit. In this case, case 1 suppose a3 b3 0 and a carry 1 comes that means c3 is 1 that means if c3 is 1 we have already seen. What is the total value total value is minus 2 plus 1 this is an unsigned number now 1 into 2 to the power 0 and then as before c2 2 inverse and c0 2 inverse 2 dot, dot, dot which is minus 1 minus 1 means c3.

So, c3 is 1 I can write it like this minus 1 I can also write as minus c3 because c3 in this case is 1 which is nothing but C by d decimal value under 2's compliment same as before. So, here also this is possible and you can take just this word as it is as a 2's compliment word find its decimal value under the 2's compliment formula that is the result do not bother about minus 2 okay.

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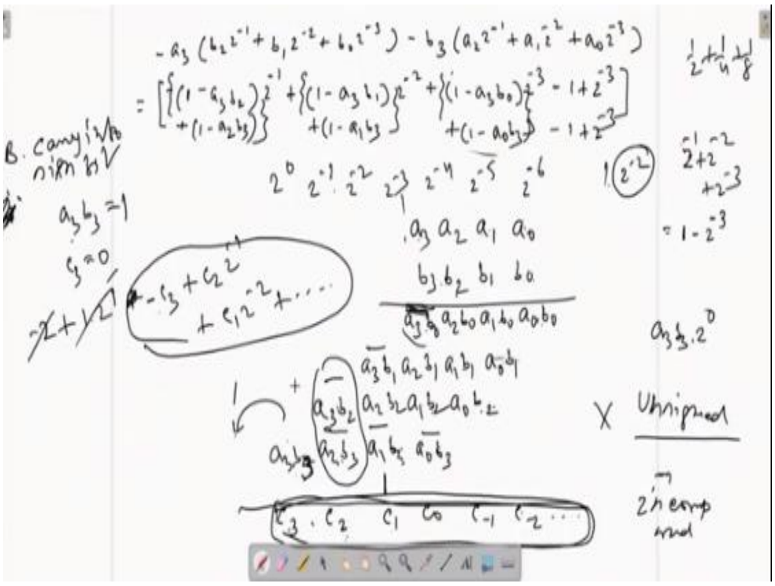

And last case forget this that is suppose a3 b3 is 1 at 1 is coming so if you add them that means c3 is 0 and 1 is going to this, but 1 is going to be this is 2 to the power 0 so next power is 2 to the power 1. So, we will have minus 2 plus 1 into 2 to the power 1 and then c3 0 so I can and c2 2 to the power c3 is 0 so I can also write minus c3 does not matter because c3 is 0 so I can write minus c3 okay because c3 is 0 and others are as it is and this two cancels.

So, again you are left with this where c3 is 0 this is a positive number okay which is nothing, but again the decimal value of this word under 2's complimentary system. So, moral of the story is if you carry out this table and all that do not bother about the minus 2 take the result as it is as a 2's compliment number and calculate it decimal value by the 2's compliment formula you get the correct result.

But here there is no scope of any overflow and all that that we have here already taken care. So it also has got an array architecture dependence graph and array this is given in PARIS's book (()))(39:04) system just see that okay. So, that is all for this Baugh Woolley Multiplier a very powerful and very you know practically used very successful multiplier used widely in practice. Okay I am nearing the end of this course I hope you enjoyed it maybe I will have another lecture session and then I will end it. Thank you very much.