VLSI Signal Processing
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Lecture 16
Loop Unfolding

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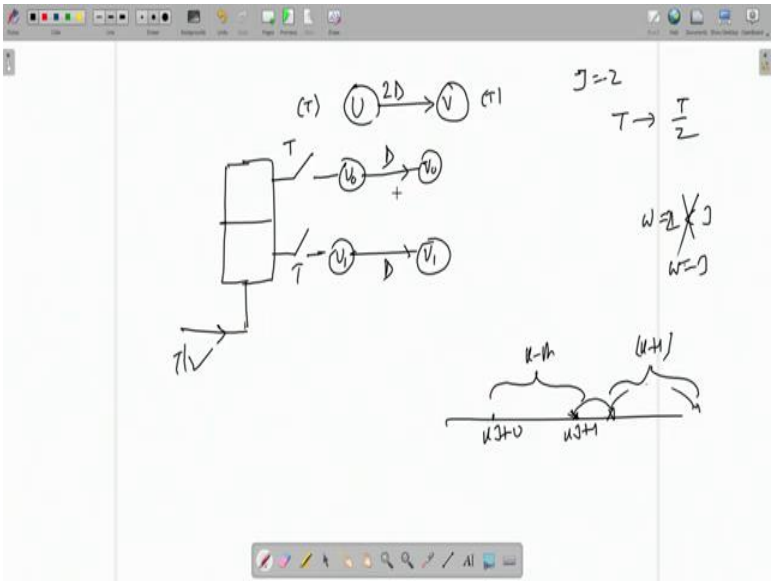

Now, with all this knowledge let us again go back to that IIR filter and then apply this formula and say see that we get the same structure which we drive their from first principles.

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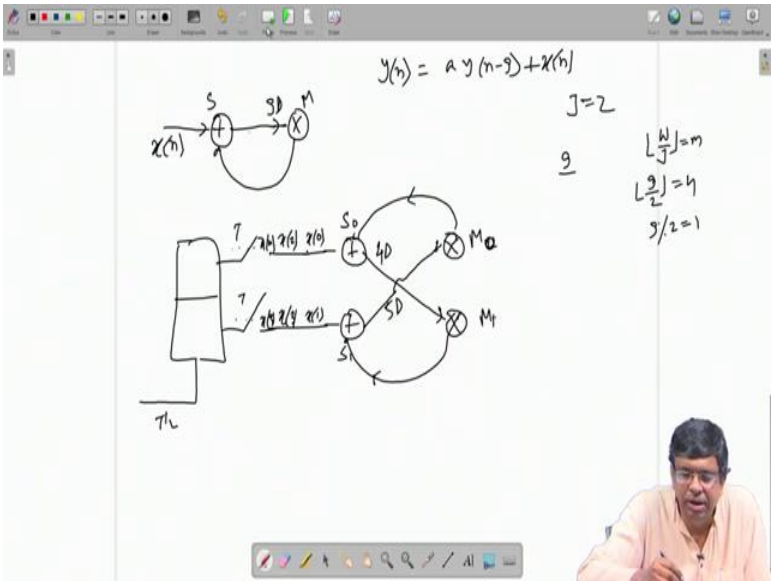

So, originally we are giving these equations is that previous example ay n minus 9 plus xn this was the DFJ this edge has 9 delay we are taking a tab from here for yn, but that I am not showing here where is that something this tab only taken to outside that is not a part of DFJ actually. So, I am not showing there is built in multiplier.

So, yn minus 9 comes multiplied by built in coast at A goes back its array with and xn new yn form. So, one node U this added as an node multiplier which is a V, U to V one edge up or up one edge from U to V another from V to U, now suppose I want to unfold by J equal to 2 as I did that time.

So, as before there will be one buffer here the speed will be T by 2. So, here we will have data you know x0 x1, so x0 that time x1 then this is followed by x2 followed by x3 followed by x4 followed by x5 dot dot dot dot dot dot T, T it is T by 2. Now, this node U is at. So, they will copy two times another will take this another will take this that we have the basic upon folding then concededly the upper edge.

I have this multiple also will be copied two times consider this edge first 9 how much is the delay W there is 9. So, 9 divided by W by J (())(02:35) cossent to was W by J that was m in one direction m and other direction m plus 1. So, 9 by 2 cossent is 4 and 9 by 2 remainder 1. So, this will go to if you call it, if give it an S to this node and m for multiply to this node this is your S 0 this is your S 1, M0, M1.

So, S 0, 0 will go to M1 because remainder is 1 from 0 th it was going to R th R was the remainder, remainder is 1. So, it will go to the last fellow only and how many delay 4. Now, it will go to the upward direction because I have already got to the bottom most 1 extra delay M plus 1. So, it will become 5, so 4D 5D and then from M to S there is no delay. So, M0 will go to S0 no delay M1 will go to S1 no delay. So, this will go here this will go here and this is what we have derive that time. You can try with the J equal to 3, 4 anything like that and get start that.

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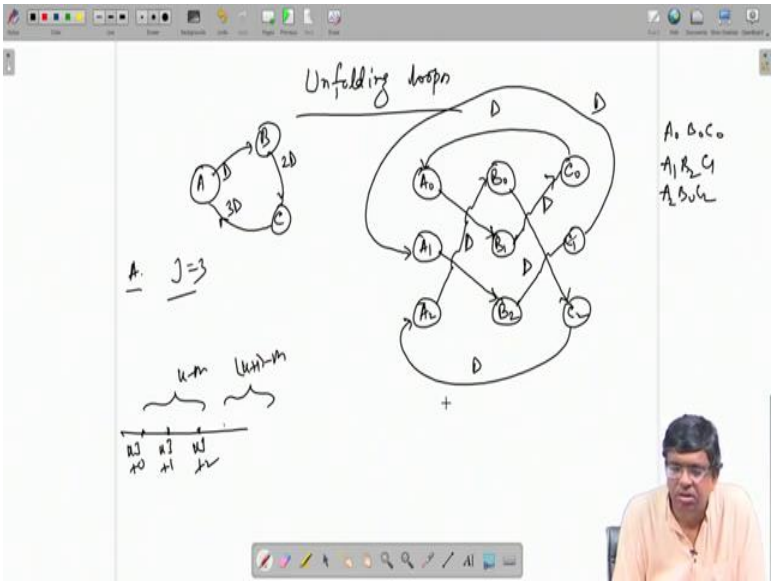

I now move to the next topic in this area of unfolding, there is unfolding loops we explain some peculiar things that happens to you when you unfold loops. I take an example you have given say A, B, C three nodes A to B, B 2D 3D first is A you take J is equal to 3 there is unfold by 3 that means A will be copied 3 times A0 A1 A2. B will be copied three times B0, B1, B2. C will be copied three times C0, C1, C2.

So, now we know very well I mean if you have the timing diagram in mind A0 index 0 we delay y input delay y 1. So, that will go to from 0 to 1 so S 0 will go to B1 it would have (())(05:44) supplies the formula of cossent and remainder from the timing diagram only you can understand that if you take the K th system clock the 3 points because 3 J is 3 means three data points are paralyzed. So, KJ plus 0, KJ plus 1, KJ plus 2. So, three points are paralyzed this will go to A0 this is will go to A1 this will go to A2. A0 will after one input cycle delay it will hit here.

So, 1 and which category A goes to B. So, it will be B1 under the same system clock. So, no system delays. So, A0 goes to B1, A1 here AJ plus 1 it will also if you go to the right by 1 delay you hit here. So, index 1 to index 2 that is why A1 to B2 same delay that is 0 delay another sees and if you move to the next starting from here if you go you are here.

So, one systems delay an index 2 to index 0. So, index 2 to index 0 that is what I have done in the general case M M M and M plus 1 M plus 1. So, this has a 1 delay and is a routine stuff down from B to C B0 but now 2 delays. So, 1 to index 0 to index 2 but delay free and then this will go

to the next cycle 1 to 2 and 2 to 0. So, 1 will go to C1 B2 C3 will go here 1 delay and 2 will go to the next one from 2 to index 0 and then index 0 to index 1 with total delays is 2 here. So, this will go to 1 delay this a structure and then C to A I have got three delays C to A.

So, KJ plus 0 if I go to right by three 1 2 3. So, I hit the next starting index of the next clock. So, 0 to 0 one system delays. So, C 0 to A 0 because C to A is the direction here, one system clock delay then from 1 if you go to the right 1 to 2, 2 to 0, 0 to 1. So, C 1 to and these at case you know where W equal to J.

So, parallel lines horizontal line all system delay and then C to will similarly go to A2. Now, you see one thing do we have any loops formed let us see we start at A category. So, A0, A0 to B1. So, one A category one B category one C category and then from C0 I come back to A0. So, this itself is a loop A0, B0, C0 similarly A1, B2, C1 and I come back to A1.

So, A I have got one loop A0, B0, C0 another A1, B1, for A1, B2, C1 next one you see A2, B0, C2 that is another loop the three loops. Now, if I do the same exercise, if I do the same exercise and you can see one thing number of delay count has gone down like this edge is a delay free where A to B already had one delay but now their delay free.

Then I have a 3D here from C to A now here only one D, one D, one D but that is with respect to original input clock, clock was faster three times period was shorter here it is wider clock because clock is lower. So, number of clock number of delay count has gone down by in terms of time it has not come down because time as expanded clock period has expanded anyway. So, this for J equal to 3. Now, if I just change the J from 3 to 4 you will see some new changes.

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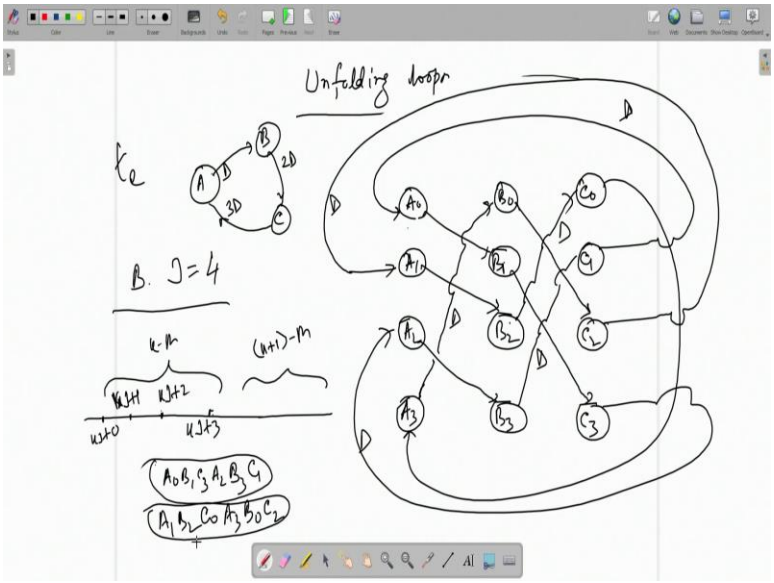

So, that is why we will do, now case B is J equal to 4 case B, J equal to 4. So, obviously each node will be copied four times. So, I will have A0, A1, A2, A3, B0, B1, B2, B3, C0, C1, C2, C3. Timing diagram wise this K th just KJ plus 0, four points. So, KJ plus 1, KJ plus 2 and KJ plus 3. So, W here is 1 less than J obviously it will lead to delay free edges.

So, we have all of them D 2D or 3D all figure are less than J. So, every edge will give rise to, every original edge will give rise to some new delay free edges anyway. So, if we started KJ plus 0 that is A0 if you just up to count to the right by one, we got one original delay. So, 0 will take you here, so A0 to B1 under the same system umbrella.

So, no system delay, 0 delay then A, A1 from here if you go to the right by 1, 1 to 2 under the same umbrella no delay and 2 to 3 under the (())(11:53) no delay... under the same umbrella no delay and from KJ plus 3 if you go to the right by 1 you hit the next clock. So, new system cycle means 1 delay at index 3 to index 0.

Now, same thing routine stuff you know I loop from B to C delay is 2 here. So, 0 will go to 2 no delay, 1 will go to 3 no delay, 2 will go where 1 2. So, it will go to the 0 th index starting index starting of the next clock with 1 delay and this will also similarly go to C1 with 1 delay.

And now lastly C to A 3D. So, C0 from here if I start counting 1, 2, 3. So, 0 will go to 3 here no delay C1 will go to 1 to 2 to 3 and 3 to 0 again next cycle. So, one delay and go to index 0. So,

C1 will go to one delay C2 will go to A1 because 1, 2, 3 next cycle. So, 1 delay and C3 will go where? 1, 2, 3 C3 will go to A2 one delay.

Now, let us see what are the loops formed after unfolding. So, start at A0 say A category, so A0, B1, C2 from C2, do I come back to A1? Last time I gave but this time A0, B1, C2 here I move further I am coming to A1 not A0 startedA0, B1, C2 that I should have come back to A0 then allowed loop would have been formed but it is not coming and A0. Now, I come somewhere else A1.

So, I proceed then A1 if I from A1 if I go I am here B2 to then from B2 if I go for the C0 from C0 if I go one minute there may be some mistake here, A0, B1, C2. This is sorry I wrote wrongly A0, B1, A0, B1. B1 goes to C3 not C that it has a problem C3 from C3 if I go further I do not reach A0 I reach A2. So, find A2 then from A to B3 and B3 to C1 from C1 I come back to A0.

So, this is one loop you see A category a process or node is executed twice. So, if A takes times say TA, TA plus TA 2 TA, B also executed twice. So, if B category takes time TB, TB plus to B 2B similarly twice TC, are we? So, it is a bigger loop or you earlier I have got only 1A, 1B, 1C and loop was form now I have got A coming twice, B coming twice, C coming twice.

So, loop competition time here if it was capital T now it is 2T or T if it is TL it is 2TL. There is one loop and now started A1, A1 B2 C0, A1 B2 C2, then A3 B0 C2 and then back to A1 so this is another loop. So, I have got two loops now earlier I have got 3. Now, I have got number of loops placed and (())(16:43) of the same node within a loop is (())(16:47).

Now, A is coming twice, B is coming twice, C coming twice like that. Remember the two loops cannot have a common node. Because you have start at A0...if you hit I mean, if you start at 1 index and you go to another index you cannot reach the same index by starting from some other point I will discuss this in detail.

So, there is some problems unfolding, you know number of loops generated and now total competition per loop they all depend on not only the delays in the edges, but depend a lot on the J factor. So, we will now try to generate the theory here to understand to tell the directly about the how many loops will be formed. What will be the composition then per loop all this after unfolding? I just wanted to, for this I just wanted to do one thing.

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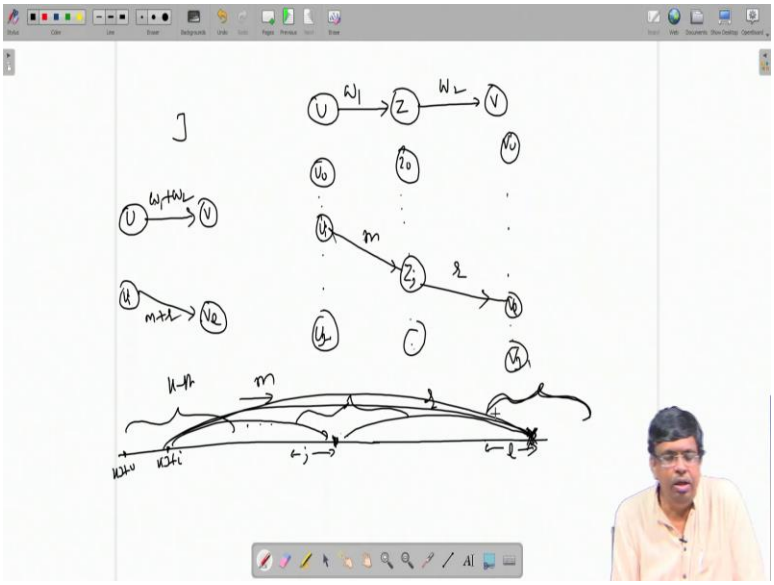

So, U one thing before I use it. Suppose I have got U going to some intermediate node the Z and then V, this has W1 this has W2 you unfold by factor J. So, we will have U0 dot dot dot UI dot dot dot U J minus 1, Z 0 dot dot dot maybe Z small j dot dot dot, there is Vl dot dot dot V J minus 1 you unfold, you just take this factor separately use that unfolding formula and of course and remainder of his formula.

So, you can from U0 you know where to go and with how much system delay and all that. So, complete similarly from U1 where have to go from Uy where to go you can complete just take any particular case. So, UI, UI supposed takes you to ZJ with may be some delay L and again Z to V this sector also you do separately. So, Z0 will take me to some V Z1 will take you to some V with some system delay and like that. So, Z0 will also take you to someplace maybe let me I have used to up L here. So, I cannot use L again.

So, let me change it to some other index maybe M sorry maybe M because already L is used up. May be ZJ, when you start it will takes you to VL with r number of delays. So, if you consider this total age total path then path is the combination of edges UI will take you to VL with total delay M plus R.

Now, suppose I just have a question that I am not interested to find out the details like this. I just wanted to know one thing that if I started UI here UI where is VI go to? If I started UI which VI go to. So, and with how many system delay, with how many system delay? Then my claim is, if I

have just instead of Z if I just have U to V with same delay W1 plus W2, W1 plus W2 forget to V and then UI will go to the same VL with delay same system delay M plus R then we explain suppose a timing diagram, there is a K th KJ plus 0 dot dot dot here you have got KJ plus I. So, this data will be giving to UI.

Originally, originally you go right by W1 amount of delay 1, 2, 3 dot dot dot wherever and you hit some other umbrella another cycle or maybe same cycle depending on the delay here. So, this will require M number... you have to go to the right by M system cycle.

So, it will be delayed by M system cycle and this much may be J is offset remainder will be J that is why you started I U go to J th from here if you started you know start counting W2. So, 1, 2, 3 dot dot dot maybe you will hit here to these took to queue here this might take somewhere here. Here we have after r number of systems cycle delay you are here and this much is L.

My question is, if I did not having this Z if I had directly U directly V or total delay is same W1 and W2. So, W1 plus W2 and then if I unfold then also if I started KJ plus I, I will hit the same point here and we under the same system cycle. That is with the same number of delays. Why? Because W1 plus W2. So, I have to go to the right by W1 plus W2 original cycle.

So, first you count W1 dot dot dot or hit here then another W2. So, if you count W1 and W2 plus here you will go to the same fellow only under the same system cycle and the same point. So, L will be L and numbers of system cycle delay required to reach this block, reach this umbrella that will be same as M plus R whether you go by hopping like KJ plus I then W1 you are here and then another W2 here if you here, or you go to the right by to the total you know W1 plus W2 hit here, this system umbrella, system clock will be remain same in both cases and at the point L also will remain same.

So, if it is to the right of this original one by m plus r. Now, once it will be to the right of with this by m plus r and if this is offset this remainder is L the index is L that time also you will have L that means in this case also UI will go to VL same I same L delay will be m plus r. Then if I am not bothered about the actual thing, but I just wanted to do some calculation then if I do unfolding started UI which VI go to you can forget Z just make an edge with the same total delay W1 plus W2 unfold wherever you land, that will be the correct answer. So, if UI texted to VL

original also will take you to VL like this total delay will be m plus r, alright. This is one property I will be making use of.

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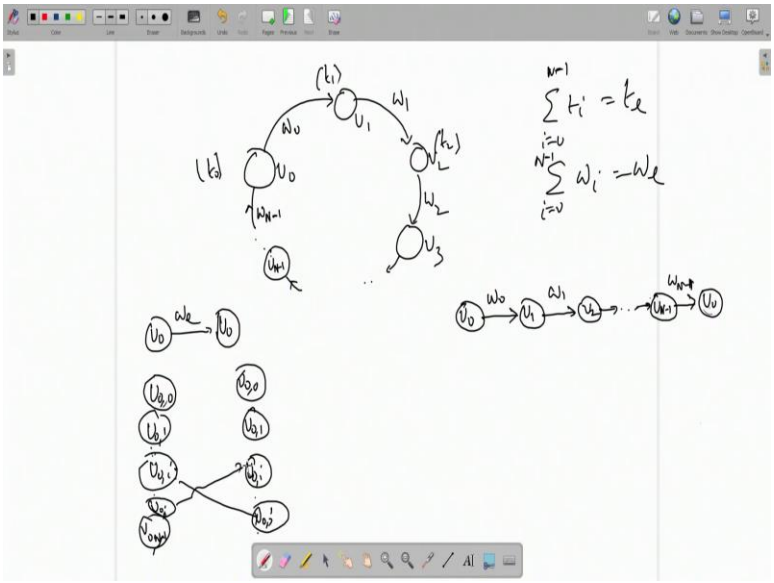

Now, consider a loop you are giving total time this is taking time T0, U1, V2 like that. So, total time in the loop ti. I equal to 02 how many I have? N minus 1, 01... this is at loop competition time tl and total delay in the loop W0 W1 all that these figures are important this called loop competition time.

Now, for convenient cycle redraw this figure in this way also U0 instead followed by U1 followed by U2 and then W0 delay W1 delay the last one is U N minus 1 and then it should come back here, but I can for analysis purpose only not for actual implementation I can take it to again the same node U0 but written separately here that is will be another way of showing loop you start at U0 go up to U N minus 1 come back to U0 but U0 is shown separately and here you have got W N minus 1. So, this also another way of showing loop you because the two terminal nodes are same.

Now, suppose I have this issue that I unfold all of them by J I want to see that if I started U0 is I th copy? Which copy of this U0 I got, to answer that I told you intermediate nodes I can forget I can just consider as I did you the previous example, there was a J th intermediate node I could

forget it, I could take the two terminal nodes U and V here the two terminal nodes are U0 and U0 and total delay will be W0 W1 up to WN minus 1 that is WL.

Now, here if I unfold them maybe I have got in fact if you excuse me. I will rather a row case. Suppose you have got U0 0 U0 is from this node U0 1 dot dot dot U0i and here also U0 0 U0 1 U0i dot dot dot. Now, if you are starting U0i which of these U0, 0 U 0 1 and all that I will go too. So, this I have to just unfold this only, I can ignore the intermediate nodes.

I can add up all the delays. So, they become WL and then if you unfold by the same factor, I will get the correct answer that I have already established in my previous page. So, if suppose I start here and it takes me here that means from the i th copy I may coming back to the i th copy then a loop will be formed.

But it may not be so, it may take me somewhere else maybe here U0J then U0J if it takes me to Ui copy then I started U0i go to U0J with some system delay as coming by the unfolding formula the same U0J if I start I will come back to the same U0i they are sound separately here, but they are basically same this U0 and U0 they are different.

They are sound separately for applying the analysis formula and all that here, but actually they are same. So, U0i takes me to U0J, U0J takes me to the same U0I. So, a loop will be formed but U0i take me to somewhere else that is from somewhere else I can come back to it U0i. So, it can be multiple loops.

So, the next class I will expand on this and I will try to develop the theory with the number of loops if it is there multiple loops how many set parallel multiple loops will formed what will be the new loop competition time how many delay parts such parallel loops, what will be the new iteration bound all that. These are important design figures. So, that is all for today. Thank you very much.