

**Electrical Machines – I**  
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**Lecture – 78**  
**Shunt Motor Basic Equation**

Welcome to lecture number 78 and we have been discussing about DC motor. Now, all said and done that a machine can have series winding, compensating winding, interpole winding, now, we will first consider simple machines and although those series field and interpole can be incorporated some problems if you solve it will be clear how to handle that.

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200 V  $r_a = 5 \Omega$   
 $I_a = 30 \text{ A}$  15 V

$E_b = V_a - I_a r_a$   
 $I_a r_a \ll V_a$

$V_{a1} = 200 \text{ V}$      $\eta_1 = 1000 \text{ rpm}$      $I_{a1} = 15 \text{ A}$      $E_{b1} \approx 200 \text{ V}$

if  $I_a r_a$  drop is neglected    torque is load constant     $T_1 = T_2$

then back emf  $E_b \approx V_a$

$\frac{E_{b1}}{E_{b2}} = \frac{200}{100} = \frac{k\phi \eta_1}{k\phi \eta_2}$     armature voltage is reduced to half     $V_{a2} = 100 \text{ V}$

$E_{b1} = 200 \text{ V}$      $T_{e1} = k\phi \times 15 = T_{e1}$      $\frac{T_{e1}}{T_{e2}} = \frac{15}{I_{a2}} = \frac{\eta_1}{\eta_2} = 1$

$\therefore \eta_2 = \frac{1}{2} \times 1000 = 500 \text{ rpm}$      $E_{b2} = 100 \text{ V}$      $T_{e2} = k\phi I_{a2} = T_{e2}$      $I_{a2} = 15 \text{ A}$

But, in my last lecture, I told you that if it is a separately excited DC motor how to work out the problem and I assumed that armature resistance drop if it is neglected then it will be very elementary calculation.

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Lec 78

Seperately excited D.C motor

Let initial steady state  
op. point

$I_{a1}$	$\eta_1$	$T_{L1}$	$I_{f1}$	$V_{a1}$
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$$E_{b1} = V_{a1} - I_{a1} r_a = K_{\phi} I_{f1} \eta_1 \quad \text{--- (1)}$$

$$T_{e1} = K_T I_{f1} I_{a1} = T_{L1} \quad \text{--- (2)}$$

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At 2nd steady operating point

$I_{a2}$	$\eta_2$	$T_{L2}$	$I_{f2}$	$V_{a2}$
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$$E_{b2} = V_{a2} - I_{a2} r_a = K_{\phi} I_{f2} \eta_2 \quad \text{--- (3)}$$

$$T_{e2} = K_T I_{f2} I_{a2} = T_{L2} \quad \text{--- (4)}$$

Now, today what I will do is this suppose armature resistance is there, separately excited DC machine how to handle the problem separately excited; DC motor we are discussing. As I told you generators are nowadays not. So, popular at all with DC, but motors are still in use and this is the diagram and this is the field winding. Let us try to understand what I am telling.

Suppose, it is motor and it is running in this direction  $n$  and this motor is to be supplied from it is rated voltage armature voltage and this is the field voltage, separately excited, this is the field current ok. Now, suppose I say we find and the in which direction electromagnetic torque is operating in case of motor electromagnetic torque is this. In which direction load torque? This load torque is a mechanical torque suppose it is running a pump you must understand that the load on the shaft of the machine either it is cutting something or it is raising water to a high raise building.

So, load torque against this load torque it has to run; let initial steady state operating point this is  $I_{a1}$ . Let initial steady state operating point is this one that is the machine draws armature current is  $I_{a1}$  and speed is equal to  $n_1$  and load torque is equal to  $T_{L1}$  and field current is  $I_{f1}$ . Let me make it general and this armature voltage applied is  $V_{a1}$ , got the point?

So, suppose the machine is found to operate steadily with these things and electromagnetic torque generated is  $T_{e1}$ . Therefore,  $T_{e1}$  and  $T_{L1}$  must be same, that is

why it is running steadily at this rpm. [FL] Then what you have to do is this suppose these are the observed things then I will write two equations armature resistance is suppose  $r_a$ ,  $r_a$  is the armature resistance.

Then I will say that  $E_{b1}$  back emf I will not now neglect armature resistance  $E_{b1}$  will be equal to applied voltage is  $V_{a1} - I_{a1} r_a$  and electromagnetic torque developed will be equal to some torque constant into flux into armature current; flux is proportional to field current  $I_{f1}$  into  $I_{a1}$ . This will be the two fundamental equation straight away write down. This is equation 1 and this is equation 2, got the point? Initially this thing is happening and also note that this  $E_{b1}$  is equal to the constant  $K_g$  into  $I_{f1}$   $K_g$  into  $n_1$ ; this is equation 1.

And, here I will write it as this equal to load torque  $T_{L1}$ . This is one thing. Now, suppose I change some applied voltage across the armature or field voltage or field current or load torque all things I have changed, perturbation I have done. Then the second operating at second steady state operating point suppose this quantities becomes suppose I find next time it is drawing armature current  $I_{a2}$ , machine is running at a speed  $n_2$ , load torque is suppose  $T_{L2}$ , electromagnetic is  $T_{e2}$  and field current is  $I_{f2}$  and applied voltage across the armature is  $V_{a2}$  ok.

In that case, I will say that  $E_{b2}$  will be equal to applied armature voltage which I have changed  $V_{a2}$  and study armature current is  $I_{a2}$ ; suppose, I have not changed armature resistance also that I could include here earlier because I could always connect external resistance in the armature anyway let us not complicate this too much and this should be equal to then,  $K_g$  back emf is  $k$  into field current  $I_{f2}$  into new steady state speed  $n_2$ .

And, the torque equation  $T_{e2}$  developed by the machine will be equal to  $K_t$  the torque constant into  $I_{f2}$  into  $I_{a2}$  and this must be equal to  $T_{L2}$ , load torque, is not? This is 3 and this is 4. So, what I am telling separately excited motor if this is the generator condition applied voltage  $V_a$ ,  $I_f$ , this one?

Suppose, I observe that the motor is running at a steady condition drawing an armature current  $I_{a1}$ , running at a speed  $n_1$ , load torque is  $T_{L1}$  and it must have developed same electromagnetic torque  $T_{e1}$  and  $T_{L1}$  are same, that is why it is running at a constant speed  $I_{f1}$  and  $V_{a1}$  at the corresponding field current and armature voltage.

The moment I know these things these two equations can be written, what is the back emf, what is the torque. Suppose, I have played with the field current and this armature voltage changed them and new condition is I find it is running once again steady state condition at rpm  $n_2$ ,  $I_{a2}$ ,  $T_{L2}$ ,  $I_{f2}$ ,  $V_{a2}$ . Once again write down these two equations – back emf and this then what will be your next step is to calculate take the ratio of this I will write it here next page.

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① ÷ ③ 
$$\frac{E_{b1}}{E_{b2}} = \frac{V_{a1} - I_{a1} r_a}{V_{a2} - I_{a2} r_a} = \frac{k_g I_{f1} n_1}{k_g I_{f2} n_2}$$
 Neglected saturation  $\phi \propto I_f$

② ÷ ④ 
$$\frac{T_{e1}}{T_{e2}} = \frac{k_T I_{f1} I_{a1}}{k_T I_{f2} I_{a2}} = \frac{T_{L1}}{T_{L2}}$$
 If  $T_L$  is constant  $T_{L1} = T_{L2}$   
 If  $T_L \propto n^2$   $\frac{T_{L1}}{T_{L2}} = \frac{n_1^2}{n_2^2}$

$T_e \propto n$   $\frac{T_{L1}}{T_{L2}} = \frac{n_1}{n_2}$

$E_b$  vs  $I_f$  graph showing a saturation curve.

So, take the ratio of  $E_{b1}$  by  $E_{b2}$ , invariably you do this. This will be equal to  $V_{a1}$  minus  $I_{a1} r_a$   $V_{a2}$  minus this is  $I_{a1} I_{a2}$  into  $r_a$  and this is nothing, but your  $K_g$  into  $I_{f1}$  into  $n_1$  divided by this one is  $K_g$  into  $I_{f2}$  into  $n_2$ . And, obviously, I have neglected saturation; neglected saturation that is I have assumed  $\phi$  is proportional to  $I_f$  then only you can do that otherwise. If  $\phi n$  is the voltage so,  $\phi$  can be replaced by some other constant into  $I_f$ . So, this is this you get by dividing 1 with 3 of the previous equation. Then divide 2 equation 2 with 4.

If you do this you will get  $T_{e1}$  by  $T_{e2}$  is equal to  $K_T \phi$  into  $I_f$   $\phi$  is proportional to  $I_f$ . So,  $K_T$  into  $I_{f1}$  into  $I_{a1}$  divided by same constant  $K_T I_{f2}$  into  $I_{a2}$  and this must be equal to  $T_{L1}$  by  $T_{L2}$ , got the point? Load torque mechanical torque present on the shaft of the machine may also depend on speed I do not know. If I say that it is a constant opposing load torque then  $T_{L1}$  and  $T_{L2}$  will be same if I say load torque is proportional

to  $n^2$ . If suppose load torque remains constant is constant then I will say  $T_{L1}$  has not changed  $T_{L2}$

It may so happen this is 1; it may so happen if load torque is proportional to  $n^2$  then  $T_{L1}$  by  $T_{L2}$  will be equal to  $n_1^2$  by  $n_2^2$ , got the point?  $N_1^2$  square by  $n_2^2$  square. So, this thing is known therefore, with this two sets of equation if I know the first operating point completely and if I say that I have changed the armature voltage as well as field current of the shunt machine what will be the new armature current drawn?

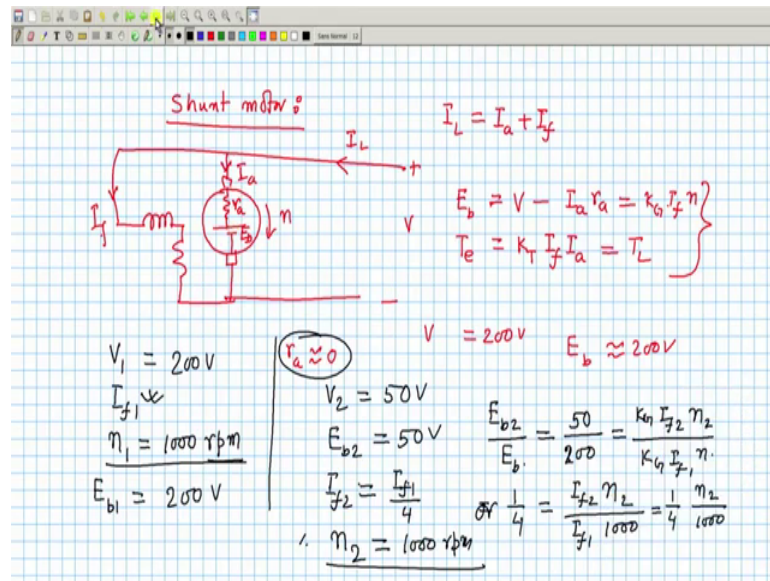
If it is like this I can it can be easily solved by using these two equation. These two are the most fundamental equations we have to solve in case of DC machine and take the ratios that is back emf equation and the electromagnetic torque equation and we can solve problem. So, with this in mind, I will now tell you so, this in fact, you can start solving several problems in DC machines by using these two equations provided  $\phi$  equal to  $I_f$ .

Only one point of question, if somebody says that  $\phi$  is not proportional to  $I_f$ , then one has to you must understand this that the equation of the generated voltage  $E_g$  is something like this it is not straight and this is your  $I_f$ . If it is not directly proportional to  $I_f$ , then I must have this information to get the ratio of back emf at this rpm this was the field current then those  $E_{b1}$  and  $E_{b2}$  are to be used because  $E_{b1}$  and  $E_{b2}$  are nothing, but the generated voltage.

Suppose, this occ so, I will refer to it is at a given rpm  $n$  then for any value of field current I will be able to tell what is the induced voltage. If it is  $I_{f1}$  and if it is running at  $n_1$  rpm from this occ only I will be able to tell that we discussed this earlier. Therefore, in that case this open circuit characteristic is to be used to correctly predict this one, but otherwise in general you can assume that  $\phi$  is proportional to  $I_f$  in the linear zone is machine is operating.

Then, these are the two equations back emf and the torque equation because in steady state electromagnetic torque and load torque must be same in case of motor. If this is the direction of rotation, this is the direction of electromagnetic torque and this is the direction of the load torque. Steady state  $T$  must be equal to  $T_L$ , so that it runs at a constant rpm  $n$  that is the thing.

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Now, what I am going to tell you that I do not care whether the machine is connected as a separately excited motor or a shunt motor, those fundamental equations will be equally true. For example, a shunt motor, what is the connection? This is the armature, this is the field current field, these two are connected in parallel and here you give supply voltage. Here no point in distinguishing between the voltage applied across the armature and the field. They are in parallel [FL], so, this is V.

Shunt generator to generate voltage there are several steps I told you in case of shunt motor do not care because we are injecting current. Here this voltage is well defined; there will be some field current flowing here. There will be some armature current flowing here and suppose the machine is running steadily at this rpm. Then there this current drawn from the supply is called the line current and this will be divided into two halves I a and I f. So, I L line current is equal to I a plus I f, this will be the load current and what is there here r a and back emf.

So, all the equations where there I wrote is valid what will be back emf here E b1. It should be equal to supply voltage V minus I a r a general when it is drawing current and electromagnetic torque will be proportional to K T into phi, but phi is replaced by field current if it is linear into the armature current. I do not write 1, 2. Now, I know what it is and this I will write it as some K G into I f into n and this one is equal to your load torque, this is the general. Now, no point in elaborating this n steps if it is at some

applied voltage, this that, armature current was one you replace it by 1, 2 etcetera take ratios.

So, the point I want to make is those two relations irrespective of the connections series, shunt whatever it is these two basic equations must be satisfied and that will be the starting point for solving any of the problem [FL]. If you are following me correctly suppose, I say that this DC machine  $r_a$  is vanishingly small suppose I say I am just asking one simple question  $r_a$  is equal to 0 vanishingly small you neglect the  $I r_a$  drop compared to the applied voltage.

Suppose applied voltage is 200 volt, then what will be the back emf of the machine? Then back emf of the machine will be 200 only and when I say 200 only no matter at what speed and what armature current it is drawing because  $I r_a$  drop is negligibly small. Suppose, I have used very good conducting material for armature that resistance is very small then you must understand that the back emf is going to be  $V$ . Perhaps I am changing playing with field current, but I will stay I will always state that back emf it is fixed.

If back emf is fixed and suppose initially you have applied 200 volt, now let me the problem let me ask you now that  $V_1$  I have applied is 200 volt, field current initially is some  $I_{f1}$  and initial speed of the machine is suppose 1000 rpm. Now, I say that I will make the and machine is found to run steadily at this rpm,  $I_{f1}$  has got some value. Suppose, I say that I make the applied voltage that is this voltage  $V_2$  to 50 volt and ask you what is the speed at which the machine is going to run.

In this case  $E_{b1}$  is equal to 200 volt only because  $I r_a$  drop is neglected 200 volt. In the second case what is  $E_{b2}$ ?  $E_{b2}$  will be how much? Once again 50 volt. I have reduced the voltage by this amount. Therefore,  $E_{b2}$  by  $E_{b1}$  will be equal to 50 by 100 approximately that will be thing and this is equal to.

Student: (Refer Time: 26:23).

Student: 200.

200, thank you and this is equal to  $K_g I_{f2} n_2$  by  $K_g I_{f1} n_1$  this will be the thing. Now, if this current was  $I_{f1}$  what is the field current here  $I_{f2}$ ? See, this is the

beauty of shunt field current; how much will be the field current? You have reduced the voltage by 4 times. So, your  $I_{f2}$  has to become equal to because field resistance I have not varied. So,  $I_{f1}$  by 4 it will be. So,  $I_{f2}$  by  $I_{f1}$  will become. So, this is equal to 1 by 4, no doubt; is equal to  $I_{f2}$ , what is the speed  $n_2$  I am looking for? K G, K G goes  $I_{f1}$  and initial speed is 1000 rpm  $n_1$ , is it not?

And, this is equal to one fourth or one fourth is equal to this, but  $I_{f2}$  by  $I_{f1}$  is also 1 by 4  $n_2$  by 1000. Therefore, this speed at which the machine will run is also 1000 rpm, speed will not change. It is no surprise because the field current is also reduced by factor of 4. Therefore, you see whether the machine is connected in shunt or separately, excited to motor only a thing you draw the circuit correctly and write down these two equations back emf and armature equations.

And, from this load torque; see here load torque does not come into picture provided  $I_a$  is equal to 0, but it will come into definitely and I have to take the equation of torque; there I do not have to write the torque equation I am getting the thing as it is. So,  $V$  minus  $I_a r_a$ ; so, often to get the quick results you neglect  $r_a$  and quickly calculate and back emf will remain same, got the point? So, this is how the calculation will go on in series and shunt machine.

Why I have told calculations first before discussing the speed control, various methods of speed control of DC motors as well as starting phenomena is and braking is that the calculations in DC machine are very simple. I have assumed it is running at steady state, how it reaches steady state that also I will discuss, but first thing is this one steady state this is the thing and you can carry on calculation. So, several problems once again you can solve on your own and solve it and enjoy reading analysing DC motors. We will continue with that.