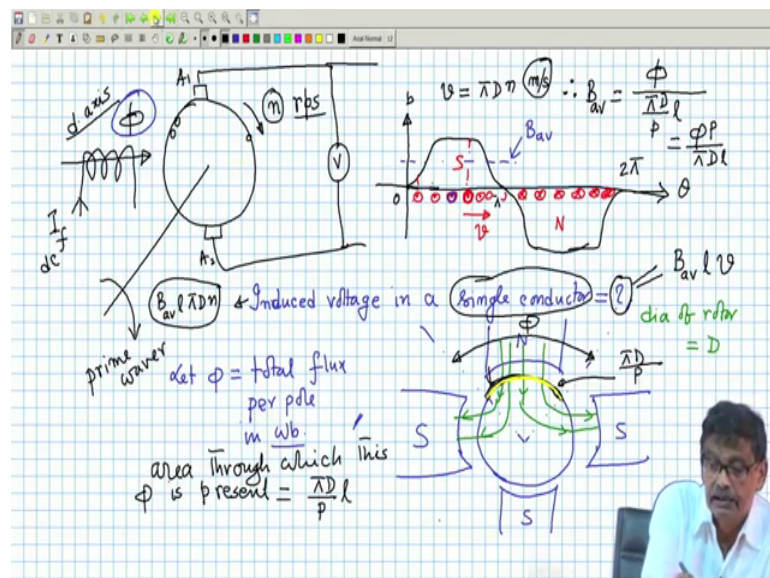


Electrical Machines - I
Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 63
Electromagnetic Torque in D.C Machine

Welcome to 63rd lecture, remember that in my last lecture we reached up to this point.

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My goal was to find out what will be the voltage available across the armature terminals when no load is connected open circuit condition we have passed a DC current in the field winding it has produced a flux per pole ϕ then my question was what will be the voltage induced in that. I know brush between the brush there will be several circuits in parallel that is all the armature conductors will be divided into several parallel pass.

My plan is I will calculate the voltage induced in a single conductor and then I will multiply this expression with a number of conductors presented in a parallel path one should not tell that this is the induced voltage in a single conductor average induced voltage. And, suppose z is the total number of armature conductors just do not multiply it with z by a z then it will be wrong because all the parallel paths have definite number of armature conductors. So, I will start from here that.

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Induced voltage in a single conductor = $B_{av} l v$

$$= \frac{\phi}{\frac{\pi D k}{p}} \times \pi D n$$

$n \rightarrow \text{rps}$

$$= \frac{\phi p}{\pi D} \pi D n = \phi p n \text{ volts.}$$

def $Z = \text{total number of armature conductors.}$
 $a = \text{number of parallel paths across the armature terminals. } = (p \text{ if lap wdg})$

So no. of conductors in each parallel paths = $\frac{Z}{a}$

voltage available across the brush or armature terminals
 $= \phi p n \frac{Z}{a} = \frac{pZ}{a} \phi n$

Diagram: A circular conductor of diameter D is shown in a magnetic field B . The conductor length is $l = \pi D$. The flux per pole is ϕ . The induced EMF is $E_{A12} = \frac{pZ}{a} \phi n$. The induced current is I_a from the brush.

Here I will say that the last days expression induced voltage we have got induced voltage in a single conductor in a single conductor we got it to be is equal to B average v velocity Blv length of the conductor B average and this I found out to be v average is flux per pole is phi divided by the area over which this flux per pole will enter into the rotor and that is equal to pi D by P into l. This is the area I got B average l is l and velocity is pi D n, where n is in rps armature rotations this is the thing.

And so, this will be equal to phi p by pi D, l cancels out into pi D n oh this also cancels out. So, it is equal to phi p n so much volts, that will be the expression for voltage in a single conductor. Now, I will say let z be the total number of armature conductors, total number of armature conductors. What do I mean by that? Armature has a coil like this, armature has a coil like this which may be multi turn which may be multi turn coil, in each slot how many coil sides will be there? 2 coil sides.

So, in a conductor in a slot like this here I draw in a slot there is the upper deck multi turn coil section of that and another coil side multi turn coil got the point. Therefore, if it is supposed having 2 turn then 2 sections you will see 2 conductors for this also 2 conductors. So, multiply each coil with that multi turn number if it is a 2 turn coil multiply with 2 then with another 2 for this one.

So, total conductors means this conductors also which because they will become seat of emf is not, they are having same velocity same flux density they will see all the coil sides in a particular slot will have same voltage B average $l v$. Suppose z is the total number of armature conductors and I hope you know how to calculate it. If s is the total number of slots how many coils are there, s number of coils are there. If s is the total number of coils how many coil sides are there, 2 into s coil sides are there and if I say each coil has 4 turns then $2 s$ into that 4 is that number z total armature conductors ok.

Let this and also this thing I have defined a be the armature parallel paths number of parallel paths in the armature that is a good English number of parallel paths in the armature circuit. That is between the brushes across armature terminals that is good across the armature terminals across the armature terminals, because we have seen at least in case of lap winding it between the brushes it is divided into as many number of parallel paths between plus and minus brushes as the number of poles and this so, I take a general case ok.

Let the number of because I have not yet investigated wave winding maybe it is number of parallel paths will be different. So, that is why a general variable name is given of the parallel paths, I am certain about one thing that this is equal to p if lap winding this much I am sure about lap winding. So, better use a general variable name parallel path, then how many conductors will be there in each parallel path?

So, number of conductors, conductors in each parallel path will be equal to z by a what else equally divided. Therefore, voltage available across the brush available across the brush or armature or armature terminals will be simply voltage in a single conductor multiplied by z by a is $\phi p n$ into z by a that is equal to p number of poles, z total number of conductors, by a number of parallel paths into ϕ into n that is all such a simple expression.

So, what I mean to say is now this if you have a DC machine like this if you pass some field current and it produce a flux per pole ϕ and you allow this to run with a speed let me change the color it is not good n , then voltage across this voltage across say $A_1 A_2 E$ $A_1 A_2$ will be $p z$ by a into ϕ into n . In this formula mind you n is in rps ϕ is flux per pole p and a are numbers z is the conductor ok. So, this is the most famous formula for a DC machine, if a DC machine you know the flux per pole, if you know at what rpm or

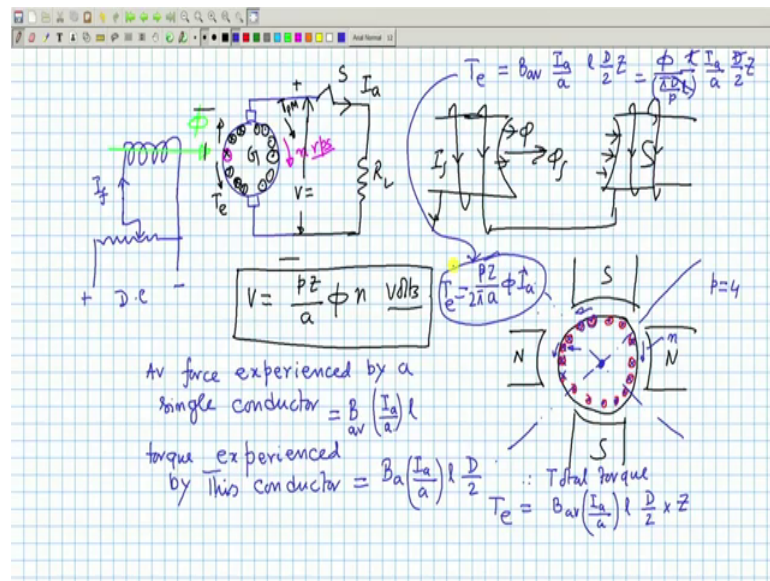
rps it is running, if you know the total number of conductors, total number of poles of the machine, you can easily calculate what will be the generated voltage very simple formula.

In case of AC machines we used to have rms voltage as $4.44 f \phi n n_1$ winding factors this that, but here everything goes very simple neat and sometimes if the field circuit is assume to be linear then this ϕ can be related to I_f by a linear relationship and we say here the induced voltage will be if ϕ is proportional to I_f which will be at least in the linear zone of bh curve then generated voltage this $A_1 A_2$ will be also can be see pz by a is a constant of the machine.

Once the machine has been manufactured number of poles z , a are fixed you cannot do anything with that. So, it is sometimes they written like this machine constant $K \phi n$ where K is this and generated voltage can be also sometimes written with this K and this ϕ with some constants say $K \text{ dash into } I_f \text{ into } n$, if it is linear magnetic circuit. So, that generated voltage is proportional to $I_f \text{ into } n$ it looks like for a fixed weight go on increasing the field current your generated voltage will increase and such an arrangement for producing voltage also I will tell you the name is called a separately excited DC generator.

That is field winding requires a separate excitation from DC source here you require a DC source to pass this DC current through the field winding to create flux and then of course, there it should be a prime mover to drive the armature and you get a voltage generated ok. This is the most one of the most important formula another formula important formula is there that is called the expression for torque electromagnetic torque ok. If there is no load connected just try to see what I am trying to tell.

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This is suppose the field winding we will now refer to this simplified diagram and this is your armature terminals, you pass some field current please try to follow me. Suppose this is the I_f and because of this I_f where you have connected perhaps say potential divider connection you do and you can vary the field current also plus minus you require a DC source like that and it is a separately excited DC machine field winding is excited.

Now, here what I am telling I of course, there will be a generated voltage, suppose the flux per pole produced is ϕ ϕ is the flux per pole and suppose you are driving the machine at n rps suppose in this direction ok. Then from this diagram I am not sure whether I will not be able to tell whether this is plus this is minus brush that is not possible, but what is possible is, there are conductors here I am drawing a representative conductor and this pole is what North pole.

If I show the poles also to complicate the matter this is suppose if over which this field winding zone it is not correct to draw like this, but this side flux is entering to the conductor better I will not show like that it will only create confusion. So, I have in my back of my mind it is like this is the field winding. If I_f is like this then this is ϕ_f that is what I mean. So, other side there is a South pole is not lines of force will go in. So, it is South pole here it is emanating.

So, this is the flux per pole, in the simplified diagram it is drawn like that and you can always think that there is a South pole which will observe this lines of force here after cutting the armature conductors. Now what I am telling in this diagram so, this is right hand rule I will apply what will be the polarity of this induced voltage. It will be right hand rule b that is from left to right ok, this is the b and direction of rotation is like this.

So, this is the right hand rule I am applying b velocity for this conductor is in this direction v it will be cross is not it will be cross and you can be rest assured if you apply the same rule velocity is same, but it is under South pole it will be dot. This is what and for all the conductors whoever is there that will be cross only is not and this will be dot. Now, out of these 2 which one which brush is plus or minus difficult to say from this, we will assume this is plus minus for that nothing is going to change because the it is the armature current distribution which matters you assume this is plus, this is minus and suppose you make an arrangement DC voltage you will get.

So, loads will be if it is generator mode that is what I have assume I will connect a resistive load switch is opened which switch opened this voltage here is the open circuit voltage is given by $p z$ by $a \phi$ into n so much volts you will get. What will be the armature current? Armature current is 0 this cross and dot here shows the polarity of the induced voltage mind you these are all dot on this side and that the South pole and these are all cross.

But it will drive a current if the switch is closed it will deliver a current external current I a will this current be same as this current no I know between the brushes there are parallel paths. So, current in the conductors will be this current divided by a and what will be that directions this directions I am certain it as it is absolutely correct with this direction of motion this being North pole this is the thing. So, this is the current and they get divided in cross and dot, it looks like a 2 parallel paths in this simplified diagram, but it is not we know that.

Anyway now the question is the moment so long with S open these conductors where not carrying any current, prime mover was running this generated at some speed in the clockwise direction. How much torque the prime mover had to apply so, as to make it run in the clockwise direction? Opposing torque was may be some little friction torque a little that is fuel consumed by the prime mover at that time was very little less just

overcome friction to run it at a constant speed. Now, you close the switch the moment you close the switch you see there appears current in this direction all the conductors here cross dot.

If that be the case then they will experience force and once again I will apply right hand rule to find out the direction of the force. So, mind you the prime mover is giving a torque in this direction TPM torque by the prime mover. Now the moment you give this try to extract power from the armature of the DC machine, this is the R L load resistance current will appear. And, then the conductors are will be carrying also current in the same direction as the induced voltage and apply left hand rule to see that it will experience this sides will experience a force in this direction.

In the opposite direction of the prime mover torque, prime mover thing is a mechanical thing who is providing you the torque electromagnetic torque and if you find the generator is supplying power running at a constant rpm or rps you will conclude the prime mover torque and electromagnetic torque are same, that is why it is running at a constant speed. Because if prime mover torque wins over electromagnetic torque it will accelerate in this case, if electromagnetic torque is greater than prime mover torque it will decelerate.

So, it is a generator mode it goes like that we will come to this aspect many a times later, but for the time being what I am telling that ok, the moment armature carries currents conductors will experience force hence torque, how to get an expression of the electromagnetic torque that is what we will be discussing. So, to find that out what we will do is this we consider suppose a 4 pole machine just what the sake of suppose imagine very interesting calculation from the fundamental we will do suppose 4 pole North, South, North, South field windings I am not showing here are armature and these are the conductors on the slots periphery of the armature like this suppose a 4 pole machine.

Now, you divide this in 4 zones that is this is the zone of magnetic neutral axis these are called where the flux density will be 0 etcetera. So, this zone is controlled by this South pole, this zone is controlled by one fourth controlled by this North pole, this one forth zone quarter zone is controlled by South pole and these are the 45 degree lines you draw and as I told you that if you assume because this diagram will not tell you the true

pictures, why because very it is a very simplified schematic diagram of the whole things, but if I want to calculate the expression of the torque I have to referred to this and say that machine is running in the anticlockwise direction with a velocity n rps same. Then I know the currents in the conductors under the influence of the North pole will have cross so, I put cross here.

Suppose the number of poles of the machine is 4. So, these are all cross are you getting cross current whichever conductors are under South pole they will have dot what else, similarly this conductors cross and this conductors once again dot you have got. Then what how the torque will be produced? I take any conductor here suppose this conductor put attention to this. What is the force it experiences $B I a$ and l . So, force experienced by a single conductor will be equal to B at this point into current through this conductor not this total current that is very important and the current through this conductor is $I a$ by a that I know that is very important $B I a$ into length of the conductor just like the emf equation I am going.

And, once again this conductor as it moves we will see different values of b , but none the less direction of the force will be like this in the anti clockwise direction therefore, it is prudent not to deal with that b , but to take what is the average B it experiences B average $I a$ by a into l this is mind you is a slightly disturbed $I a$ this will be the force. So, average force experienced by a single conductor is this, what is the torque? Torque about the origin or the about the shaft torque experienced by this conductor will be equal to the force into D by 2 that is B average $I a$ by a into l into D by 2 radius force into distance about this point.

The interesting point is that it is true for all the conductors, if it is here under south pole it will see once again same v average and what will be the direction of the force of this say conductor dot south pole right left hand rule. So, this is 4 finger is the direction of the current, it will also experience a force like this. What will be the torque contributed by this conductor, this into D by 2 what about this one, same B average, same $I a$ by l D by 2 D by 2 everyone contributes there is no conductor experiences force which is the directions of which will change.

Therefore, all the torques will adopt is not therefore, total torque experienced by a single conductor is this therefore, total torque electromagnetic torque will be $B a I a$ by a into l

into D by 2 into how many conductors should I multiply should it be z by a should, it be z , what it must be z , because each conductor is contributing to the torque unlike the emf calculations where there are several parallel paths z by a z by a and to calculate emf. We have to consider between the brushes we have to consider only one parallel path good enough so, z by a conductors.

But in this case all the conductors each of them one of them is carrying a current of I by a and there contributing torque in the same direction therefore, it must be multiplied with z not with z by a . Therefore, finally, I will say that electromagnetic torque will be equal to v average this is v average sorry B average B average I a by a is the current flowing through each conductor into l into D by 2 into z , which will be equal to I know, B average is ϕ by πD by P into l area and this is I a by a into D by 2 into z therefore, the expression of the torque will be.

Student: (Refer Time: 33:13).

Student: (Refer Time: 33:16).

Torque expression will be.

Student: l .

Student: (Refer Time: 33:22).

l l oh over l I missed l is not that is what you (Refer Time: 33:31), so there is an l also on the top. So, this will become equal to if you see several things cancel it out is not, D l D l cancel out D l D l cancel out and it will be equal to and it is 2 . So, it will be equal to p z p z number of poles p z , p is there know p will go up p z by a not only a 2 π a into ϕ into I a this is the expression of the torque, we will continue with this in the next lecture.

Thank you.