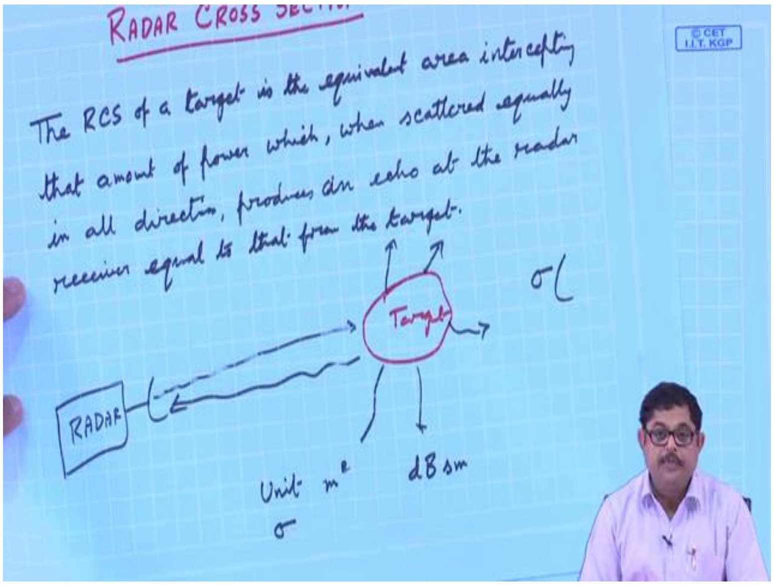
Principles and Techniques of Modern Radar Systems
Prof. Amitabha Bhattacharya
Department of E & ECE
Indian Institute of Technology, Kharagpur

Lecture - 06
Some Basic Concepts of Pulsed Radar
(Contd.)

Key Concepts: Mathematical Model of RCS, Characteristics of RCS of a Sphere

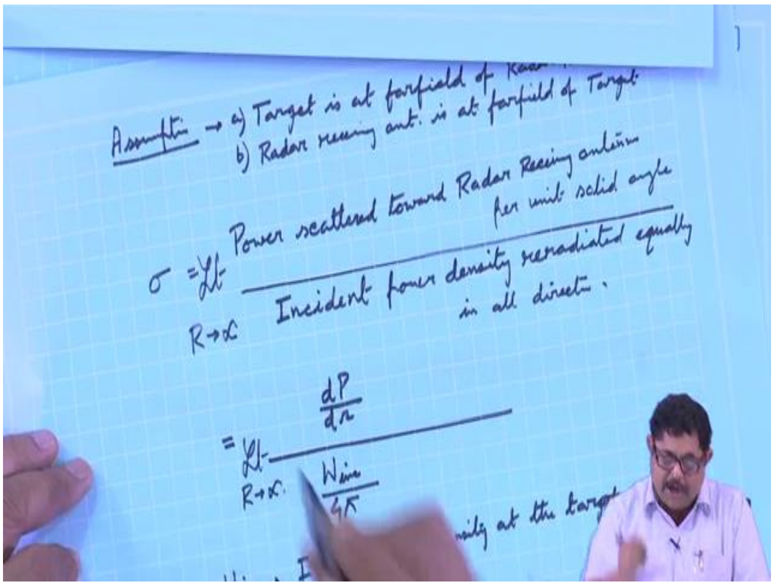
Welcome to this lecture actually we were discussing radar cross section.

(Refer Slide Time: 00:23)



So, in the last class, we have defined radar cross section. Now we will make some mathematical equations for this radar cross section. So, whatever is said here, now radar cross section its symbol is sigma.

(Refer Slide Time: 00:43)



So, I can write that whatever is said here also can be said like the way we define the directivity of the radar because this scattering is also directed in nature that it is not equally scattering in all direction. So, with respect to an isotropic scatterer, RCS thing will be detected that had this target been an isotropic scatterer what would have been and compared to that how much is the actual case happening.

So, the definition; this from the definition, it follows that it can be defined like this. Again I am writing the exact thing; you please follow this concept that power scattered toward radar receiving antenna per unit solid angle because it is a changing thing per angle. So, powers scattered toward radar receiver per unit solid angle and that should be compared with because radar cross section is the target is taking the incident power, then it is scattering; then in which direction it is scattering. So, the denominator is; that means, this thing is normalized to that had this been an the scattering energy been fully ready sorry the incident energy is fully scattered.

That means the whole energy is getting scattered and scattered equally in all direction. So, isotropic as well as the scatterer is scattering the whole power that is incident on it which is not the case actually that is why numerator, but the normalization thing is that ideal thing. So, incident power density reradiated equally in all direction. You see the numerator is a power and power per unit solid angle, but that is power whereas, this is power density. So, how they will be related? You see this is an area. So, incident power density multiplied by the area will give you the power scatter. So, this is the definition.

So, can I say that actually this it is coming from this definition. If you read it carefully, you will get that these are the things. So, sigma you see it is embodying various physical things. First is incident power is falling, how much effective area it has to take that power electrical area, effective electrical area that is taking that power then how much is getting scattered then in the desired direction relatively how much power is coming so, all that is embodied in this.

Now we will take the help of radar equation and try to manipulate these because still with all these how we will express it. So, this whatever is said can I say that that is dP d omega where omega is the solid angle because a it is scattering differently in different direction; different amount of power in different direction.

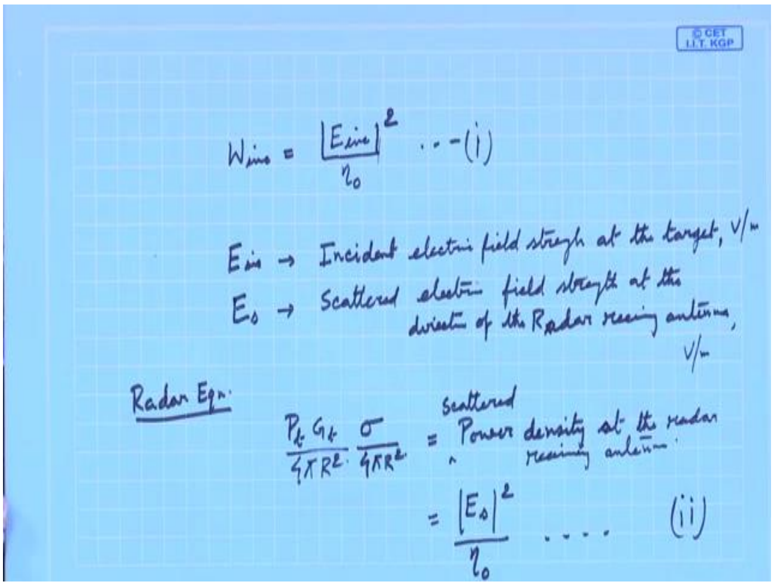
So, if I want to know that in the radar direction, how much exactly power is coming; obviously, it is a changing quantity. So, from the concept of calculus, it will be dP d omega. So, that is the numerator and here incident power density. So, let us call that incident power density is generally be W incident W incident power density that divide.

If it reradiates equally in all direction, so, what is the total that power in all direction, how much it will be power density that will be W inc by 4 pi because 4 pi is the total solid angle for a 1 meter at a distance. So, W inc by 4 pi.

$$\sigma = \frac{\frac{dP}{d\Omega}}{\frac{W_{inc}}{4\pi}}$$

So, this is a definition where what is W incident; let me say, it is the incident power density at the target and its unit is Watts per meter square. Now, I will take the help of the model that we have derived radar equation and try to find out an expression for this which is usable.

(Refer Slide Time: 06:13)

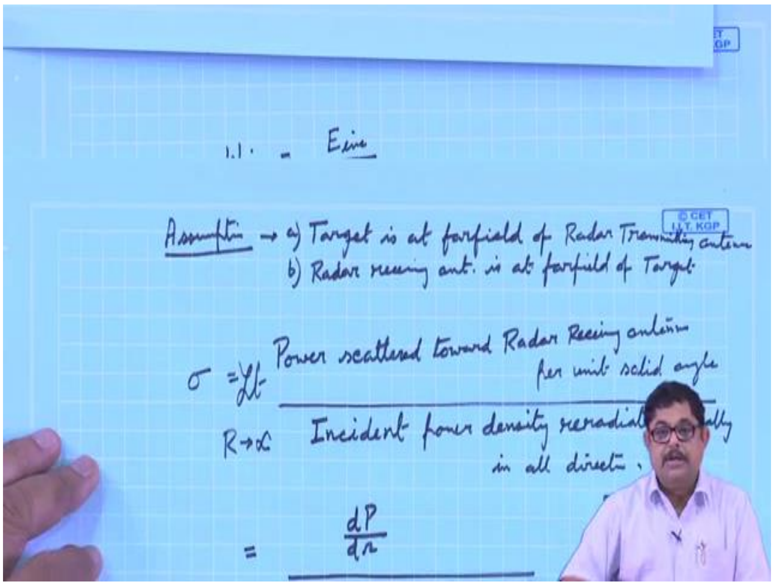


So, from radar range equation can I write that what is W inc; that means, the incident power density that we know that that is

$$W_{inc} = \frac{E_{inc}}{\eta_0}$$

Because, this is here there is an assumption that assumption we have not said, but in the radar always that in our radar equation that assumption was there. So, let me write that assumption that target is at far field.

(Refer Slide Time: 06:47)



So, this is assumption a target is at far field of radar transmitter; transmitting antenna and b, the radar receiving antenna is at far field of target. What is this meaning? The first one is obvious, the second meaning means the target when it is scattering. So, as if it is a radiator so, it also has a far field that can be found out from 2 d square by lambda t.

So, the radar receiver or radar receiving antenna that is at far field of the; that means, both these things are very much separated. And so, with this assumption I can say actually plane wave is falling on the target and when the radar is sensing the echo scattered by the target that is also in the form of plane wave.

So, we can use plane wave expressions then only that. So, I will say that all these things actually says that mathematically, this means limit R tends to infinity. Actually in actual cases not infinity; actual case is far field far away so, in all these definitions I should have limit R tends to infinity.

Now, if that is there, then can I say that what is the power density is the electric field incident; its magnitude square by the intrinsic impedance of free space. So, let me write what is E incident; E incident is incident electric field strength at the target. So, what is its unit electric field unit? Electric field intensity that is volts per meter; now this thing also I am here will be requiring another quantity which is called E s let us say; E s if we name it. So, E s will be the scattered electric field strength at the direction of the radar receiving antenna. So, both of these I will also write the units volts per meter; this unit will be volts per meter.

So, already we have seen that this is a for plane wave I can write that what is the power density for the incident wave that is the E square by eta. This is in the far field; this relation is there for plane wave. Let me call this equation, equation one. Now, let us find out what is the power density that is scattered back at radar receiver and that will express in terms of this scattered electric field because these quantities are measurable. I can measure what is the electric field strength incident on the target.

If I could have been on the target and put a probe I could measure similarly here I could measure, what is the radar receiver there. If I put an power though they are generally not made, but let us develop the model. So, now I take the help of the radar range equation. So, I say that from radar equation that we have developed, there we have seen that what is the scattered power. Here you see that transmitter is again you think of this model that time we have seen. So, radar is now transmitting. So, P t G t by 4 pi R square is falling here P t G t by four pi R square is the following there, then a portion of the radar cross section we have multiplier.

So, that is the power scattered there and then towards the receiver and again it is travelling as a spherical wave and at the far field. So, again it is travelling a distance of four pi R square. So, this is what I can say that this is nothing, but the power density or I can say scattered power density scattered power density at the radar receiving antenna and scattered power density. So, this is in the form of plane wave. So, I can say it is again can we say that the magnitude of the field.

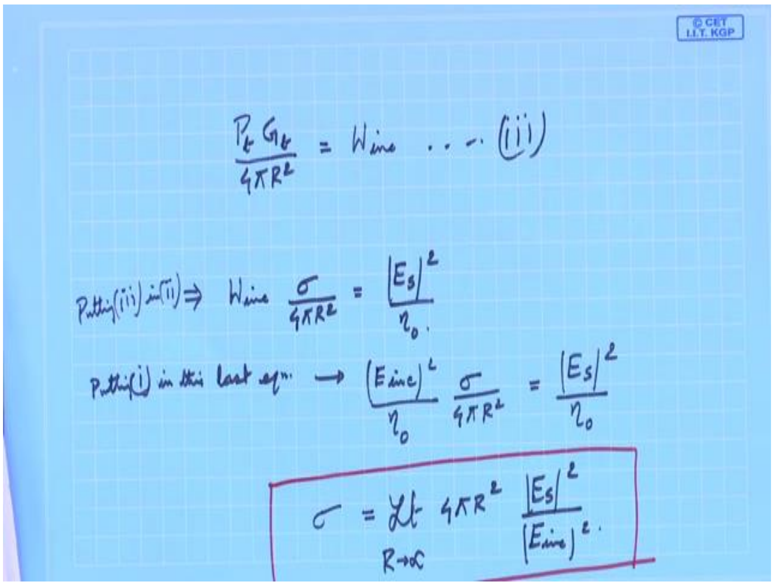
Field is a vector quantity. So, we will have to take the magnitude; magnitude square by eta naught.

$$\frac{P_t G_t}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2} = \text{Scattered Power density at the radar receiving antenna}$$

$$= \frac{|E_s|^2}{\eta_0} \quad \ldots \quad \text{(ii)}$$

So, this is my second equation ok. Also one more thing I can write that actually you see that what is the this is one sided value of these, but from radar range equation, I can have a third equation that you see the it is transmitting radar is transmitting.

(Refer Slide Time: 13:41)



So, P t gain of the transmitting antenna G t by 4 pi R square. What is it? It is nothing, but I can say W incident.

$$\frac{P_t G_t}{4\pi R^2} = W_{inc}$$

I have given it a name to simplify this, but actually this is my third equation. So, now I have got three equations. One is this ii is this and iii is this. Now, this third equation; if I put into second equation, what we get? So, putting iii in ii, I can write

$$W_{inc} \frac{\sigma}{4\pi R^2} = \frac{|E_s|^2}{\eta_0}.$$

and I have this i. So, this I can put in place of this W i.

Now I can say putting i in this last equation, I can say that

$$\frac{(E_{inc})^2}{\eta_0} \frac{\sigma}{4\pi R^2} = \frac{|E_s|^2}{\eta_0}$$

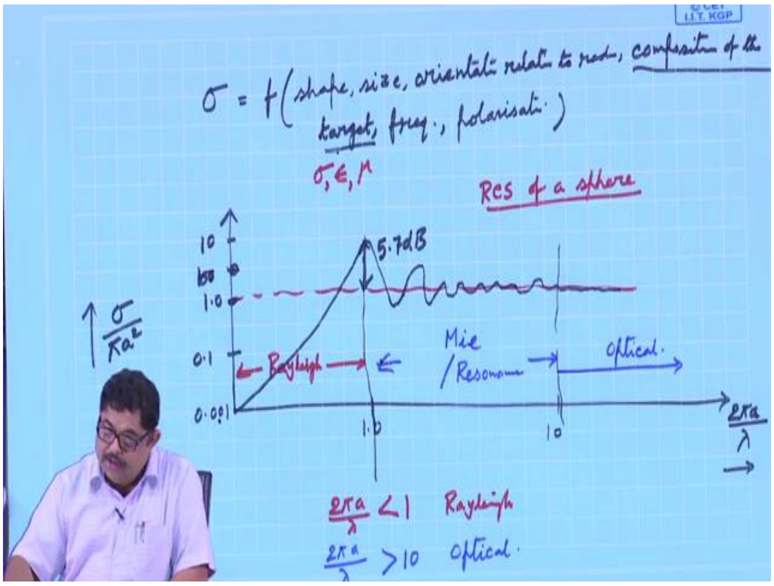
So, if I simplify I can write sigma is

$$\sigma = 4\pi R^2 \frac{|E_s|^2}{|E_{inc}|^2}$$

Only thing is the assumption that I will put in the form of a far field assumption. So, this is a much simpler thing. What it says that four pi into R square and then you have the E s magnitude square divided by E inc square.

This actually is an usable formula. So, this actually we will be using many times. That you can find out RCS of any body by measuring these. Actually people do that or people also simulate etcetera to do this. As I already said that what is sigma's physical significance, it is the scattering effectiveness of any body. So, anybody how much it is scattering in a particular direction when it is getting illuminated? that is the measures of this.

(Refer Slide Time: 17:07)



And it please remember always that it depends on various things that sigma is a function of I can say shape of the body, the size of the body, the orientation of the body relative to the radar orientation relative to radar, then composition of the body, the body of the body or target. Then it also depends on the illumination things. That means what is the frequency of the illuminating electromagnetic wave, then what is polarization of the electromagnetic wave etcetera.

And when I say composition of the target basically I mean that electrical parameters for that. So, composition of the target, this thing actually means what is the sigma; that means, the conductivity of the target, what is the epsilon the permittivity of the target, what is the permeability of the target. All those affects this thing.

So, it is depends on all these so; that means, if you want to find out the RCS, there are ways either you can calculate; that means, apply Maxwell’s equation. There you know all these things of the body. So, a given picture is there that this is the orientation, this is the medium etcetera and there you apply Maxwell’s equation solve that. But it is very difficult to do.

I mean analytically it is not tractable for most of the things, only for some canonical objects; canonical means which has a proper shape like sphere, plate, cylinder, cone. In those cases, people have found the RCS things and, but it is good that actually from this canonical things, actual objects suppose an aircraft or a ship. If they can be approximated by these canonical objects, then people have try to find their RCS. Now a days with numerical techniques, the simulation etcetera are performed and there are moment methods and other methods numerical methods by which people try to find this out and.

But later see that a closed form expressions of this RCS that also give some interesting things like there are closed form approximations for those plates spheres etcetera outer. But all those cases you see that orientation relative to radar, we call it aspect. So, aspect angle is important. So, generally this RCS is a strong function of this aspect, but there is one canonical object which is a sphere which is invariant to this aspect.

That is why sphere it’s easier because whatever aspect you take the RCS is same because sphere does not by its shape actually it is independent of the aspect. So, people have found out closed form expressions for this and that let us discuss that that RCS of a sphere. So, I will say that RCS of a sphere.

Now, what is in the x axis? It is generally the diameter sorry periphery of the sphere, but that is normalized usually to the wavelength because actually this means the electrical perimeter or a electrical length of the perimeter. And this side is the RCS, but usually that RCS is normalized to the physical area. So, physical cross section of a sphere that is a circle and the radius of the sphere is a. So, I can say pi a. So, this is y axis, this is your x axis. So, RCS is plotted here; that means, what you are plotting that what is RCS value for various sizes of the sphere because here you are changing. So that means, you are actually changing the diameter and finding out that what is the RCS.

So, there are points. Let us say this is our 1, this is our 10. So, and this value let us say that here in this direction 0.001; let us say here it is 0.1. So, it will be one here and I can say this is 10 here. So, if you plot what color I am using; so, change 1 because it will be somewhere here. So, you see, it is something looks like a second order function that when we have a second order circuit and then a second order LCR circuit; you see that we have this that there is a peak and then gradually dies down.

So, this is the steady state. Now, based on these actually these sizes of scattering or the scattering phenomena is broken into regions. One region is this actually this 1 actually this zone and this 10 this zone. Now this region is called; that means, when 2 pi a by lambda is less than 1, this zone is called Rayleigh region. This is a Rayleigh region what is it is when the 2 pi a by lambda is much less than one that is Rayleigh. When it is greater than 1; let me use other color. So, this zone is called optical region.

What is it 2 pi a by lambda is greater than 10. This is optical and what is this zone? This is called Mie region; Mie or also various people call resonance. So, this zone now what it says you see in Rayleigh, the RCS is quite small. So, as I am increasing the object, RCS is not much. You see the values they are not much; that means, the physical area pi a square, but your RCS is let us say typically 0.1, 0.2; that means, what if you have a 1 meter square, but RCS will be much less than that; so, very small amount of power is scattering here.

Whereas you see actually this here there are ups and downs so; that means, the object has an it is changing. So, it can be going high then coming down. So, it is a resonant sort of thing that it is changing, but in the optical region it is going flat and reaches a steady state value of 1.

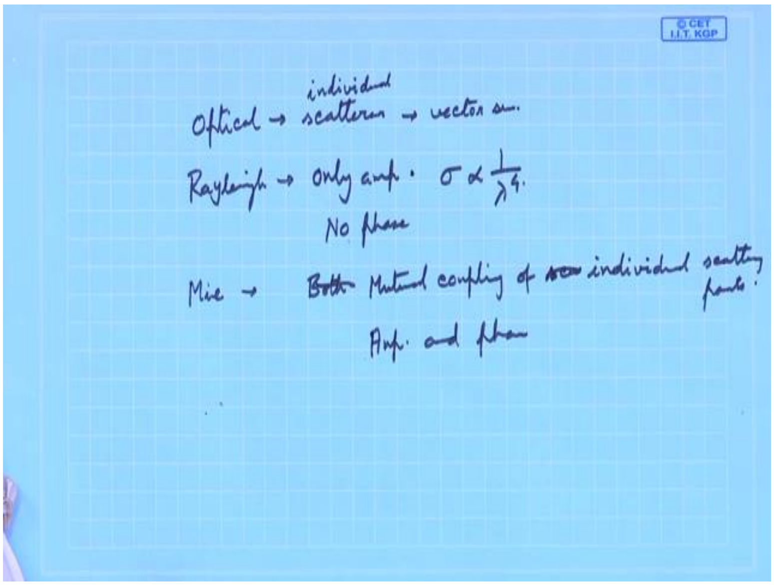
What does that mean? That means, in this region the electrical area and optical area, they are same; that means, there not much power compared to the; that means, the area is more or less as if the whole power it is taking as if it is a physical area so, not much scattering is there. But here you see actually this peak; this peak happens this peak is around 5.7dB.

So, compare to 1 this value is so, I can say that this is 5.7 dB up at the maximum. So, that is important; that means, here it is a very good scatterer. If you have this size is 2 pi a is equal to 1, then it goes up; that means, it is a very good radiator here also, you see it a good radiator.

So, there are various sizes where it is good and generally the microwave illumination of various objects, they come here. Now this is one of the problem. Here you see there is an interference sort of pattern. Why that is coming? If we people have explored that and found that in Rayleigh region actually the various scattering centers of a sphere, they are not interacting with each other.

And also they are very weak so, their total sum is same. In the optical region, this scattering centers of the sphere or of anybody actually this classification is true for anybody. Though we are discussing it with sphere, but for anybody you can have this three separate regions usually microwave come in this Mie region. There is and the interference pattern and in the optical region the scattering centers, they do not interact with each other; they simply behave as a individual localize thing individual scatterers. So, RCS is vector sum of that.

(Refer Slide Time: 29:17)



So, I can say that the in the optical region, in the optical region the scatterers they are I can say individual scatterers and only their vector sum is to be taken. So, and it is a that is why it is a steady value because they are not interacting. And, in Rayleigh region this thing actually not even the vector sum only the amplitude factor their sum and only amplitude and that amplitude is it can be shown that the RCS is there almost proportional to 1 by lambda by 4.

And no, phase while addition, there is no phase addition also, but in the Mie region the whole problem is both when the scatterers there is mutual coupling. So, I will say that mutual coupling of all scatterers of individual scattering points individual scattering points and so, the total vector sum that is have a strong function of amplitude and phase and that is why that this type of interference sort of thing. That is why it is very difficult to model it analyze it etcetera. But you see that due to this Rayleigh region if you can make that the target is very small compared to the lambda, then the RCS is very small.

So, you think that suppose I am trying to detect an aircraft; now raindrop suppose generally microwave region means typically I can say, the 10 gigahertz. Suppose 3 centimeter is the wavelength. So, anything which is less than this 2 pi means 6; so, anything which is of millimeter of size; suppose a raindrop its 1 millimeter diameter typically.

So, that will give you give me very small return. So, that is why we say that radar is invisible to rain radar is invisible to snow, radar is invisible to fog. This is an advantage that rain etcetera; they do not create any problem clouds rain, there are. So, this is the chief advantage of radar in bad weather.

On the other hand; if you wants to observe raindrop, then you will have to increase the RCS of that. So, what we will do? You now go to very high frequency; that means, you reduce this then you are going to Mie region you observe it. So, that is why weather observing radars they are of high frequency cube and much higher etcetera higher frequency. Now already I said in Mie region the maximum, RCS is 5.7 dB. So, always be careful that you can have for a sphere maximum RCS, you can get 5.7 dB up from its physical area.

Now, let us now come to the RCS measurement. If we want to do RCS measurement generally for a complex target like in aircraft ship etcetera, then we will have some more things to discuss on these. Actually we are spending much time on this RCS because, there are lot of conceptual things here and measuring RCS is a very important thing. Also in military technology you want to conceal your RCS that is called stealth so, that also is a very important application that how you have your aircraft etcetera aircraft, ship, missiles, but I want to conceal that i; that means, to the radar I want to become invisible.

That means I want to reduce my RCS. So, that is called stealth technology. So, RCS measurement RCS evaluation reducing RCS in certain cases even enhancing RCS all these are very important considerations and lot of work goes on there lot of research is still going on in this field. So, we will discuss something more also in the next lecture

Thank you.