VLSI Signal Processing

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Lecture – 31

Hardware Efficient, 3-Parallel FIR Filters

Well, in the last class we are considering parallel relation of course we started with this that if there is suppose some job which require some time say capital T that is its critical path is capital T and you make parallel copies J number of parallel copies then by properly parallelizing which is called unfolding your input, output speed can go up by factor of J that is input clock period and output clock period will go down by a factor of J okay.

But hardware goes up also by a factor of J then we said that if the process is nothing, but an FIR filter then using some DSP tricks particularly using something called Polyphase Decomposition of sequences we can restrict this increase in hardware somewhat that is it will not go up by a factor of J, it will of course go up, but by factor less than J. So, for that I consider 2 parallel structures that is J is equal to 2 that is it was unfolded by 2.

And we brought in also then Polyphase Decomposition of factor of 2 that was one example, but if you want to have further improvement here that is if you want to see that you parallelize it further okay to make it still faster.

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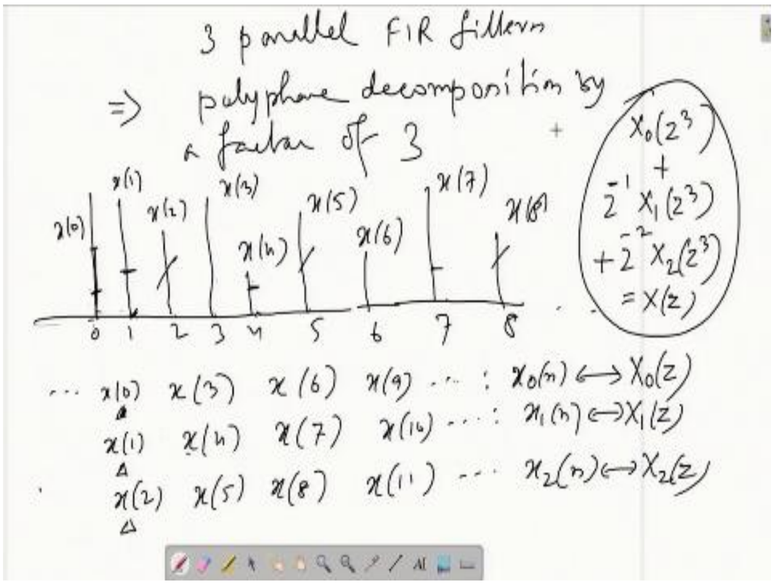

We will consider another example that is J is equal to 3 that is 3 parallel okay, and obviously we will consider Polyphase Decomposition by a factor of 3 or if it is 3 parallel it will be 3, if it is 4 parallel it will be 4 like that. We all know what is Polyphase Decomposition of a sequence, but quickly if it is a factor of 3 we can do a quick recap. Suppose, we have a sequence like this, suppose x of 0 then x of 1, x of 2, x of 3, x of 4 that is 0, 1, 2, 3, 4 x of 5, x of 8 dot, dot, dot, dot.

If you take (())(3:11) this guy then throw away the 2 intermediate samples take this guy okay and again so 0 then 3 then 6, 9 like that. So, you get basically decimated version decimated by a factor 3 you get a sequence x0 followed by x3 followed by x6 followed by x9 dot, dot, dot from this side. If you go to the left hand side it will be x minus 3 and so on and so forth. This will be called the 0th Polyphase factor that is capital 0 N dot, dot, dot, dot which is the 0th Polyphase factor x0 n and its z transform is X0 z.

Then if you take this guy 1 then go ahead by 3 4, then go ahead by 3 7 and put them side by side okay x1 let me put a notch here to indicate this is the time origin. So, again notch time origin x4, x7, x10 like that. This is called x1 n these are fast Polyphase factor. If I plug in the 0 that is if I plug two 0 in between x0, x3 again two 0 in between x3 and x6 and so on and so forth then it will be its Z transform will be capital X0 z to the power 3.

That is capital X0 z to the power m means between every pair of samples m minus 1 0. So, if it is z to the power 3 between every pair of samples two 0 that is here 0 here 0 x of 0, x of 3 then again here 0, here 0, x of 6, here 0, here 0, x of 9 so on and so forth similarly here, similarly here, but at the moment I am not considering the 0 so low clock rate sequence where (())(5:20) x0th sample I have x3 so this much is the sampling period 0 to 3, 3 to 6 likewise or 1 to 4, 4 to 7 okay low rate sequence.

The theory for all this had been done earlier this is just a quick recap and the last one is x2 started 2 go ahead by 3 5, 8 and like that x2, x5, x this is the second if Z transform is X2z. Obviously using this we can reconstruct the original one. First plug in the two 0s here so after x0 we have got two 0s then x3 then two 0s then x6 like that.

The corresponding z transform x0 z to the power 3 that is 3 minus 1 two 0s between every pair of samples. Every pair of samples difference between x0 and x3 there will be two 0s, between x3 to x6 two 0s so 0 here 0 here x3 0 here 0 here x6 like that. Then here also plug here between x1 to x4 two 0s, between x4 to x7 two 0s, x7 to x9 two 0s, so that will be x1 z to the power 3 then you shift to the right by 1 so that x1 comes at this position.

Right now it is at origin x1 comes here two 0s, then x4 comes here. So, they come in their respective places x1 comes back to its original place, x4 comes back to its original place, x7 comes back to its original place okay and then in between them two 0s and you add them all the theory was explained in the previous class and again here also putting two 0s between x2 and x5, between x5 to x8 between x8 to x11.

And shift them by 2, so x2 which is at origin now it will come at position number 2 and so on and so forth. So, everybody will be coming into their respective position shifting by 2 this is your Xz where you get back your original sequence if Z transform Xz is this okay. This is the factor of 3 Polyphase Decomposition. We will use it for 3 parallel FIR filters that is when you are unfolding your FIR filter by a factor of 3.

We will make clever use of a Polyphase Decomposition by a factor of 3 so that hardware does not go up by 3, but by a factor less than 3 quite less than 3. Okay this is what we are going to do.

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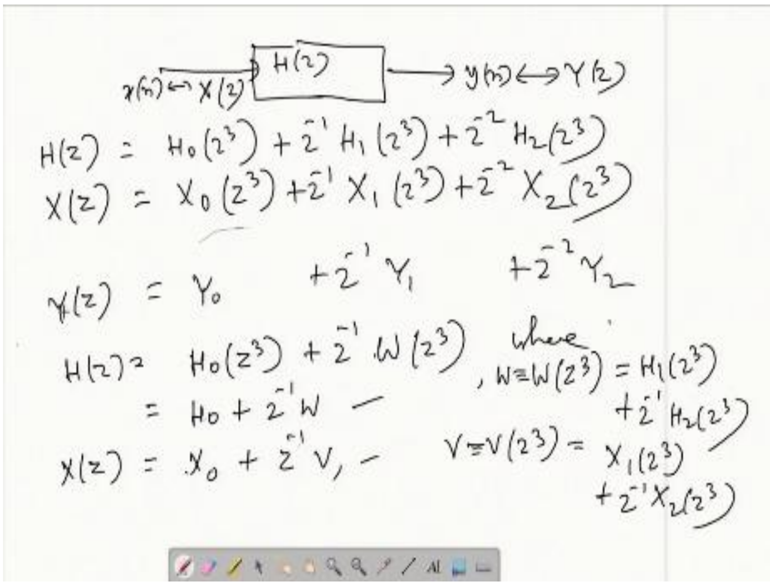

So, you basically have a FIR filter whose transform function is Hz, input is xn whose z transform is Xz, output yn whose z transform is Yz. So, if you go for factor of 3 Polyphase Decomposition for both Hz Xz and Yz so Hz is H0 z to the power 3 plus z inverse H1 z to the power 3 plus z inverse 2 H2 z to the power 3. Xz x0 z to the power 3 plus z inverse X1 z to the power 3 plus z inverse 2 X2 z to the power 3 okay.

Okay if I multiply the 2 I will get Yz and Yz should be of the form that is when I multiply the 2 first find out a polynomial in terms of z to the power 3 only that under this then find out terms from which z inverse can be taken common and remaining part is again a function of z to the power 3 that whole thing will come here then out of the remaining terms z to the power minus 2 should be common and remaining fellows should be function of z to the power 3 again so they will come here.

Thereby, I know I will find out what is Y0 z to the power 3, what is Y1 z to the power 3, what is Y2 z to the power 3. If I multiply Hz and Xz I will get like this. Let me write Hz as H0 z to the power 3 plus let me take z inverse common here so remaining part let me say V z to the power 3 or not V have a rotation for this z inverse I will call it W z to the power 3 where is nothing where W z to the power 3.

If I take z inverse common it will be H1 z to the power 3 plus z inverse H2 z to the power 3 okay. Now, one thing I have been having this bracket z to the power 3 coming everywhere and it is kind of you know I mean making things very congested. So, for the time being I might just drop it I mean call it H0 or I might call it W, H0 means H0 is a function of z to the power 3, W means W as a function z to the power 3, Y0 means Y0 as a function of z to the power 3.

Okay because every time if I write H0 then bracket z to the power 3, W bracket z to the power it will become very more clumsy. So, taking your permission I will just drop this so I will just write like H0 plus z inverse W which is this. Similarly, in Xz I take X0 common as X0 z to the power 3 let me straight away write X0 because I told you whenever I take X0 actually I write I mean X0 bracket z to the power 3.

I am just dropping it for the sake of you know I mean convenience so that there is no congestion, there is no clumsiness and again Z to the power z inverse I take the remaining part common that part is called V where V means v is equivalent to V z to the power 3 which is actually if you take this common okay this is one, this is one Hz is H0 plus z inverse W, X is X0 plus z inverse V and Hz into Xz is Yz. Yz is Y0 you can even write as this way, you can erase this.

You can call it Y0, Y1, Y2 means Y0 as a function of z to the power 3, Y1 as a function of z to the power 3, Y2 as the function of z to the power 3 just I am not writing that okay it is in our mind. So, H into X is Y.

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$$Y(z) = H(z)\,X(z)$$
$$= H_0 X_0 + z^{-2} W V + z^{-1}\left[(H_0 + W)(X_0 + V) - H_0 X_0 - W V\right]$$

$$W V = \left(X_1 + z^{-1} X_2\right)\left(H_1 + z^{-1} H_2\right)$$
$$= H_1 X_1 + z^{-2} H_2 X_2 + z^{-1}\left[(X_1 + X_2)(H_1 + H_2) - H_1 X_1 - H_2 X_2\right]$$

Now, if you do that I will do that right here. So, Yz will be H into Hz into Xz okay and if you substitute Hz into Xz it will be H0 X0 okay as a function of z to the power 3 then z inverse 1, z inverse 1 okay z inverse 2 W V and the two cross terms, but we will write it in this manner H0 X0 plus z inverse 2 W V and when it comes to cross terms we will not carry out the product X0 W or H0 V.

We will follow the same trick which we did earlier that is we will avoid this multiplication X0 W or H0 V by introducing more addition we will just take z inverse common take H0 plus W H0 plus W X0 plus V and from this subtract this H0 X0 which is already there so no need to compute it and WV which is already there so that will give you the cross terms W X0 H0 V okay WX0 H0 V with z inverse common.

So only this product minus this product and this product so H0 X0 already outside, already obtained WV already obtained okay so I do not need to re-compute this that is why I am writing in this form. This is the form which I told you is used again and again in this, this is a trick we use to minimize number of multiplication so products. So, this is what I will write where this is a bit messy, but you can cross check and verify that what I am doing is correct.

Now, WV what is WV? W was one of them is X1 plus z inverse X2 okay that is V, other one is H1 plus z inverse H2 that is W, this is W, this is V okay. This again we write in this way X1 H1 that is H1 X1 and another term z inverse 2 H2 X2 or when it comes to cross terms take the z inverse common I will write again in a similar manner X1 plus X2 H1 plus H2 minus the same H1 X1 same H2 X2.

These are already calculated here so need to re-compute okay so that (())(17:34) computation like above this how I find WV so this WV has to be put in here. If I put in the WV here what I get is we will just substitute this here what I get I will write in the fresh page in fact I am just writing that from the book by (())(17:54) but it is just a substitution there, nothing conceptual and just rearrangement of terms.

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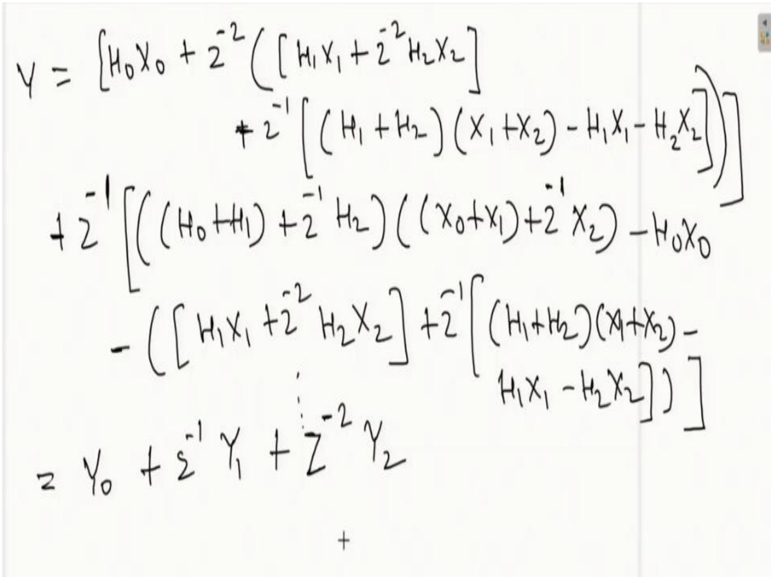

So, what we will get there is this if we substitute that Y will be this bracket ends and this also ends minus it is just substitution (())(19:12) not minus plus z inverse 1 it is direct substitution only nothing else okay and then we simplify it so that we finally after some simplification which are given in (())(21:10). Finally everything you can write as Y0 plus z inverse 1 Y1 plus z inverse 2 Y2 okay where.

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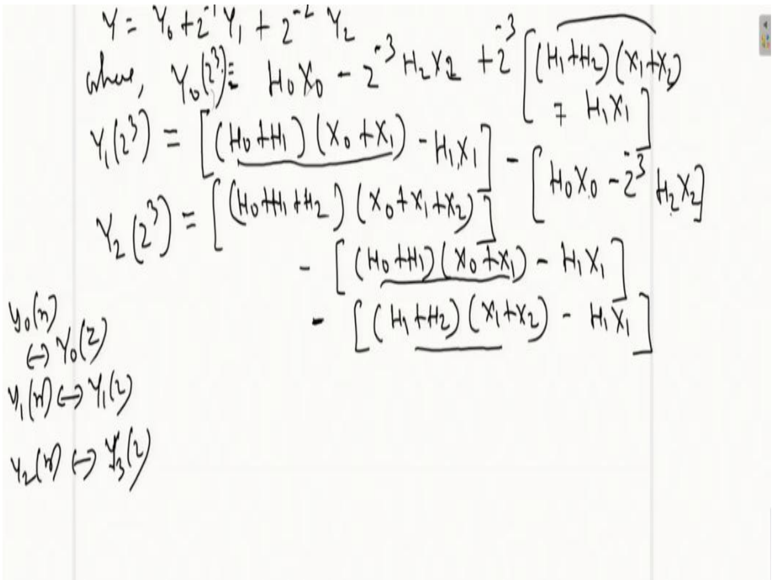

Let me write in the next page. So, again I rewrite Y is Y0 plus z inverse Y1, Y0 is a function of z to the power 3, Y1 is a function of z to the power 3, Y2 is a function of z to the power 3 sorry okay where from those (())(22:16) expression Y0 as a function of z to the power 3 it is nothing, this comes by calculation it is given in (())(22:28) so I am not doing this (())(22:30) algebra.

These are all function of z to the power 3 okay this H0 is a function of z to the power 3, X0 is a function of z to the power 3, actually this is nothing but a function of z to the power 3 all these are function of z to the power 3 then Y1 that is a function of z to the power 3 will be H0 is a function of z to the power 3, H1 is a function of z to the power 3 same everywhere X0, X1 all and last one a bit complicated, but you can see one thing.

We can now unscramble things there is one term H0 X0 present here H2 X2 H1 X1 H1 X1comes here also H0 X0 again comes here, H2 X2 again comes here H1 X1 comes here, H1 X1 comes here so on and so forth. So, these terms though used in many places we do not have to recalculate again and again H0 X0 is nothing but passing the Polyphase component X0 n through a filter H0 n.

Now, right now what we will do is this instead of obtaining this as a function of z to the power 3 we will write them as a function of Z, remember if Y0 z to the power 3 means between every pair of samples of this sequence small y 0 n there will be two 0s that is z to the power 3, but suppose we consider only Y0 n whose z transform is now Y0 z. Similarly, Y1 n and Y2 n, Y3 z.

So, now it will be everything will be function of just Z, which means what is H0 X0 I will pass the polyphase sequence x0 n small x0 n through a filter, FIR filter of impulse response small h0 n. Okay which is the 0th and polyphase component of the original Hn similarly for H1 X1, similarly for H2 X2 whenever you have got this term H0 plus H1 basically it is the new filter whose impulse response will be what summation of the two Polyphase components small h 0n and small h1 n.

Okay (())(27:25) does not change the support that is the position where H0 n and H1n located they remain same just they get added. For the new filter before processing the input I will take the 0th Polyphase component and first Polyphase component add them to create a new sequence, pass it through this filter so on and so forth that will be the philosophy, but we will be reusing terms that is the advantage.

For instance you see H0 plus H1 into X0 plus X1 I am sure it will be occurring somewhere like it is occurring here again. So, I will be doing this calculation only once though it is occurring twice. Similarly, this term H1 plus H2 which is a filter its processing signal X1 plus X2 this will be coming here also. So, I will not be doing it again and again I will be just taking common and likewise, and this will give rise to the simplification. So, this part I will do in the next class, next section. Thank you.