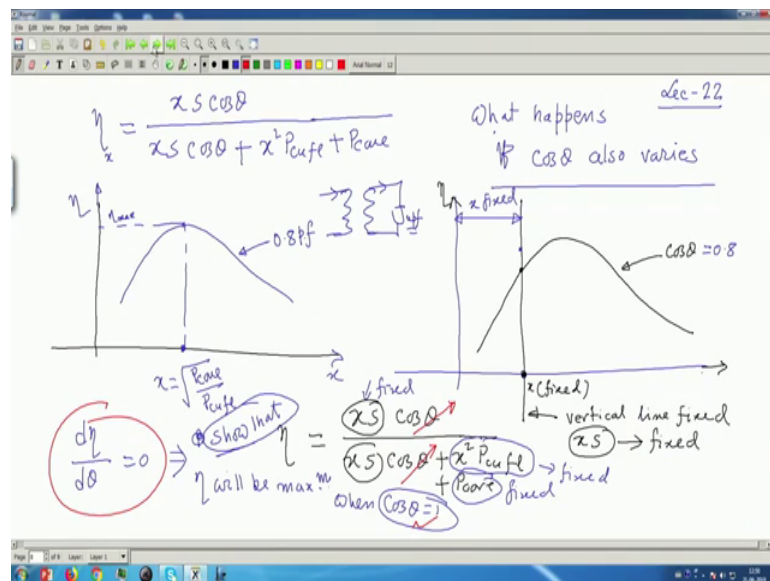


**Electrical Machines - I**  
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**Lecture – 22**  
**Family of Efficiency Curve at Various Power Factor and Energy Efficiency**

Welcome to lecture 22nd and we were discussing about efficiency of transformers.

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And there are efficiency which is called power efficiency as you know we at a degree of loading  $x$  is I am writing many a times, so that it really you always remember this. So,  $x$   $S$  and based on this I can tell so many things  $x$   $S$  square  $P_{cu}$  full load plus  $P_{core}$ . And what I saw with this expression of efficiency is this thing that you at a particular power factor if you sketch it, it will be like this and this is efficiency etc.

And this is the load at which maximum efficiency will occur  $P_{core}$  by  $P_{cu}$  full load and this is your  $x$  this is  $\eta_{max}$  this was found. And I told you also this is important to note that this curve has been obtained at what fixed power factor, so tag that 0.8 power factor

Now, the question is power factor of the load may also change. Suppose I repeated the same exercise I want to get the efficiency versus  $x$  curve for say 0.6 power factor or unity power factor or 0.2 power factor. All these efficiency versus  $x$  curve will change will it is position will remain same I am not sure right now. But I am sure about one thing

whatever be it is position the value of  $x$  at which maximum efficiency will occur it only depends on  $P_{\text{core}}$  and  $P_{\text{copper}}$  full load it will be here only.

Now, let us see how to then find out what happens what happens, if  $\cos \theta$  also varies also varies. A how to find out this that is what this problem can be framed at this way I will frame the problem. This is your efficiency curve at a given power factor say  $\cos \theta$ , what I am going to do is now I will fix the load on the transformer that is I will see that kVA I want to see the effect of variation of power factor angle of the load on efficiency that I want to do.

So, to do that what I will do this is my  $x$  no doubt I will keep this  $x$  fixed and vary  $\theta$ . So, at a given power factor angle suppose at a given  $x$  this I will now this vertical line I will keep fix, line fixed means what  $x$  into  $S$  this I have fixed I am not now going to vary  $x$  and vary  $\theta$  in this expression.

But the point that that is what I mean efficiency is now  $x$  into  $S \cos \theta$  divided by for a given kVA  $x S \cos \theta$  plus  $x S$  square  $P_{\text{cu}}$  full load plus  $P_{\text{core}}$ . This is the efficiency expression what I am telling now to understand how this curve how efficiency will vary if you vary  $\theta$  what I have done is I will keep this constant at a given kVA I want to see I will vary power factor at this kVA.

What is this kVA? A fixed value of  $x$  into  $S$   $x$  is fixed now I will fix  $x$ . And then vary  $\theta$  that is what I will do power factor I will vary and I want to see at a given fixed power factor at a given kVA then I will be able to tell at what power factor efficiency will be maximum.

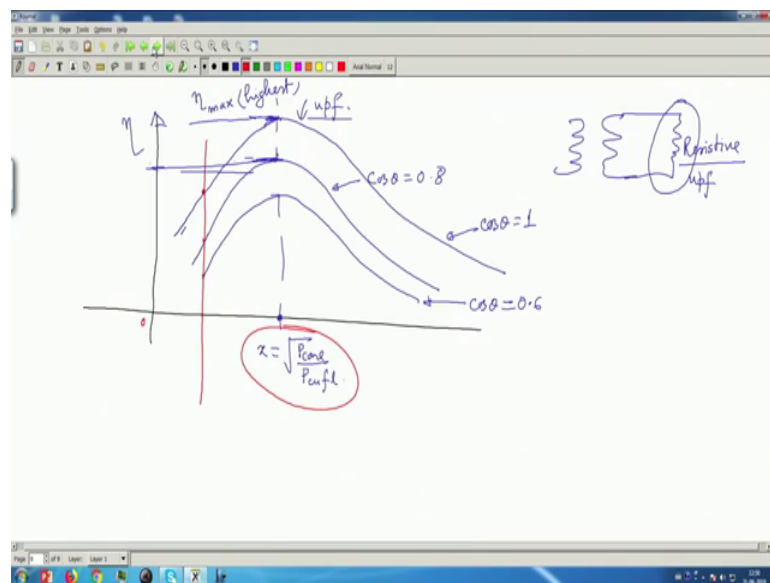
Now, this thing that is what I will do next is this expression I will do  $d \eta$  by  $d \theta$ . In this expression in this expression in this modified problem statement What I am telling  $P_{\text{core}}$  is fixed  $x$  square for  $P_{\text{cu}}$  fuel fixed.

Because I am not going to change  $x$  I will be on this line only fixed, I am not going to vary  $x$ . At a given  $x S$  kVA the transformer is handling tell me at what power factor efficiency will be maximum. So, to do this  $x S$  I will keep fixed that is this line  $x$  is fixed, keep  $x$  fixed and vary  $\theta$  other things are fixed  $x$  being fixed.

Now, So, it will be not very tough if you set this to 0 and see that prove that this is proved. So, that not proved, so that, so that eta will be maximum, when cos theta is equal to 1 this is the thing. You go on vary for a given kVA this is corresponding to say 0.8 power factor. What this expression will tell you this efficiency will be maximum when cos theta equal to 1 that is at this point it must be above this curve.

Therefore, I can now draw, so this you please try to prove on your own you have to simply differentiate it and, so that at a given kVA that is what I am telling this is the transformer. You always pass a given fixed current by varying the load, but in this case I will vary only cos theta to change the magnitude of the load. And then I am telling if it is unity power factor then efficiency at this fix load as you vary theta power factor efficiency will be maximum here.

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In other words next page I will draw this is quite interesting to note that now I can complete this efficiency versus x curves by drawing a family of curves. Suppose this is at cos theta is equal to 0.8, what I am telling the curve will be above it and we draw a vertical line here.

This will be corresponding to cos theta is equal to 1, it will be above cos theta equal to 0.8. If cos theta equal to 0.6, it will be this you can draw a family of curve. Now, showing really that is I am now restricting that cos theta is 0.8 now the full picture is clear to me.

But none the less the that is this thing remains this one P copper full load. And this maximum efficiencies are corresponding to different power factors  $\eta_{\max}$  at 0.6  $\eta_{\max}$  at a 0.8 it is this much. And this is  $\eta_{\max}$  at one unity power factor, so this axis is  $\eta$  efficiency.

Now, we come to know oh the transformer will have maximum efficiency possible efficiency this much at 0.8 power factor if that 0.8 is kept fixed that will give you some maximum. But you can still operate the transformer at higher efficiency than the load these provided a load is in power factor got the point. This is the highest maximum efficiency possible  $\eta_{\max}$  highest this is an interesting point to note that is at unity power factor.

But nonetheless whatever be the power factor the load at which this maximas will occur that is fixed. So, this curves a family of curves now bring out the total picture what is going on in a transformer when you are changing load both power factor and impedance.

So, highest possible efficiency is when you say the load on the secondary of the transformer is resistive unity power factor. Then only you will get the highest efficiency for a given power factor if the load power factor is 0.8. It will still give you maximum efficiency here, but that will be lesser than the maximum efficiency which is still possible if the impedance is purely resistive that is what I want to tell

But here is a catch, the catch is the load on the secondary of a transformer will be several kinds of motors in industry this will supply some motors. Motor means rl there will be other rl type of load, rl type of load are most common load that is not in my hand you know.

You must be knowing that the load power factor people try to maintain  $\eta$  at say 0.8 is a good number ok. You connect rl load this that load, but try to see the power factor is close to unity, but at least 0.8. And if you load power factor is a below that then what supply authority says that you will be penalized.

Because you will be drawing you are utilising lesser power out of the total kVA you are for drawing from the lines. Therefore, it is the condition imposed by the supply authority that you are the unit of electricity try to see that load power factor is not below this

maybe 0.8 say. And if you are found to draw power for loads which is below 0.8 say 0.6 you will be charged extra, these are the things which coming to play now

But as I told you unity power factor is not the usual case. So, that is why even if your load is having low power factor that is why people connect capacitor at the beginning capital at the beginning of your supply. So, that power factor is improved you know those techniques from your circuit analysis and this that. Therefore, although resistive load will be a desirable load to be connected across the secondary of a transformer to have always the highest efficiency.

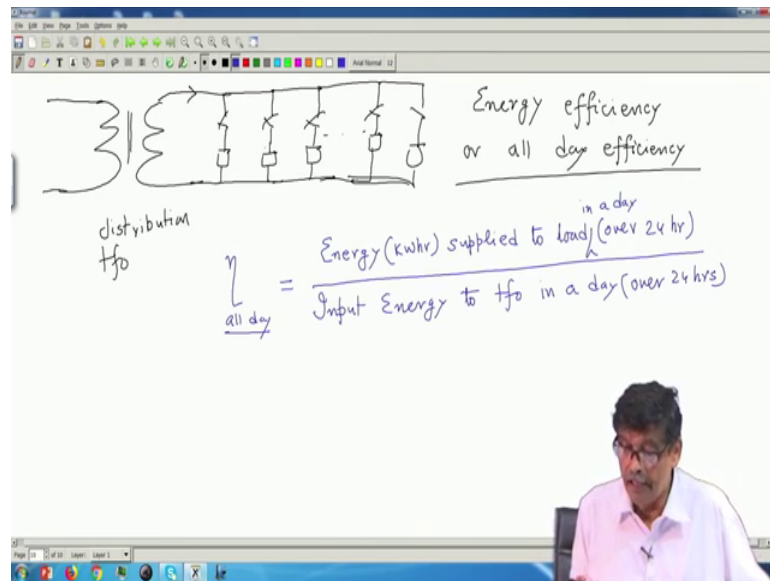
For any kVA you mind you if you are operating the transformer at this kVA fixed kVA, then you also you see efficiency will be maximum among these maxima among these available efficiency at different power factor that will be maximum that is a very good thing. So, it is an important point to be noted nothing is better than unity power factor load.

At whatever kVA it is discharging you will always get highest this thing highest efficiency with unity power factor load. And the highest most efficiency will be when the kVA is this much at this kVA you are operating and the unity power factor load it is. You must think about this you think yourself what I am telling.

So, that things will these are quite interesting I mean there is some logic in that nothing out of nothing something is being told to you. So, so this curves then I have drawn a family of curves with different power factors. Then I told you now I will keep the kVA fixed and would like to see these are the efficiency which of these will become maximum.

And you have been asked to find it out that at  $\cos \theta$  equal to 1 you have to differentiate this expression by noting that. Now you will not change kVA only power factor you will change differentiate it set it to 0 and come to the conclusion that if  $\cos \theta$  equal to 1 that efficiency will be maximum it is very interesting to note, so that is all.

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Now, we will come to the as I told you if in a transformer. Suppose here is a transformer a distribution transformer say, what is going on in a distribution transformer here are loads, how the loads are connected I am a consumer I will connect my load. All loads are connected in parallel you know, here is another consumer he will connect load like this. There is another consumer you connect load like this dot and, so many consumers and perhaps when all the consumers, because my habit of using electricity will not be matching yours.

So, I will sometime switch on my load or not at any point of time that may change many complicated situation. So, far as the loading pattern on the secondary of the transformer is concerned we really do not know. But what happens is this with large number of consumers it is found that there is a general type of variation of load on the secondary of the transformer, when you close these two consumers then current supplied is this plus this,.

So, load increases as consumers are connecting their loads you understand that. Now, the question is a in distribution transformer there will be a variation of load pattern over 24 hours. There will be some evening peak, there is some morning peak offices are open electricity's are used. In the evening once again as I told you there is a peak load demand, because most of the users will switch on their lights and fans and what not.

So, so, there will be a variation of load. And if there is a variation of load then certainly I am sure about this point that no point in asking that for this transformer I will demand

power efficiency maximum power efficiency for this transformer should occur at x equal to 1, because at full load condition this is will be operating at some point of time over certain time period.

But certainly not for 24 hours that must be understood before I calculate that is why this type of transformer should be judged their performance should be judges not from the point of view of power efficiency. But from the point of view of what is called energy efficiency or called all day efficiency.

And as I told you I wrote last time that eta all day is equal to energy in kilowatt hour. You must be knowing energy is in kilowatt hour, supplied to the load to load over 24 hours load in a day in a day that is over 24 hours that is why it is called all day efficiency divided by input energy to the transformer input energy to transformer.

In a day in the same day in a day that is over 24 hours, very simple calculation nothing very complicated ok. Now, so what it is what are the information's needed to find out the energy efficiency.

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In distr. tfo primary must be kept energised for 24 hrs.  $\textcircled{S} \text{ kVA} \rightarrow \text{Rated kVA}$

distr. tfo	time intervals hrs	chg. by loading	Pf by load	Calc'd	conclass
	$T_1$	$x_1$	$\cos \theta_1$	$x_1^2 P_{\text{cufl}}$	$P_{\text{core}}$
	$T_2$	$x_2$	$\cos \theta_2$	$x_2^2 P_{\text{cufl}}$	$P_{\text{core}}$
	$T_3$	$x_3$	$\cos \theta_3$	$x_3^2 P_{\text{cufl}}$	$P_{\text{core}}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$T_n$	$x_n$	$\cos \theta_n$	$x_n^2 P_{\text{cufl}}$	$P_{\text{core}}$
	$T_1 + T_2 + \dots + T_n$				$24 \text{ hrs}$

Energy output (kWh)

$$\eta = \frac{(x_1 S \cos \theta_1 T_1 + x_2 S \cos \theta_2 T_2 + \dots + x_n S \cos \theta_n T_n)}{[ (x_1^2 P_{\text{cufl}} T_1 + x_2^2 P_{\text{cufl}} T_2 + x_3^2 P_{\text{cufl}} T_3 + \dots + x_n^2 P_{\text{cufl}} T_n ) + 24 P_{\text{core}} ]}$$

all day efficiency

$T_1 + T_2 + T_3 + \dots + T_n = 24$

So, suppose I say that this is the primary mind you a distribution transformer this a primary is energized for 24 hours V 1 whatever it is the voltage at frequency f in distribution transformer. Primary must be kept energized for 24 hours continuously for 24 hours it must be.

Because you do not know when your consumer wants power that is not known to me I must be ready at that door steps with the power available. Whether they will use it or not that is their botheration I do not know. Now, suppose I say I have to do some statistical observations how the load pattern changes etcetera, but we will not go to that level of statistical distribution.

But I will say this much suppose for this distribution transformer this information is known, what is that information? That is for the first I will make a column time intervals. Very simple calculation mind you, see that is first T 1 hours T 1 hours.

Suppose they starts after midnight or say 6 AM say, from 6 AM we start counting. See first T 1 hours I find that degree of loading, next column degree of loading. I say it is  $x_1$ , because it is not full load always. So, first T 1 hours intervals T 1 in hours T 1 hours  $x_1$  is the degree of loading and S is the kVA rating s kVA is the rated kVA rated kVA. Similarly I will write another column my make power factor of load.

Suppose it is given as cosine theta 1. In the same way I will do for the next T 2 hours lead the degree of loading is  $x_2$  and power factor is cosine theta 2. T 3  $x_3$  is the degree of loading and cosine theta three is the power factor.

In this way I will say that the last T n hours  $x_n$  is the degree of loading and cosine theta n is the power factor angle. This T 1, T 2, T 3 this sigma T 1 plus T 2 plus dot T n is 24 hours over a complete day that I must see. First 6 hours, next 12 hours, next 6 hours such that it is like this, this will be the thing.

So, what I have to calculate and make S is the rated kVA, so energy output over 24 hours I have to calculate [FL]. Next, what I do is the copper loss, in the next column, in the first T 1 hours degree of loading is  $x_1$ . So, what is the copper loss? It is  $x_1^2$  into P copper full load, what is the copper loss here?  $x_2^2$  square P copper full load, this is  $x_3^2$  square P copper full load. In this way this is  $x_n^2$  square P copper full load, P copper full load is constant.

At full load condition what is the copper loss, which I get from short circuit test data. So, this is the copper loss at various time intervals taking place, what is core loss? Core loss as you know is independent on the degree of loading. So, it will remain P core P core all the time no matter whether  $x$  is 0 or  $x$  is 1  $x$  is 0.8 no.



So, it is always  $P_{core}$  fixed, got the point these informations unit. Then you can straight away write down I will write it here all day efficiency as equal to what I told I must write energy output over 24 hours. What is the output energy in the first  $T_1$  hours, what is the kilowatt output? It will be  $x_1 S_N \cos \theta_1$  basic kilowatt into  $T_1$  are you getting into  $T_1$ . In the first  $T_1$  hours, so much kilowatt hour plus  $x_2$  next  $T_2$  hours, what is the energy output  $x_2 S \cos \theta_2$  into  $T_2$ .

This is not  $x$  multiplication into  $T_2$   $x$  is here(Refer Time: 31:24). In this way you go on adding  $x_3$  you add and finally, you add to this plus  $x_n S T_n \cos \theta_n$  into  $T_n$ , so over 24 hours this is the energy output energy output what is the unit kilowatt hour. Then below what should I write, this is the output energy.

So, this bracketed term I am not rewriting this numerator this thing this is the energy output that is this thing here plus the losses energy losses. Now, the question is what will be the energy losses? See of which you see the core loss remains constant all the time it does not depend on degree of loading.

So, it will be 24 into  $P_{core}$  how this 24 comes? It is. In fact,  $T_1$  plus  $T_2$  plus  $T_3$  plus  $T_n$  into  $P_{core}$ , but that thing is 24, so 24 into  $P_{core}$  kilowatt hour plus. So, core loss will always take place in the distribution transformer plus the copper losses. Copper losses energy dissipated will be how much? I will write it in different colour it will be first  $T_1$  hours.

It will be  $x_1^2 P_{cu}$  full load into  $T_1$  plus next  $T_2$  hours  $x_2^2$  square because degree of loading changes full load into  $T_2$ . Plus  $x_3^2$  square plus  $P_{cu}$  full load into  $T_3$  plus dot dot dot last term  $x_n^2 P_{cu}$  full load into  $T_n$  this will be also in kilowatt hours. So, so this whole thing is in the denominator.

So, this divided by this will give you the energy efficiency or all day efficiency of the transformer. And it is based on this number that is while purchasing a distribution transformer you should rather tell the manufacturer this is my degree of full loading pattern on the secondary of the transformer will be and I would like to have energy efficiency of this much. Rather than, so your transformer has a specific rating here  $s$  kVA is the rated kVA fixed voltage ratings are there.

But you will say I do not like to have the maximum efficiency to occur at rated condition, knowing fully well that the load on the secondary of the transformer is not in my hand it is going to change over time. And after doing some statistical observations I have seen on an average first  $T_1$  hours so much demand of power degree of loading is  $x_1$  then degree of loading  $x_2$  then. And power factors of the expected load on the secondary of the transformer at that time will be  $\cos \theta_1$  for the first  $T_1$  hours and so on.

And as degree of loading changes copper loss in terms of rated  $P_{cu}$  full load will go on changing and I know how to calculate that I will calculate, core loss remains same. Only thing instead of output kilowatt by input kilowatt it is output kilowatt hour divided by this whole thing is the input kilowatt hour. What is the input kilowatt hour? This bracketed term once again I have not repeat it, it is this one numerator plus core loss remains same. So,  $watt_{in} \times hour_{24}$  plus this, mind you  $T_1$  although it is written  $T_2$  plus  $T_3$  plus dot dot dot plus  $T_n$  is equal to 24 hours ok.

Thank you we will continue with this next time.