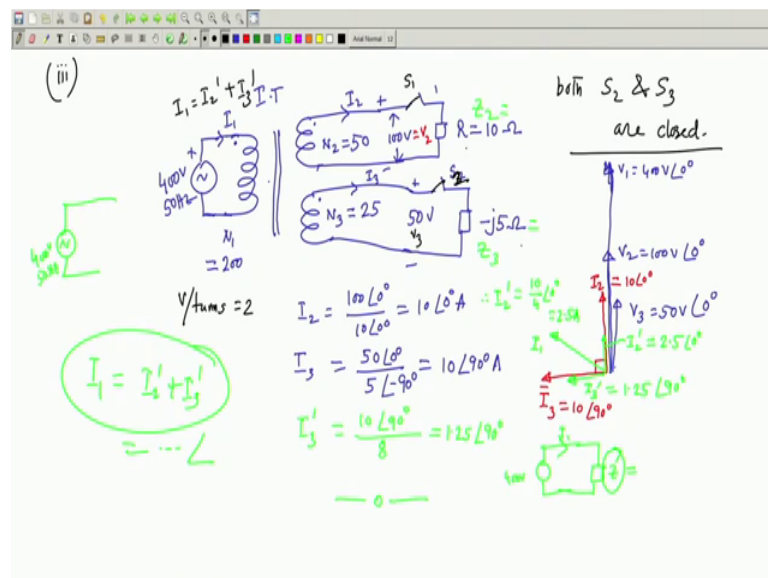


Electrical Machines - I
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Lecture - 08
Modelling of Practical Transformer - I

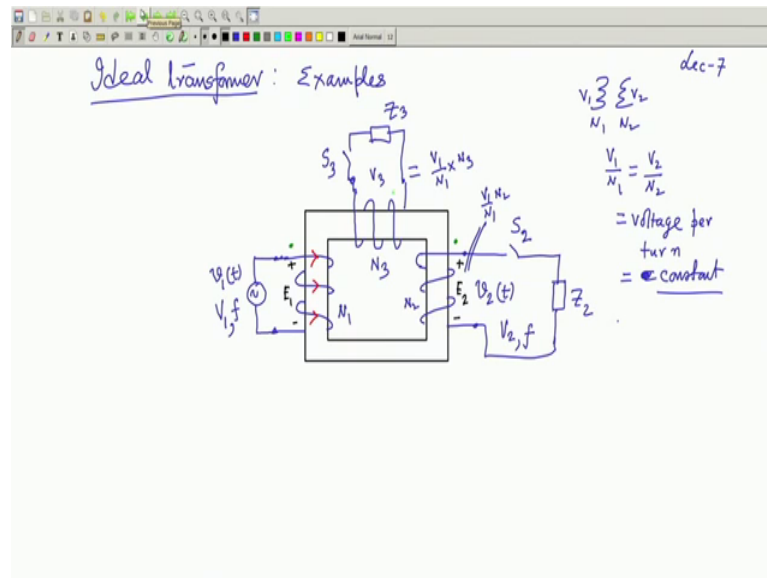
Welcome to this lecture number 8 on Electrical Machines I.

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And you remember that in my last lecture; we were considering a problem interesting problem that is suppose a transformer has got 3 coils wound on it that is like this; this was the transformer core.

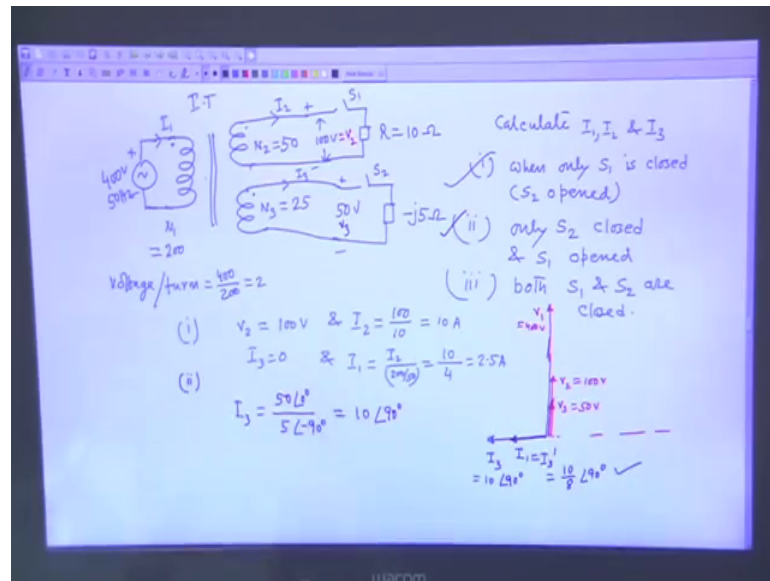
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So, one common magnetic circuit what which 3 coils have been wound; then I told volt per turn is a useful concept; in such cases there are two secondary's; one primary. If you apply some voltage V_1 number of turns N_1 and it is ideal transformer; no leakage flux no winding resistance; therefore, V_1 and E_1 are same.

Then what you do is you calculate voltage per turn V_1 by N_1 and you can quickly calculate if that number is known V_1 by N_1 into N_3 will give you V_3 ; V_1 by N_1 into N_2 will give you V_2 . And our problem was we know if there is only one secondary if I close this what will be the current and how to draw phasor diagram. But here the problem is when both the switches will be closed, both the secondary's will supply their respective loads Z_2 and Z_3 how to calculate the primary current that I told you.

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In a simplified way, this previous thing could be drawn; instead of drawing elaborate diagram involving core; this two lines will indicate core ideal; transformer it is none the less and then I was considering this problem.

So, part 1 and part 2 of the problem were simple, but part 3 was a little bit involved in the sense that both the switches are closed how to calculate the current. Then what I told that if V_1 is known then V_2 , V_3 all the voltages will be co phasor and they are all shown on the vertical lines. Then if you know V_2 ; then you can calculate I_2 , impedance is known it can be position.

Similarly V_3 ; I_3 can be known and each of this current I_2 will have its reflected component I_2' dashed; I_3 will have its reflected component I_3' dashed. So, I_1 ; I_1 will be actually comprise of I_2' dashed plus I_3' dashed; that is what exactly we did. While calculating the reflected current be careful about the turns ratio for this it is 200 by 50 with that number you have to divide. Similarly, for this it will be 200 by 25 we have to divide here; like that you have solve the problem.

Now, one thing of course, should not be; the same problem could be done in a much simpler way by making the; making use of the energy balance equation or power balance equation on both the sides ok.

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The slide contains the following content:

- Top Left:** A transformer circuit diagram with primary turns N_1 , secondary turns N_2 and N_3 , and loads Z_2 and Z_3 . The primary current is I_1 , secondary currents are I_2 and I_3 , and the primary voltage is V .
- Top Right:** A simple AC circuit diagram with a voltage source V , current I , and a load impedance Z .
- Bottom Left:** A phasor diagram showing voltage \vec{V} and current \vec{I} relative to a reference axis. The voltage angle is α and the current angle is $\alpha - \beta$. The magnitude of the current is $I = I \angle \beta$.
- Bottom Right:** Equations for complex and real power:

$$P = VI \cos(\alpha - \beta)$$

$$\text{Re}(\vec{V}\vec{I}^*) = VI \cos(\alpha - \beta)$$

$$\neq VI \cos(\alpha + \beta)$$

You will know that in any AC circuit suppose air is the source ok; that is power. From power also it is much more easier and you need not go to the; I mean intricacies of transformer, flux is constant these that like that. What I am telling is suppose this is V ; the same problem here is your transformer ideal transformer primary.

And there are two secondary's and there are impedance is connected here; in general I am telling now and there is another impedance connected here. Suppose this is z_2 , this is z_3 and I know the number of turns N_1 , N_2 and N_3 . And suppose these are the dots of the terminals; then what you do is this and my problem is to calculate this currents I_2 then I_3 and the current turn from the supply. This is the problem and this is the transformer code which is ideal transformer mind you ideal.

Now, in this case I can also do it in this way; see this is the two points; I have injected power into the circuit. There is no other source which is pumping power into the circuit and these are the things; two loads; I am consuming power on the secondary side. Therefore, the power drawn from the supply must balance the some of the powers given to z_2 and z_3 ; that is the very simple way of looking at it.

And you know that in AC circuit suppose we have a source and you have an impedance here; z a source is there V and this is suppose the current the deduction of the current I have assumed. Then I will write it like this; power complex power, delivered by the source is given by $V I^*$.

See in language I have written complex power delivered; delivered is the keyword because here is a source through the positive terminal of the voltage; current is living. So, like a battery; so it is really delivering power and take voltage phasor and current phasor multiply with I^* . And you must be knowing, but still I am very quickly telling; it is not that it is $V I$ do not and this is usually denoted by the letter S . Real part of heat will give you real power, imaginary part of heat will give you reactive power.

But only thing is do not just multiply $V I$ take; the real part, it is simply because of the fact that in general voltage phasor could be $V \alpha$ and current phasor could be $I \beta$ and that is the reference phasor; reference phasor. So, this angle is α suppose this angle is β therefore, real power as you know it should be $V I \cos(\alpha - \beta)$ is not; this angle is $\alpha - \beta$. So, real power is $V I \cos(\alpha - \beta)$; that is the reason you should do like this and reactive power will be $V I \sin(\alpha - \beta)$.

So, if you simply; so V is $V \alpha$, this is $V I$ mean I am omitting this bar all the time, but this is the phasors. Therefore, if you simply do $V I$ in this case; it will be $V I$ that is if you simply multiply this two complex number; it will be $V I$, angle of $\alpha + \beta$ it will become. And a real part of heat is not real power that is the reason for general case; you do not know voltage might have also angle, current might have also some angle with respect to some other reference phasor.

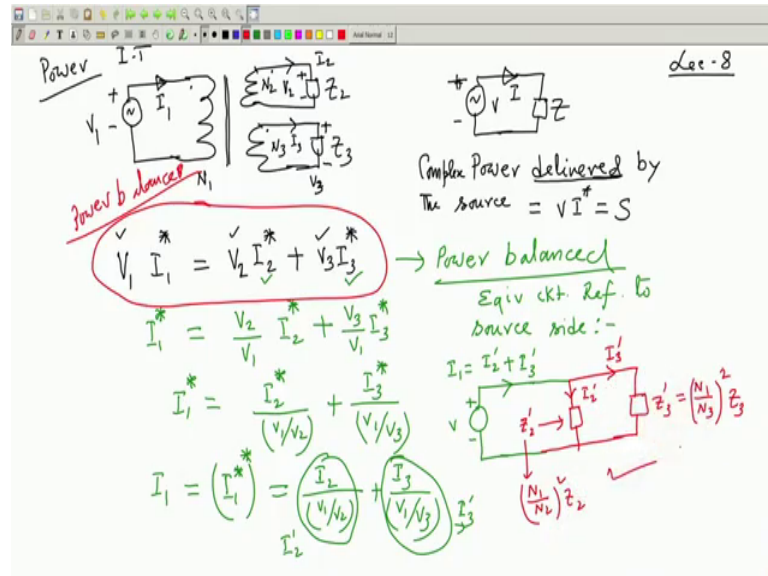
Therefore, more correct thing will be; so real part of this is not equal to $V I \cos(\alpha - \beta)$. Therefore, you should just remember this; anyway so this is just a side remarks of this one. Therefore, $V I^*$ you have to take mind you V and I ; although I am not putting bars to they are complex numbers and this gives you the complex power delivered by the source.

Now, after calculating this if both real parts and imaginary parts become positive numbers; then I will tell real power is delivered by the source; reactive power is delivered by the source. Because in any circuit you know current deduction is your choice and if even if you have to chosen it wrongly then the it will be reflected in this equation.

It might so happen $V I^*$ after you calculate; it will give you real part becomes becoming negative. So, I should not be surprise then I am telling complex power delivered by the source its some minus real power 100; means it is really absorbing real

power, you know all these things. So, this is the thing equation; I will be using here to balance the power and I will demand that.

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See what is the power delivered by the source; it is $V_1 I_1^*$; I am so sorry; this is $I_1 I_1^*$; $I_1 I_1^*$ is the power delivered by the source to the system. And this must be equal to see to the load if you see this is the voltage drop because I_2 deduction, I have assumed; I cannot then say and voltage drop then has to be upper one plus, lower one minus.

You can select the deduction of I_2 in whichever way you like, but once you select that deduction of I_2 ; remember the voltage drop polarity across that element is decided then plus minus I can do. Similarly, this fellow voltage drop across Z_3 is I_3 ; Z_3 upper one plus lower one minus. And through the dots current come out that way in the transformer you will assume, so, that MMF will be compensated by the current drawn by the primary through the dot that is the whole implication.

But what I am telling this must be true; $V_1 I_1^*$ must be equal to this voltage is suppose V_2 ; this voltage is V_2 this side upper one plus lower one minus. So, power delivered; power absorbed by Z_2 is $V_2 I_2^*$ plus $V_3 I_3^*$; this is $V_3 I_3^*$ plus minus $V_2 I_2^*$ very simple; it has to be from power balance. Even if you do not go into the complexity of transformer how flux remain same, but this has to be if it works like that you cannot do. So, in this case I know V_1 , V_2 and V_3 because number of turns are

known, those voltages are known. What else I know? I know I_2 , I know I_3 therefore, I_1 star can be calculated.

Because my problem is what is the current drawn from the supply it will be equal to V_2 by V_1 is not into I_2 star. If I make any mistake please point it out to; V_3 by V_1 into I_3 star, this will be the thing. But V_2 , V_1 , V_3 , V_1 all are co phasor numbers; so this will give you the turns ratio that is it is essentially another way of showing that reflected current of this one is V_1 by V_2 ; bring it down this is nothing, but N_1 by N_2 . Similarly this is I_3 star by V_1 by V_3 that is N_1 by N_3 ; I_2 by a is this one.

So, I will get I_1 star; is not, but I want to know I_1 ok; if you have calculated I_1 star once again take conjugate of both the sides. So, I_1 then will be equal to I_1 star; take once again conjugate and the conjugate of the respective terms it will simply become V_1 by V_2 plus I_3 ; another conjugate I am taking; so it is becoming $I_3 V_1$ by V_3 .

Mind you this I_1 you will get magnitude an angle therefore, what is the effective impedance seen by the source it will be simply V_1 by I_1 and so on; everything is known now. Therefore, although it looks like if there are several coils not even 2; there may be 3, 4 coils on the secondary side and each one of those coils are supplying impedances; here is a simple way of calculating the current distributions in the transformer. And also the power factor at which the transformer will be operating and still we are in ideal transformer.

So, ideal transformer otherwise is so simple to handle with that is what I want to point out. And in one stroke you will get both to real and reactive power supplied by the source and this one. So, it is a very useful method; if you know the transformers are ideal, one way of calculating the currents is to apply this power balance. Mind you power balance means both reactive and real power should be balanced like that.

So, here is a nicer way of handling the situation; you have understood this. Only thing is if I want to draw the equivalent circuit; refer to the primary side what it will be? This equation will help me out equivalent circuit; refer to source side; what it will be? Source side, so V_1 will remain V_1 that is there; then this current I_1 is I_2 star; I_2 reflected is not this is I_2 reflected and this thing is I_3 reflected. So, I_2 plus I_3 reflected this is your I_1 is not.

Therefore, this impedance must appear in parallel such that one is I_2 dashed and everything will be this is the I_3 dashed. And this impedance I will write z_2 dashed this first one as this one; I will write it as N_1 by N_2 whole square into z_2 ; this impedance. And this impedance reflected impedance on the primary side; I will write it as N_1 by N_3 squared into z_3 .

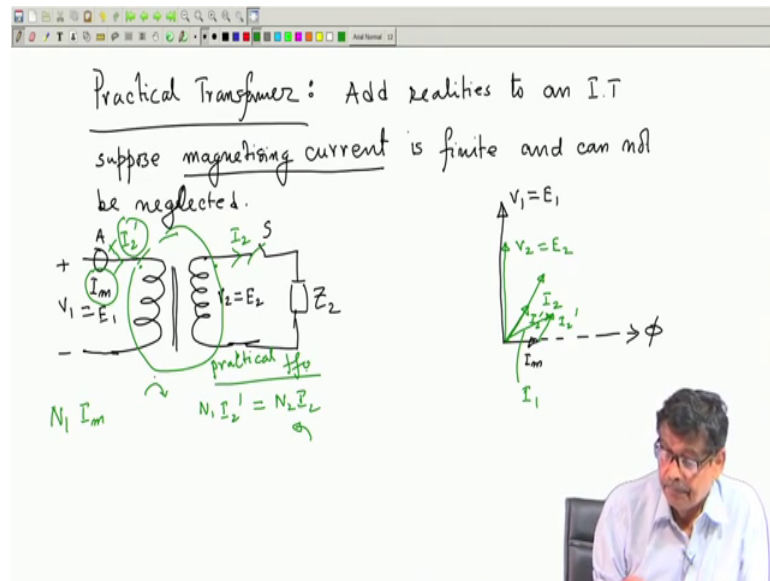
So, you see if there are several secondary coils and each one of them is supplying load; then the equivalent impedances referred to the primary side; they will be not in series they will be in parallel. So, this a lot of problems; we will be able to handle when ideal transformers are connected in different ways supplying different kinds of load.

So, this one is very useful power balance; try to use that power balance means complex power balance, both real and reactive power delivered by the source. That must be equal to some of the real powers delivered to z_2 z_3 plus some of the reactive powers based on that this equation is written and you get everything. And please go through this concept the $V I$ star; solve these get I star, but do not forget to take once again the complex conjugate so that it becomes I_1 and in the system ok.

So, this is the thing I wanted to tell about ideal transformer. So, after doing this then we ask ourselves that what about a practical transformer. So, what I will do now? I will now bring the realities of a practical transformer into an ideal transformer. I will go on adding the realities what are present in a practical transformer; start thinking that initially it was ideal transformer; then you have a practical transformer what are the things there.

For example, there will be router register I mean resistance of primary secondary windings; which I assume 0, there may be leakage flux which I assume 0 and there may be core losses which may be 0 and there will be a finite magnetic magnetizing current.

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So, a practical transformer will start with. Add realities to an ideal transformer. So, first reality what I will add is suppose magnetizing current; current is finite; is finite and cannot be neglected.

Suppose it is like this what I mean by this? This is my practical it is somewhat a practical transformer still lot of ideal qualities are present, still resistances are neglected core loss is not there, but only I am telling the core material is having a finite μ or values. So, magnetizing current cannot be neglected and I want to model it. So, this is a somewhat practical transformer having a finite magnetizing current.

Now, let us try to understand what is happening with this switch opened; this we discussed earlier with this switch is opened secondary impedance is z , what will be the ammeter reading now? It will be magnetizing current; in case of ideal transformer absolutely ideal transformer with secondary no current; primary current will be 0, but with this switch when even if it is open, primary will draw a current I_m finite magnetizing current.

So, it is like this then in the phasor diagram if I draw this is suppose V_1 ; there is no finite magnetizing current I_m . And which will be lagging the supply voltage V_1 by 90 degree after all inductance and this is the axis along which flux phasor can be shown. So, this is the open circuit equivalent circuit and if this is V_2 ; V_2 and E_2 are same because still drops are neglected V_1 is equal to E_1 .

So, all voltages will be in phase; so this is V_1 is equal to E_1 same phasor and depending upon the ratio your V_2 phasor will be here; if it is a step down transformer this is your V_2 . What is the secondary current with S opened nothing I_2 ; what is the reflected current? Nothing, but what I am telling the moment you close it; there will be a secondary current I_2 , what was the MMF? Acting in the circuit before S was closed, it was $N_1 I_m$.

What will be the MMF acting when S is closed? Net MMF once again has to be $N_1 I_m$ because flux remains same; therefore, if you close it dependent upon the power factor angle I_2 will be here. But primary immediately will draw additional current I_2 dashed; I have assumed low voltage side to be this. So, I_2 dashed will be of lesser length; high voltage side current is less I_2 dashed.

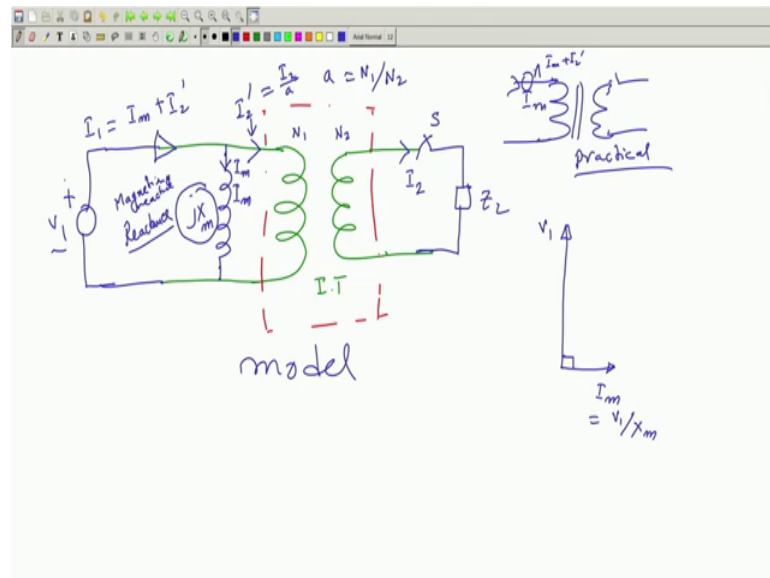
And I_2 dashed is such that $N_1 I_2$ dashed which produces flux in the clockwise direction, if it is $N_2 I_2$ which will produce flux in the counter they will balance of and once again MMF will remain $N_1 I_m$, but the question is what will be the current drawn from the supply? So, current drawn from the supply will now have two components.

So, one is the magnetizing component I_m plus this I_2 dashed. And this I must say this will decide about I_1 are you getting? So, there was I_2 dashed plus I_m you add and you get the primary current; this is a crucial point open the switch ammeter will read I_m , close the switch ammeter will read I_m plus I_2 dashed this plus this.

Now, the question is this is fine, but how to model this, how to put this to an ideal transformer I should add something to the ideal transformer so that this model; this happening whatever is happening in this transformer is correctly modelled in a; what do I mean by modeling? I means that I will add some parameter some components in an ideal transformer so that whatever I have seen is happening in a practical transformer.

I will add some components in that ideal transformer which will explain that thing correctly that is the idea. See it can be very nicely drawn; so this is practical transformer mind you, this is practical transformer what I will do now?

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I will say consider this is ideal transformer; I will tell, I will put a dotted mark around it. This is ideal transformer and what I will do across it; I will connect a reactance $j X m$ fixed reactance. Mind you this is not primary coil; this is a reactance added to an ideal transformer and I am saying it will represent correctly a practical transformer which is having only finite magnetizing current; still it is not having any resistance, leakage flux, any core loss.

So, this is a reactance; added by me to an ideal transformer which I have shown in a box and with same turns ratio $N 1$ is to $N 2$ like this. So, $j X m$ is external to this ideal transformer and it is no winding I am emphasizing this; this is the primary coil. That is what I am telling that your practical transformer is like this fine and here you observed you; nothing is connected you will see ammeter you showing magnetizing current connect a load it will draw a current $I 1$ which is $I m$ plus $I 2$ dashed; that is what we have seen.

Let us see whether really it can be; it can really faithfully follow the rules of this practical transformer, it can faithfully represent the happenings whatever is happening in a practical transformer. So, here is your $V 1$ fine applied voltage; you see if there is a load connected like this $z 2$ with S opened; with S opened this as I told you is a reactance I have model the transformer thought I will represent.

So, what is this current? V_1 by $j X_m$; that is V_1 is here with S opened I know magnetizing current 90 degree lagging. And this I_m ; I will choose the value of X_m means such a way that it is X_m ; V_1 by X_m 90 degree lagging.

So, X_m is called magnetizing reactance; therefore, how do I find out that? With S opened in a practical transformer with S opened measured I_m ; V_1 by I_m gives you the value of X_m and I write it like this with S opened; with S closed listen carefully. Suppose I have closed S ; this current is I_2 fine, but this portion is an ideal transformer with S opened, there was no current here, there was no current here; this transformer which is ideal has no magnetizing current. And we know that when S was even opened, current drawn from the supply was vanishingly small for this transformer 0.

But when I close this; this current has to be I_2 dashed because it is an ideal transformer we have spent so much time on that and this is equal to I_2 by a ; where a is N_1 by N_2 . I mean it should not disturb you or to anybody that I have applied a voltage; something is connected between these two; why there is no current when S was opened.

This we have discussed that length previously; this is an ideal transformer whatever little current I mean ideal means ideal me what is infinitely large; you do not require any magnetizing current for this portion. And we know ideal transformer how it behaves connect load there will be reflected current here; I_2 dashed.

And then the current drawn from the supply will be this magnetizing current; I_m plus I_2 dashed. And this will be your I_1 you have to add this and this is exactly correctly representing what I have observed in a practical transformer with S opened practical. S opened current drawn is I_m ; is it?

Yes, in this model this is model mind you; this model comprises of an ideal transformer with a reactance with S opened; current drawn is I_m because there was no I_2 there is no I_2 dashed I_1 is equal to I_m . With S closed this current in a practical transformer we have seen it is I_m plus I_2 dashed. Yes, it is correctly representing; close the switch from the knowledge all of this portion is ideal. So, here now we will appear a reflected current I_2 dashed and I_2 dashed plus I_m will give you I_1 .

We will continue with this in the next lecture, but please go through this portion very carefully.

Thank you.