Principles and Techniques of Modern Radar Systems
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Lecture - 09
Tutorial Problems on Basic Concepts of Radar (Part II)

Key Concepts: Tutorial Problems on Basic Concepts of Radar

Welcome to this NPTEL lectures on Principles and Techniques of Modern Radar Systems. So, in last class, we have done tutorial I on Basic Concepts, we are continuing that.

Actually what I scheduled earlier that one such problem left many one important problem.

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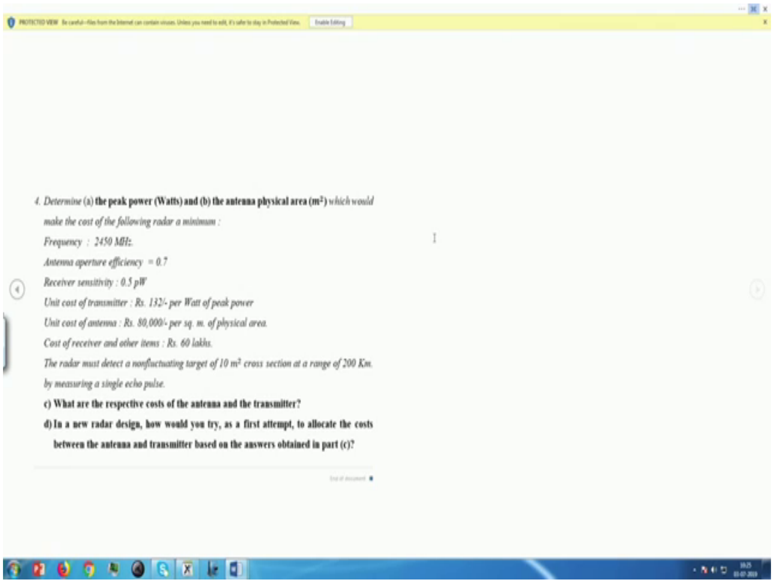
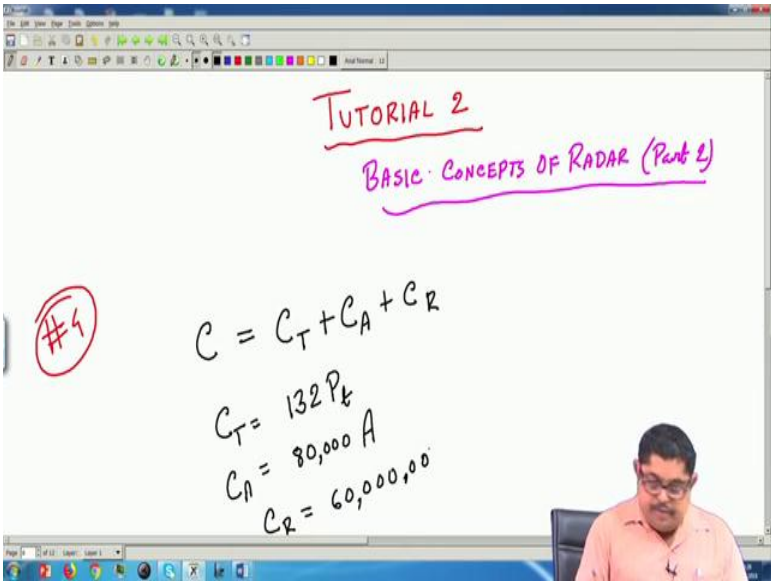


So, that we will see now you see this 4th problem of the previous tutorial. It is a very very interesting problem. Determine the peak power and the antenna physical area which would make the cost of the following radar a minimum.

So, frequency is given antenna aperture efficiency is given, receiver sensitivity is given. Unit cost of transmitter is rupees 132 per Watt of peak power, unit cost of antenna rupees 80000 rupees per square meter of physical area, cost of receiver and other cables etcetera other items 60 lakhs. Also the radar must detect non a fluctuating target. What non fluctuating target means that later we will see. Let us say that it has a constant RCS of 10 meter square cross section at a range of 200 kilometer by measuring a single echo pulse.

So, you will have to find what are the respective cost of the antenna and the transmitter and then based on that in a new radar design, how would you try as a first attempt to allocate the cost between the antenna and transmitter based on the answers obtained in part c. So, you will have to find you see peak power and antenna physical area which would make the cost of the following radar a minimum; also you find the respective cost of the antenna and the transmitter. So, four things need to be found out. Now apparently it looks like an economy problem, but, no, to solve this problem also you need a model.

Actually you see in anything you need to analyze in engineering, you need a model. So, radar is a system. So, to evaluate its cost also scientifically, you will have to have a model and that model is nothing, but radar range equation. So, you see that very elegantly if we use the radar range equation model. So, I am writing this is problem 4. And so, since it is a question of cost let us say the total cost, I am breaking into three parts as suggested by the problem that cost of transmitter plus cost of antenna plus cost of receiver.



Now, it is said that C T that is 132 rupee into P t that is said that per each Watt. Similarly for C A. It is said 80000 into area of the antenna and for C R it is say it a fixed cost.

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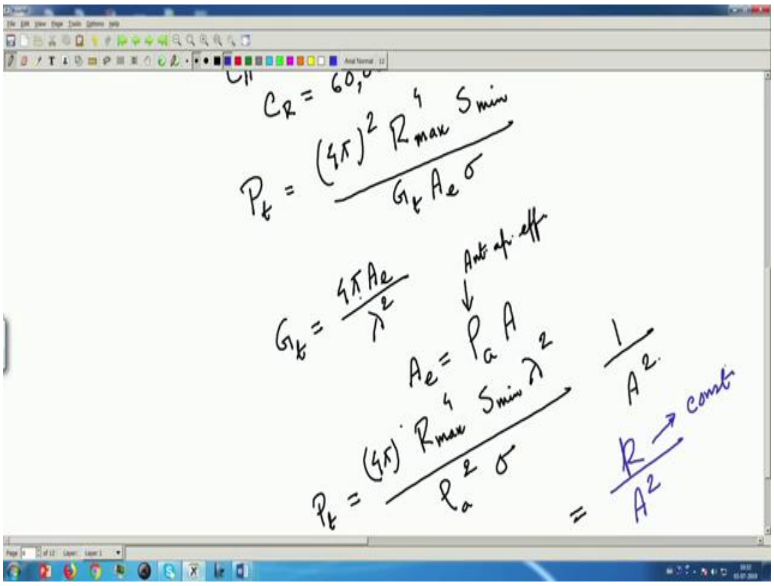

So, now let us come to the radar range equation. We know P t is 4 let us select P t to R max is given. So, let us connect to R max; R max whole to the power 4 S min then G t A e sigma. So, you see deliberately I have taken this A e in the receiver receiving antenna portion because it will be then related to these physical area of the antenna.

Now, what are the things what is now G t is not said in the problem, but area of the problem will come out. So, we know G t can be converted to area effective area of the antenna by this formula 4 pi A e by sigma square. So, that we will do and also we know what is the connection between effective area or electrical area of the receiving antenna and the physical area that connection is through the efficiency which antenna aperture efficiency. So, that I think this we are calling antenna aperture efficiency. This is in all antenna classes this is said. So, A e is equal to a where this is the physical area of the antenna.

So, putting all these in this equation, we will make it more simple in terms of the our required quantities. So, it will be 4 pi we will see. So, one 4 pi will come from here. So, 4 pi R max whole to the power 4 S min lambda square by rho a square into sigma. So, this is will be then I am writing it as 1 by A square. Now you see for the radar, this first ratio that is all are constant terms because R max is fixed, S min is a, for a receiver it is fixed the carrier frequency is fixed, aperture efficiency of the antenna is fixed, RCS of the target is fixed.

So, I can now write it as that some constant k by A square where k is a constant ok. So, P t; that means, is inversely proportional to A square.

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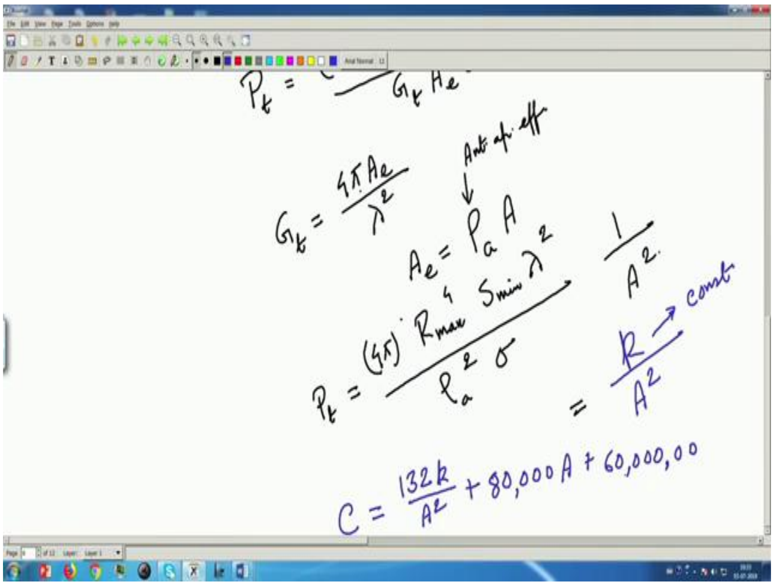

So, we can say that if this is the case. So, we can say that C is 132 k by A square; this one I am now writing plus 80000 A plus 60 lakhs.

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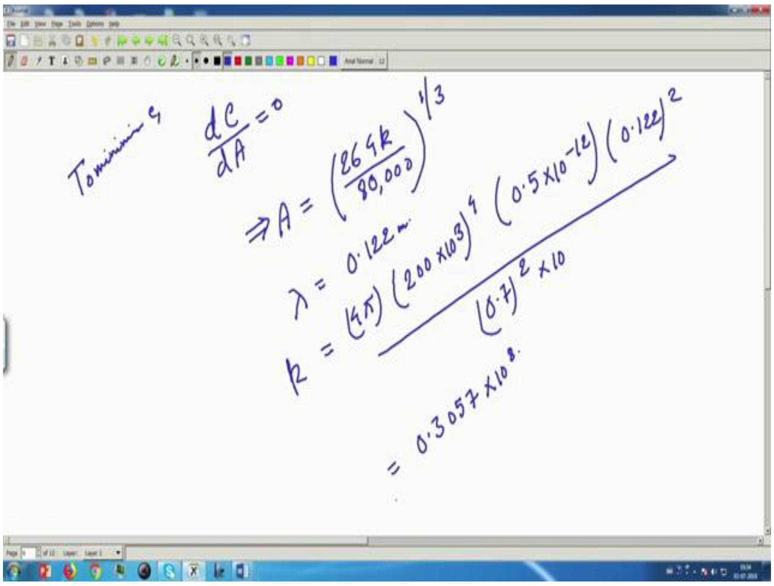

Now, if I want to minimize C so, there it is only depends on A you see. So, with respect to A, I will have to differentiate; that means, to minimize C d C d A should go 0 and that gives me A is equal to 262 4 k by 80000 whole to the power one -third.

So, lambda is given we know what is the value of lambda is 0.122 meter. So, k now we can find out we can evaluate the value of k because k previously we have that expression there we now can put because now this k needs to be found out. So, k is 4 pi into 200 into 10 to the power 3 whole to the power 4 into 0.5 into 10 0.122 square by 0.7 square into 10. So, this will be 0.30507 into 10 to the power 8. So, we have got k, we have got the value of A which is making the cost minimum.

So, now one by one we can answer various parts of the question that what is part a part a already we answer that A is so much what is a physical area.

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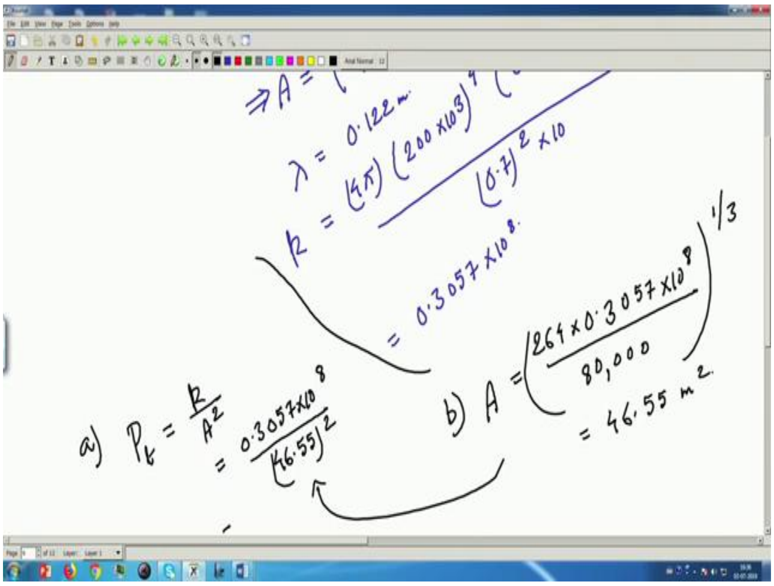

So, now, a will be then P t is equal to k by A square and so, it is 0.3057 into 10 to the power 8 by 46.55 whole square now. Where from A is coming? Actually first we will have to find A then we can come here. So, actually better b can be first found. What is b? b is A is equal to 264 into 0.3057 into 10 to the power 8 divided by 80000 whole to the power one-third. So, that will 46.55 meter square. Once we get this A, we can put it here and so, this value will be.

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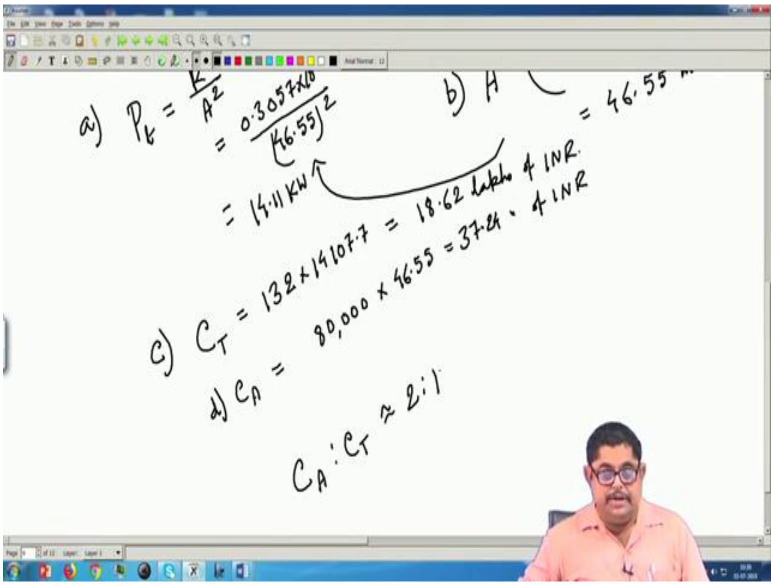

So, putting that we will get is 14.11 kilo Watt. You seen that by radar range equation, we are not only getting the what is the transmitter power required? That is 14 kilowatt peak power and what is the antenna size physical area 46. So, both are coming. So, it is not an actually economic question; it is a question of correct modeling. Then C can be now easily found what is C T?

So, C T will be; that means, transmitter power 132 into 14107.7 or 14.11 or so. So that will be 18.62 lakhs of Indian rupees. And what is C A? C A will be 80000 into 46.55. So, it is 37.24 lakhs of INR. Now this is an interesting thing, please see this is not so, obvious that an antenna cost 37 lakhs. The transmitter we think, it is a high value high power thing; it is having you see 14 kilo Watt power, but actually the technology why is antenna technology is much sophisticated that is why its cost is more.

So, what is the outcome of these that actually that is what you said you always remember these that what is this ratio typically C A is to C T. Roughly we can say this is 2 is to 1. So, while allocating fund, always you should give more fund for making the antenna because antenna's technology is more sophisticated for any microwave things. So, that's this problem then let us come to the other problem.

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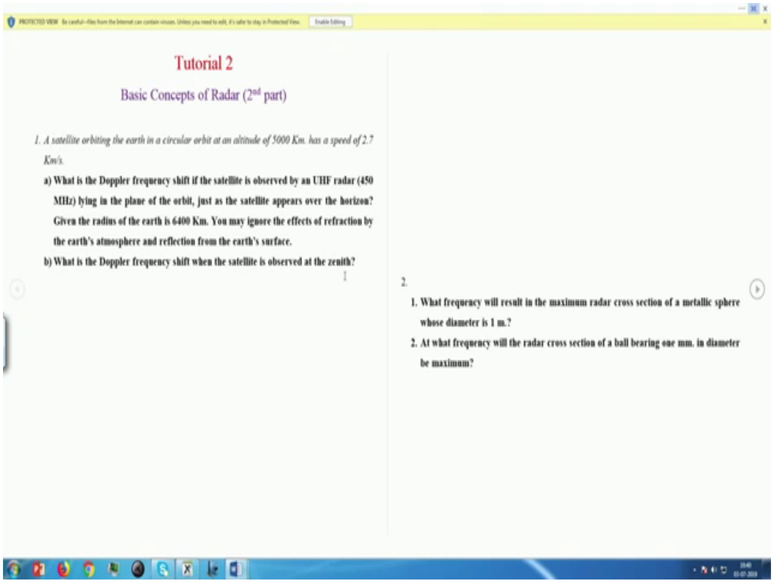

You see this is also the problem 1 of tutorial 2. A satellite orbiting the earth in a circular orbit at an altitude of 5000 kilometer has a speed of so and so.

What is the Doppler frequency shift? If the satellite is observed by an UHF radar operating on 450 mega Hertz lying in the plane of the orbit just as a satellite appears over the horizon. Given radius of earth is so much you may ignore the effects of refraction by the earth's atmosphere and reflection from the earth surface. The same thing is asked what is the Doppler shift when the satellite is observed at the zenith. So, interesting this is on the Doppler shift concept. So, let us see this (Refer Time: 16:07).

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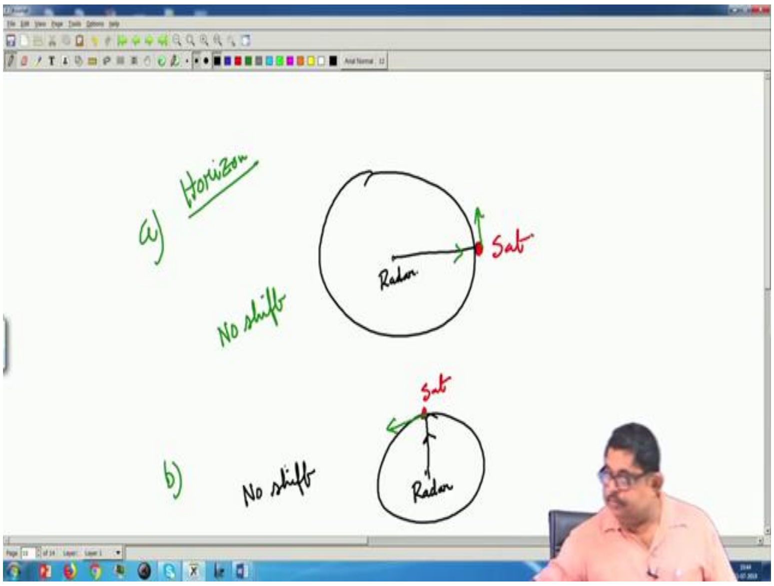

So, here you see if this is the earth and so, let this is the radar and the satellite now horizon. What is called horizon? If you look from the horizontal direction, the point where you are seeing the satellite so; that means, a satellite is here. So, let me use some other color this is a satellite ok. Now the question is what is the direction of the satellite's velocity. You see if it is here. So, satellite is this radar is this, so, radar is looking here these velocity vector is perpendicular to this.

So, what will be the Doppler shift? Doppler shift will be 0. So, this is at the horizon. Now the question is second part. So, this was the first part, this is the horizon part. So, there will be no Doppler shift. Please remember this thing that it appears that there may be some shift etcetera. But the two vectors, they are perpendicular the range vector; this is the range victor, this is the velocity vector. So, there projection on these will always be 0 that is why no shift.

The second part is that you have the same problem radar. Now the satellite is at the zenith; zenith means at the top; that means, just on top of your head. So, this is the satellite, again what is its velocity? Its velocity is this and this time what is the radar range direction this? Again you see these two are perpendicular. So, always you will see that these two are perpendicular that is why here also no shift; remember this ok.

So, next problem what frequency will result in the maximum radar cross section of a metallic sphere whose diameter is 1 metre? So, what frequency will result in the maximum radar cross section of a metallic sphere whose diameter is 1 metre? So, you have a metallic sphere now at what frequency because we have seen that graph the how a sphere's RCS varies with the radius of the sphere. So, based on that, we will have to answer this.

Similarly, in the part 2, at what frequency will the radar cross section of a ball bearing whose diameter is 1 millimetre will be maximum; two ways of saying so; that means, same thing, but you see one is 1 metre diameter metal, another is 1 millimetre metal. So, just to give you an idea what is a frequency there.

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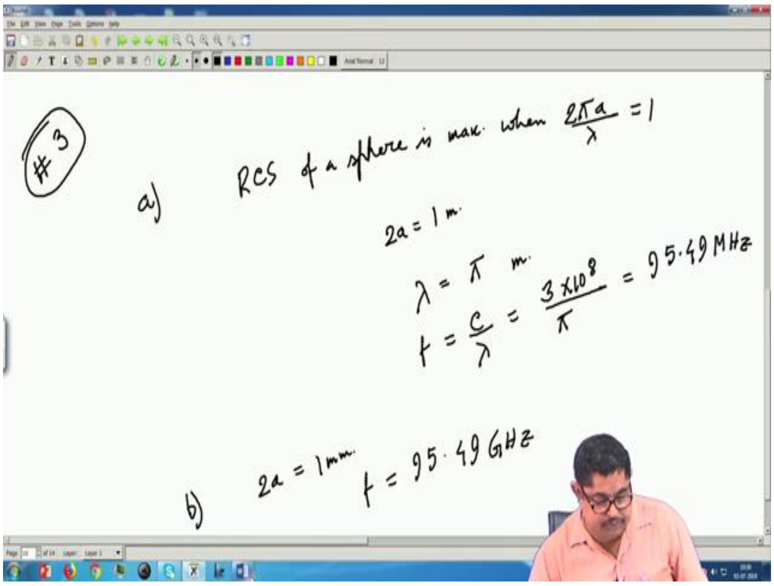

So, what I will call this is problem 2 problems 1 2. So, third problem, let us say. So, here we can say the in part 1 RCS of a sphere metallic sphere is maximum RCS of a sphere is maximum. You can see that graph we have discussed in the class when 2 pi a by lambda is 1. So, here it is given that 2a is 1 metre. So, we can say that lambda will be pi metre and. So, lambda pi, now I will have to convert it to frequency. So, frequency C by lambda so; that means, C is 3 into 10 to the power 8 and lambda is pi is value pi. So, that will give me 95.49 mega Hertz.

So; that means, if I have a 1 metre diameter things at 95 mega Hertz roughly; it will be very good; that means, if you want to observe a 1 metre diameter something so, frequency you should choose. If all other things are not considered just from choosing frequency you will choose a VHF type of radar. Now the same thing I want to observe now, a ball bearing of 1 millimetre. So, here only change will be that 2a is 1 millimetre. So, now so, rest of the thing you do and so, f will be 95.49 giga Hertz.

So, this is an important thing that you see that roughly the smaller, I want to observe I will have to go high in frequency. Suppose rain generally normal radars like UHF radar or microwave radars, rain do not affect them much. But if I want to observe rain, I said also in the class that you like to scale up the frequency. So, if I want to observe rain, if I want to observe weather, I will have to go high in frequency. So, that then the RCS of the thing becomes. So, this is also an important lesson. Next problem let us go.

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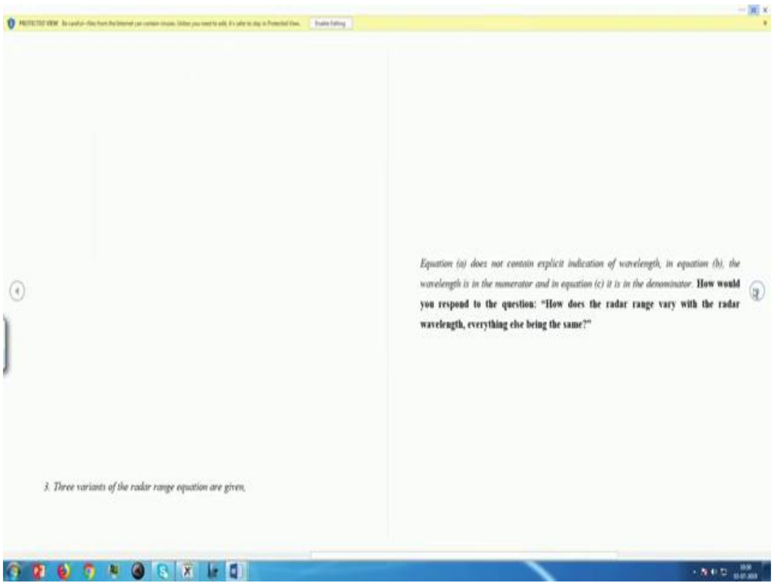

So, this one we have done, then three variants... So, I will have to give you those formulas actually we will show you in the books etcetera. If you just manipulate there are three radar range equations are given. Out of that one will be does not contain explicit indication of wavelength. In equation b, the wavelength is in the numerator and in equation c it is in the denominator. How would you respond to the question? How does the radar range vary with the radar wavelength everything else being the same?

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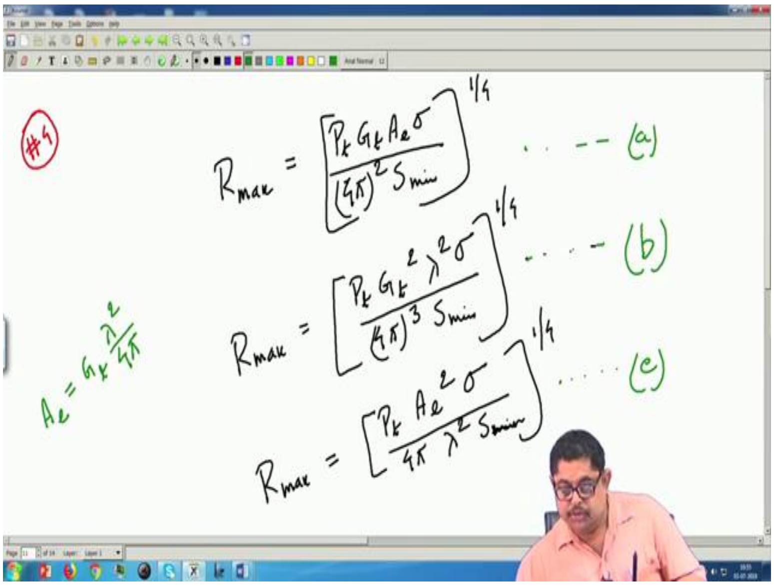

So, what I will do; let me call this 4 now. So, and I will give you that, you can see in the books the R max, you can write it as P t G t A e sigma by 4 pi whole square S min 1 by 4. Let me call this one equation a. Also if I just make this A e put its value in terms of G t, I can also write this as P t G t square lambda square sigma by 4 pi whole cube S min whole to the power 1 by 4.

On the other hand, if I eliminate G t; that means, put this G t in terms of Ae, you see from this equation this equation is both G t and Ae are present. Now in sometimes we make that ok; if G t is known we can convert A e to G t or G t to A e, we know that conversion. So, if we write it in terms of A e, then the equation becomes P t A e square sigma by 4 by lambda square S min whole to the power 1 4th.

Now, interesting thing is you see that in the first equation, the lambda is not present explicitly in the second equation, lambda is in the numerator. In the third equation, lambda is in the denominator. So, the question is how does the radar range vary. This is the range how radar range vary with lambda. So, you see you can do this three things by remembering that what is A e; A e is G t lambda square by 4 pi. So, actually A e and G t r related. So, by that these are the three variations. Now the question is then what is the actual dependence? So, what we will do we to solve this? (Refer Time: 28:45).

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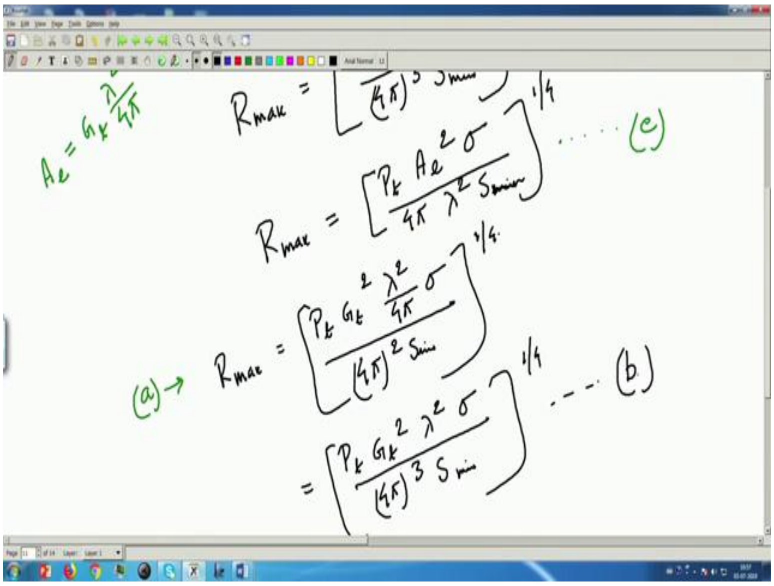

So, if we start from equation a, we can write that R max is P t G t. So, we are putting the value of A e, G t square lambda square by 4 pi sigma by 4 pi whole square S min to the 1 4th. So, this will become P t G t square lambda square sigma by 4 pi whole cube S min whole to the power 1 by 4 which is nothing, but equation b.

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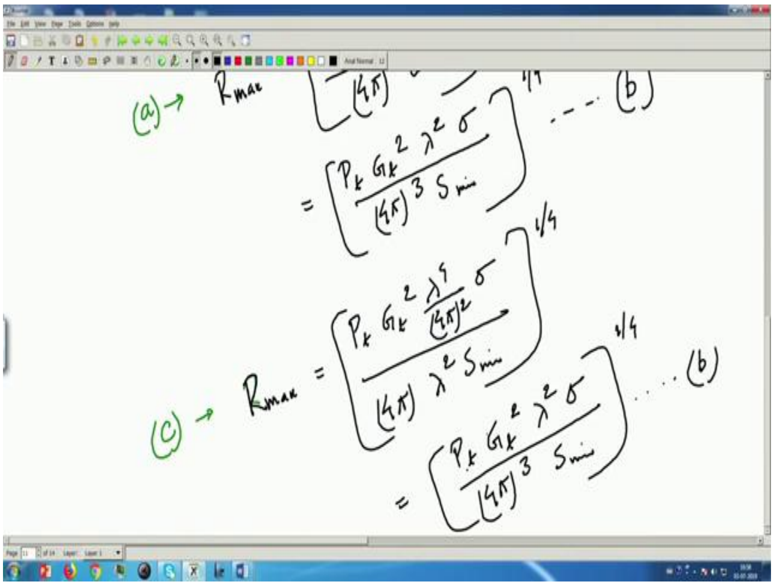

So, starting from a we can go to b and again from starting from equation c, we can say that R max is equal to P t G t square lambda 4 by 4 pi square sigma by 4 pi lambda square S min whole to the power one-fourth is P t G t square 4 pi whole cube S min 1 by

4th which is again your equation b. So, equation b is the one you see, P t is does not have any lambda dependence; G t also once an antenna is there it does not have any a dependence.

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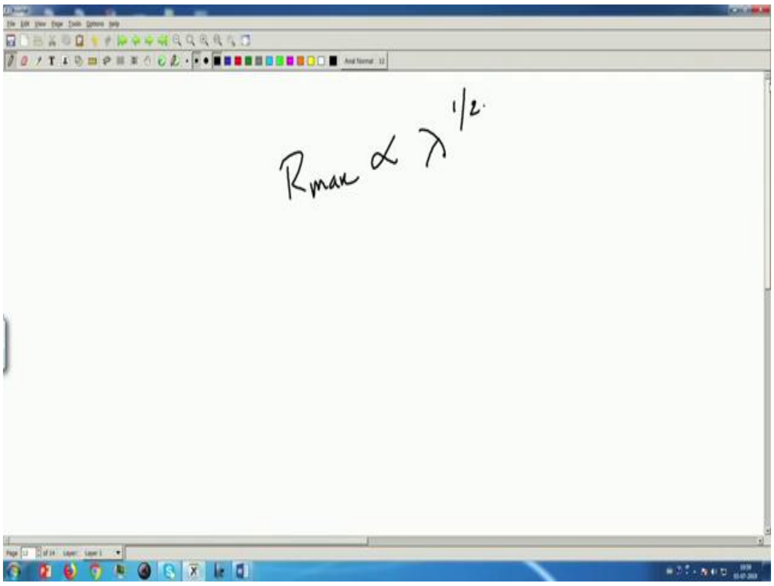

So, basically we can say that all equations are same and so, R max that is varies with lambda to the power half, you see lambda square 1 by 4th. So, it is square root proportional to lambda. So, that is the main thing; that means, if you scale up lambda in a square root fashion the R max will scale up. So, that was the third problem I thing time is up. So, we have seen enough problems on this basic concepts that what is RCS, this radar range equation various thing.

Always remember in this last problem that actually A e, this is a electrical concept; A e is electrical concept that is why the lambda is directly affecting this effective area. It is a known from antenna theory etcetera. So, do not think that effective area is of an antenna is independent of lambda. It is highly dependent on lambda and that is why you are getting all these apparent anomalies, but actually there is no anomaly the radar range that varies as a square root of lambda ok. So, thank you from next day we will start the CW radar.

Thank you.