

**Principles and Techniques of Modern Radar Systems**  
**Prof. Amitabha Bhattacharya**  
**Department of E & ECE**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 59**  
**Ground Penetrating Radar**

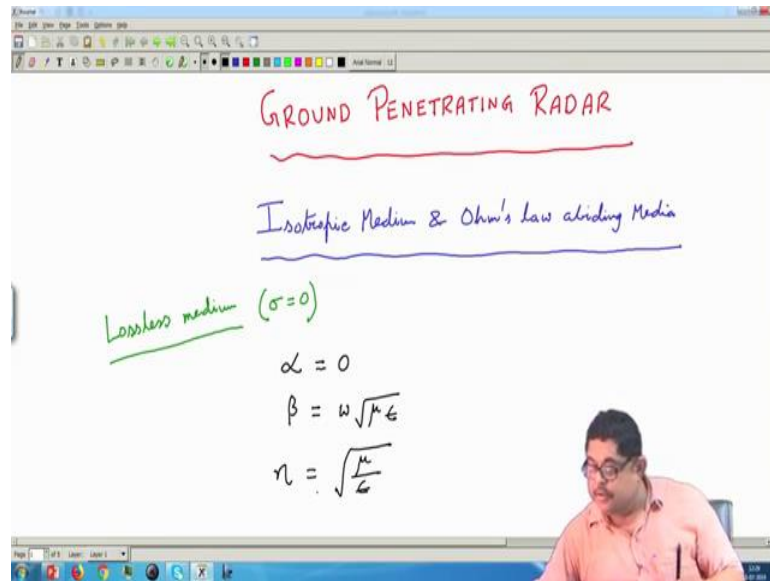
**Key Concepts:** Introduction to the ground penetrating radar (GPR), Maxwell's equations for Ohm's law abiding medium, analytical characterization of Ohm's law non-abiding medium

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar Systems. We have seen the various radars. Actually today we will see a new thing which is happening for some time this radar is mainly for civilian application. And actually in the coming decades, this radar will proliferate our daily life. This is called Ground Penetrating Radar.

Generally radar sees through free space, now these radar can see through mediums. Basically it started with ice under ice whether anything is they are polarized measurement etcetera. Now it is seeing inside the or under the surface of the earth. And there are in future it will see through anything see through walls, see through buildings, see through debris, see through forest etcetera.

So, this is a new type of radar modern type of radar; obviously, this research is still going on in this thing. So we will see this ground penetrating radar the basic principles today.

(Refer Slide Time: 01:50)



So, actually you see that in electromagnetic theory, we have seen that if an electromagnetic wave falls on a medium. Then depending on the permittivity permeability and conductivity of the medium, we can characterize the medium. And there are losses in the medium, there are phase changes as the way propagates through it the velocity of the wave gets changed and also there are the current density that gets changed. Also polarization effect come so, the electric field also gets changed etcetera.

So, we will just rephrase those things and then see that for if a radar wants to see below ground then the theory. Or certain assumptions that are made for normal materials that needs to be changed so, that we will see first. So, first I will start that basically we generally deal with isotropic medium isotropic medium and Ohms law abiding cases Ohms law abiding medium.

Actually Ohm's law is a model generally in our normal encounter in electromagnetic theory we see that the wave falls on mediums which obeys Ohms law. So, and also we generally see isotropic mediums; isotropic media means in everywhere you will see the same properties and epsilon mu etcetera they are constant they are not tensors etcetera that you have seen. Now for these there is a generally we see first a lossless medium; the medium does not have loss. This means that basically the conductivity of the medium is 0.

So, in that cases in electromagnetic theory all of you have learned in your undergraduate that the attenuation constant becomes 0 so, no loss. And beta is a real number it is given by omega into mu by mu epsilon. And the intrinsic impedance of the medium which is the ratio of the electric field by the magnetic field, that also is real and that is given by mu by epsilon. So, it's a real quantity and as it's a real quantity.

$$\alpha = 0$$

$$\beta = \omega \sqrt{\mu \epsilon}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\angle \theta_n = 0$$

(Refer Slide Time: 05:25)

The screenshot shows a whiteboard with the following handwritten equations:

$$\beta = \omega \sqrt{\mu \epsilon}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\angle \theta_n = 0$$

$$\tilde{\mathbf{J}} = \tilde{\mathbf{J}}_c + \tilde{\mathbf{J}}_d = 0 + j\omega \epsilon \tilde{\mathbf{E}} = j\omega \epsilon \tilde{\mathbf{E}}$$

$$\nabla \times \tilde{\mathbf{H}} = j\omega \epsilon \tilde{\mathbf{E}}$$

So, I can say that the phase angle of this impedance that is 0; theta n angle is the phase angle. And so, the phasor current density that which is always we know that it is the conduction current density plus, the displacement current density. Now in this case since sigma is 0 and it is Ohm's law abiding. So, J c becomes 0 and it is mainly the J d and J d we know is given by j omega epsilon electric field. So, basically the current density is solely the displacement current density.

$$\tilde{\mathbf{J}} = \tilde{\mathbf{J}}_c + \tilde{\mathbf{J}}_d = 0 + j\omega \epsilon \tilde{\mathbf{E}} = j\omega \epsilon \tilde{\mathbf{E}}$$

And also the magnetic field, because due to these current density there will be a magnetic field created that is also given by  $j \omega \epsilon \mathbf{E}$ ; that means,  $\nabla \times \mathbf{H}$  is equal to  $\mathbf{j}$ . So, this is the characterization of the lossless medium.

$$\nabla \times \widetilde{\mathbf{H}} = j \omega \epsilon \widetilde{\mathbf{E}}$$

(Refer Slide Time: 06:52)

Lossy case ( $\sigma \neq 0$ )

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2}} \left[ \sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} - 1 \right]^{1/2}$$

$$\beta = \omega \sqrt{\frac{\mu \epsilon}{2}} \left[ \sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} + 1 \right]^{1/2}$$

$$\widetilde{\eta} = \frac{\omega \sqrt{\mu \epsilon}}{\sqrt{\sigma^2 - \omega^2 \epsilon^2}} \sqrt{1 + j \frac{\sigma}{\omega \epsilon}}$$

Now, the moment we have lossy case; that means, our sigma is non 0 and that time all of you who have solved this that attenuation constant is a frequency dependent term. Beta is given by the same beta the phase constant; remember this is plus one.

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2}} \left[ \sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} - 1 \right]^{1/2}$$

$$\beta = \omega \sqrt{\frac{\mu \epsilon}{2}} \left[ \sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} + 1 \right]^{1/2}$$

Then intrinsic impedance that becomes.

$$\widetilde{\eta} = \frac{\omega \sqrt{\mu \epsilon}}{\sqrt{\sigma^2 - \omega^2 \epsilon^2}} \sqrt{1 + j \frac{\sigma}{\omega \epsilon}}$$

$$\angle \theta_n = \frac{\tan^{-1} \frac{\sigma}{\omega \epsilon}}{2}$$

(Refer Slide Time: 08:32)

$$\angle \theta_n = \frac{\tan^{-1} \frac{\sigma}{\omega \epsilon}}{2}$$
$$\tilde{J} = \sigma \tilde{E} + j\omega \epsilon \tilde{E}$$
$$= (\sigma + j\omega \epsilon) \tilde{E}$$
$$= j\omega \epsilon \left(1 + \frac{\sigma}{j\omega \epsilon}\right) \tilde{E}$$
$$= j\omega \epsilon \left(1 - j \frac{\sigma}{\omega \epsilon}\right) \tilde{E}$$

And the angle theta n the angle of the intrinsic impedance that is tan inverse sigma by omega epsilon by 2 and so. The current density, that since its still in Ohm's law abiding case. So, we can say this is the conduction part and this is the displacement part. So, it becomes sigma plus j omega epsilon in to E. So, we can take j omega epsilon out and that gives us 1 plus sigma by j omega epsilon into E and that we can write it as 1 minus j.

$$\tilde{J} = \sigma \tilde{E} + j\omega \epsilon \tilde{E}$$
$$= (\sigma + j\omega \epsilon) \tilde{E}$$
$$= j\omega \epsilon \left(1 + \frac{\sigma}{j\omega \epsilon}\right) \tilde{E}$$
$$= j\omega \epsilon \left(1 - j \frac{\sigma}{\omega \epsilon}\right) \tilde{E}$$

(Refer Slide Time: 10:02)

$$\begin{aligned} &= j\omega\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)\tilde{E} \\ \nabla \times \tilde{H} &= j\omega\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)\tilde{E} \\ \epsilon_{eff} &= \epsilon - j\frac{\sigma}{\omega} \\ &= \epsilon' - j\epsilon'' \\ \epsilon' &= \epsilon \\ \epsilon'' &= \frac{\sigma}{\omega} \end{aligned}$$

And if J is like this, then the magnetic field that becomes this.

$$\nabla \times \tilde{H} = j\omega\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)\tilde{E}$$

So, you know that now to make the whole thing simple, we define by comparing with the lossless case that the permittivity that becomes effective. So, effective permittivity we say it is nothing but sigma minus j, sorry epsilon minus j sigma by omega.

And so, this is a complex number now so, that is the thing due to the presence of the non 0 conductivity we have the epsilon complex. And so, we call it epsilon dashed minus j epsilon double dashed.

$$\begin{aligned} \epsilon_{eff} &= \epsilon - j\frac{\sigma}{\omega} \\ &= \epsilon' - j\epsilon'' \end{aligned}$$

So, easily we can say what is epsilon dashed? Epsilon dashed is nothing but our original epsilon. And epsilon double dashed is a frequency dependent term and depends on conductivity.

$$\begin{aligned} \epsilon' &= \epsilon \\ \epsilon'' &= \frac{\sigma}{\omega} \end{aligned}$$

(Refer Slide Time: 11:44)

$$\begin{aligned}\epsilon_{eff} &= \epsilon - j\omega \\ &= \epsilon' - j\epsilon'' \\ \epsilon' &= \epsilon \\ \epsilon'' &= \frac{\sigma}{\omega} \\ \tan \delta &= \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega \epsilon} \\ \vec{J} &= \tilde{\sigma} \vec{E}\end{aligned}$$

So, culprit here is the presence of this conductivity that is why the epsilon is effectively called a thing and so, we define that loss tangent is given by epsilon double prime by epsilon prime and that is sigma by omega epsilon.

$$\tan \delta = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega \epsilon}$$

So, what is physically the effect of this lossy case? So, here also we say that if an external field is applied then the dipoles those are present inside the material, that aligned to the external field but when frequency is increased then they have a lag.

So, the polarization vector is not in time phase with the applied electric field and it lags by a factor of tan delta, sorry factor of delta so this we know. Now the problem is that when the radar sees tries to see under the ground; now ground is not isotropic in various layers it has different permittivities etcetera. Because soil water content of the thing is different the composition of the thing is different. Also the Ohm's law simply does not hold.

Actually what is Ohm's law? Ohm's law says that J is equal to conductivity into these the inherent assumption of Ohm's law is this conductivity sigma that is a real number.

$$\vec{J} = \tilde{\sigma} \vec{E}$$

Now actually it is not this conductivity, this should be phasor also so let us see. Now actually this also becomes complex in actual cases.

(Refer Slide Time: 14:13)

Case of lossy, dispersive, non-ohm's law type ground

$$\alpha + j\beta = \gamma = + \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$= + \sqrt{j\omega\mu \left\{ (\sigma' + j\sigma'') + j\omega(\epsilon' - j\epsilon'') \right\}}$$

$$\alpha = \left\{ \frac{\omega^2\mu\epsilon'}{2} \left[ \sqrt{\left(1 + \frac{\sigma''}{\omega\epsilon'}\right) + k_0 n \delta \left(1 + \frac{\sigma'}{\omega\epsilon''}\right)^2 - \left(1 + \frac{\sigma''}{\omega\epsilon'}\right)} \right] \right\}^{1/2}$$

So, we will see that now that when e m wave falls through ground we can again redo the e m theory cases I am not going into that; but we need to determine the attenuation constant. So, the case of ground I can say the case of lossy dispersive non Ohms law type ground. So, we here we cannot assume anything; so but in all these cases we can go to our basics. And say that what is the propagation constant gamma?

Gamma is we know it is the positive root of these j omega mu in to sigma plus j omega epsilon where everything is complex. And this plus root is necessary because otherwise radiation condition does not get satisfied. So, now so I can write it as plus j omega now I am assuming that the medium ground is generally non magnetic. So, mu I am not disturbing permeability; if it is a magnetic materials are present in that case is these also needs to be changed but we are not going there.

But the sigma that we are saying it is nothing, but its a complex quantity and it has two parts real and imaginary part. Similarly so, I will have to put a second bracket here j omega this is as it is like a normal lossy case.



$$\gamma = + \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$= + \sqrt{j\omega\mu\left\{(\sigma' + j\sigma'') + j\omega(\epsilon' - j\epsilon'')\right\}}$$

So, now, you need to extract the alpha and beta from here; because this gamma is nothing but alpha. So, I should write that it is alpha plus j beta. So, you can solve for it and that will give you that alpha is omega squared mu.

(Refer Slide Time: 17:55)

The image shows a handwritten derivation on a whiteboard. At the top, the expression for gamma is repeated:  $\gamma = + \sqrt{j\omega\mu\left\{(\sigma' + j\sigma'') + j\omega(\epsilon' - j\epsilon'')\right\}}$ . Below this, the real part alpha is derived as  $\alpha = \left\{ \frac{\omega^2\mu\epsilon'}{2} \left[ \sqrt{\left(1 + \frac{\sigma''}{\omega\epsilon'}\right)^2 + \tan^2\delta \left(1 + \frac{\sigma'}{\omega\epsilon''}\right)^2} - \left(1 + \frac{\sigma''}{\omega\epsilon'}\right) \right] \right\}^{1/2}$ . This is then simplified to  $\alpha = \omega \sqrt{\frac{\mu\epsilon'}{2} \left[ \sqrt{\left(1 + \frac{\sigma''}{\omega\epsilon'}\right)^2 + \left\{ \frac{\sigma'}{\omega\epsilon'} \left(1 + \frac{\sigma'}{\omega\epsilon''}\right) \right\}^2} - \left(1 + \frac{\sigma''}{\omega\epsilon'}\right) \right] }^{1/2}$ . Finally, the imaginary part beta is given as  $\beta = \omega \sqrt{\frac{\mu\epsilon'}{2} \left[ \sqrt{\left(1 + \frac{\sigma''}{\omega\epsilon'}\right)^2 + \tan^2\delta \left(1 + \frac{\sigma'}{\omega\epsilon''}\right)^2} + \left(1 + \frac{\sigma''}{\omega\epsilon'}\right) \right] }^{1/2}$ .

So, this can be written as omega. So, this is just to prove my some point these should have been square plus plus ok.

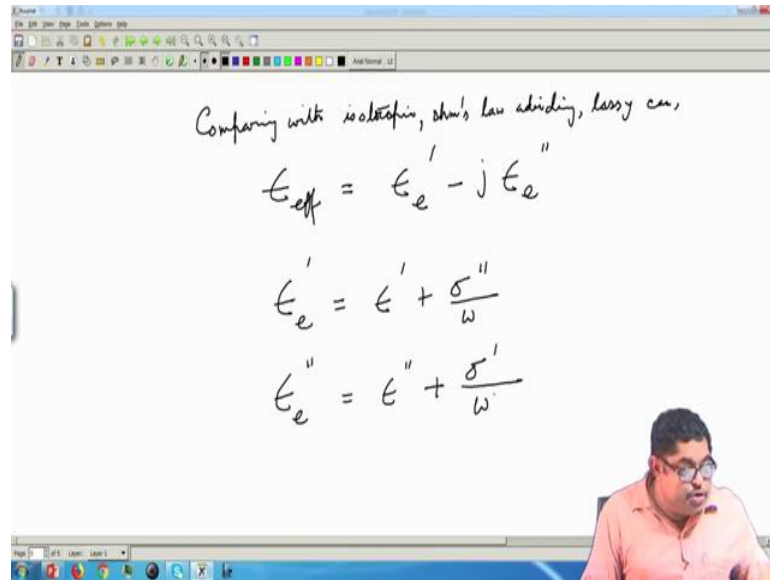
$$\alpha = \left\{ \frac{\omega^2\mu\epsilon'}{2} \left[ \sqrt{\left(1 + \frac{\sigma''}{\omega\epsilon'}\right)^2 + \tan^2\delta \left(1 + \frac{\sigma'}{\omega\epsilon''}\right)^2} - \left(1 + \frac{\sigma''}{\omega\epsilon'}\right) \right] \right\}^{1/2}$$

$$= \omega \sqrt{\frac{\mu\epsilon'}{2} \left[ \sqrt{\left(1 + \frac{\sigma''}{\omega\epsilon'}\right)^2 + \left\{ \frac{\sigma'}{\omega\epsilon'} \left(1 + \frac{\sigma'}{\omega\epsilon''}\right) \right\}^2} - \left(1 + \frac{\sigma''}{\omega\epsilon'}\right) \right] }^{1/2}$$

Similarly beta will be omega ok.

$$\beta = \omega \sqrt{\frac{\mu \epsilon'}{2}} \left[ \sqrt{\left(1 + \frac{\sigma''}{\omega \epsilon'}\right)^2 + \tan^2 \delta \left(1 + \frac{\sigma'}{\omega \epsilon''}\right)^2} + \left(1 + \frac{\sigma''}{\omega \epsilon'}\right) \right]^{1/2}$$

(Refer Slide Time: 20:57)



Now, if we compare with comparing with isotropic Ohm's law abiding lossy case; we can define an effective permittivity as now I am calling it is different from the previous Ohm's law abiding case. So, that is why I am giving an subscript again e minus j where this what is epsilon e prime that is these plus and double prime is.

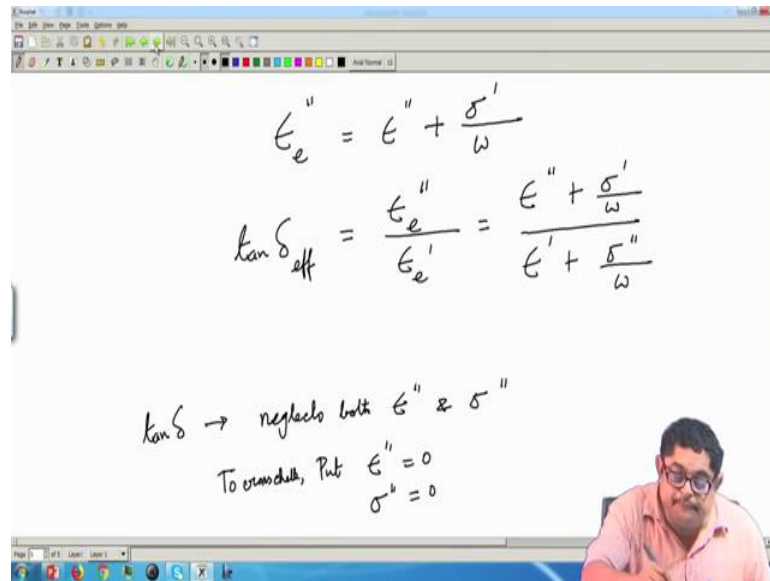
$$\epsilon_{eff} = \epsilon_e' - j \epsilon_e''$$

$$\epsilon_e' = \epsilon' + \frac{\sigma''}{\omega}$$

$$\epsilon_e'' = \epsilon'' + \frac{\sigma'}{\omega}$$

So, you see that the sigma double prime affects this thing it affects the permittivities prime a thing and also this one comes here.

(Refer Slide Time: 22:47)


$$\epsilon_e'' = \epsilon'' + \frac{\sigma'}{\omega}$$
$$\tan \delta_{\text{eff}} = \frac{\epsilon_e''}{\epsilon_e'} = \frac{\epsilon'' + \frac{\sigma'}{\omega}}{\epsilon' + \frac{\sigma''}{\omega}}$$

$\tan \delta \rightarrow$  neglects both  $\epsilon''$  &  $\sigma''$   
To crosscheck, put  $\epsilon'' = 0$   
 $\sigma'' = 0$

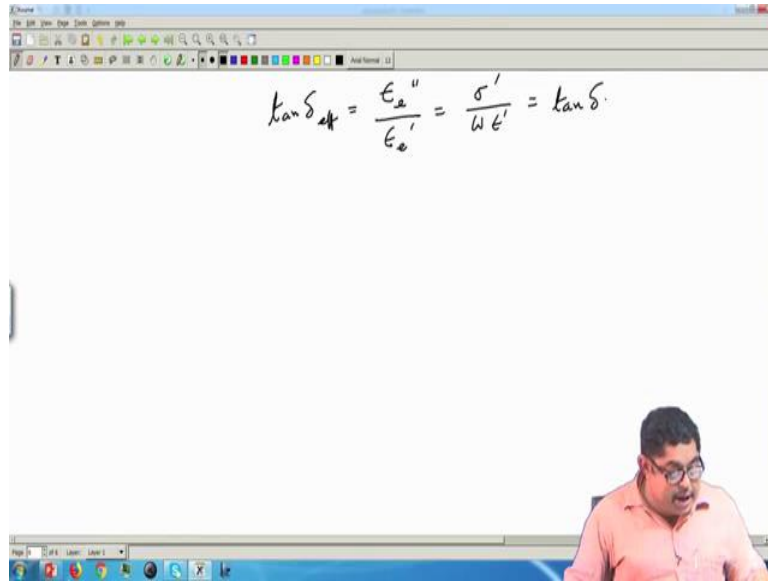
So, we can say that what will be the tan, but now we cannot call delta; delta was in the previous case. We call it tan delta effective that is nothing but epsilon e double prime by epsilon e prime and that is given by this.

$$\tan \delta_{\text{eff}} = \frac{\epsilon_e''}{\epsilon_e'} = \frac{\epsilon'' + \frac{\sigma'}{\omega}}{\epsilon' + \frac{\sigma''}{\omega}}$$

So, physically we can say that the loss tangent that we say; that means, the tan delta this was for the Ohm's law abiding case.

Tan delta is a model, which neglects both epsilon double prime and sigma double prime; that means, it calls the material isotropic and Ohm's law abiding; whereas, in our case actually for ground this should be the case. Now this model, if you put neglects these two things; that means, put these to 0 in this expression. So, you see that this tan delta effective, now put or this is to crosscheck to crosscheck put sigma dash is equal to 0, sorry epsilon dashed is equal to 0 and sigma dashed is equal to 0 in our tan delta expression.

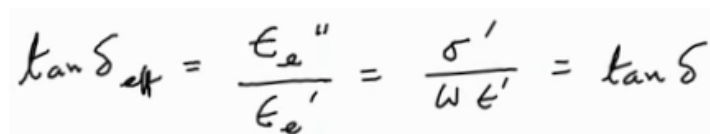
(Refer Slide Time: 24:44)



A screenshot of a whiteboard with a hand-drawn equation:  $\tan \delta_{\text{eff}} = \frac{\epsilon_e''}{\epsilon_e'} = \frac{\sigma'}{\omega \epsilon_e'} = \tan \delta$ . The whiteboard is part of a video recording, with a person's head and shoulders visible in the bottom right corner.

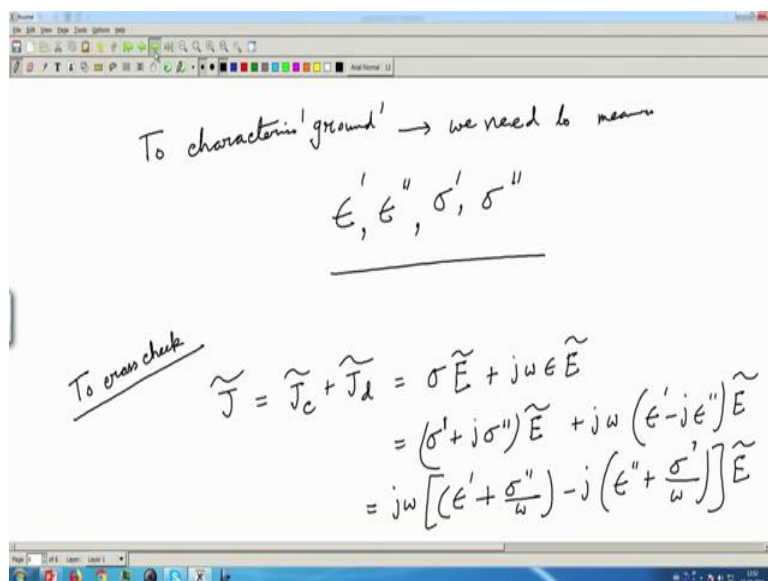
So, tan delta effective then will become epsilon e double prime by epsilon e prime and it will boil down to sigma dashed by omega epsilon dashed, which is our normal tan delta.

So, we are our expression is correct.



A handwritten equation on a white background:  $\tan \delta_{\text{eff}} = \frac{\epsilon_e''}{\epsilon_e'} = \frac{\sigma'}{\omega \epsilon_e'} = \tan \delta$ .

(Refer Slide Time: 25:27)



A screenshot of a whiteboard with handwritten text and equations. The text reads: "To characterize 'ground' → we need to measure  $\epsilon', \epsilon'', \sigma', \sigma''$ ". Below this, under the heading "To cross check", the following equations are written: 
$$\tilde{J} = \tilde{J}_c + \tilde{J}_d = \sigma \tilde{E} + j\omega \epsilon \tilde{E}$$
$$= (\sigma' + j\sigma'') \tilde{E} + j\omega (\epsilon' - j\epsilon'') \tilde{E}$$
$$= j\omega \left[ (\epsilon' + \frac{\sigma''}{\omega}) - j(\epsilon'' + \frac{\sigma'}{\omega}) \right] \tilde{E}$$

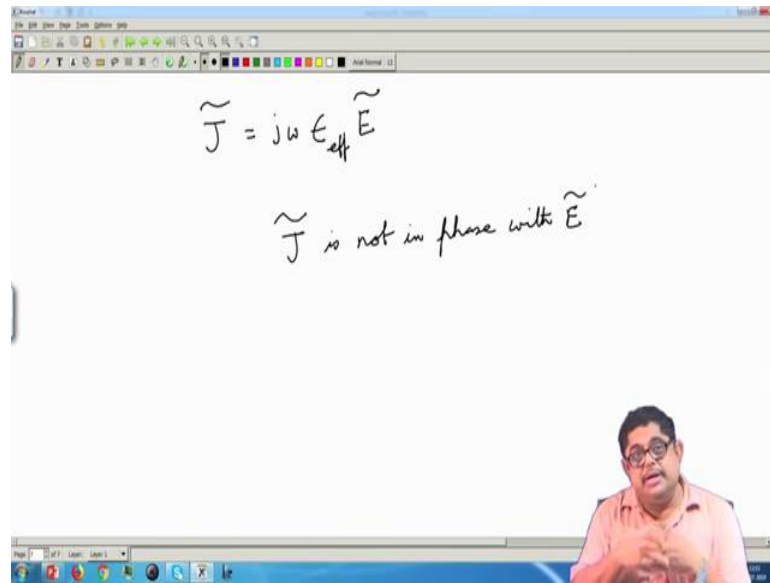
And now, once this is done the ground dielectric that becomes like our normal em theory characterizations will hold there. But we should remember that to characterize ground, we need to measure four quantities epsilon prime epsilon double prime sigma prime and sigma double prime. So, this is the additional thing that we need to do in case of ground and you see that these are if you see the expressions. So, this is the actual expressions.

So, they are there is a cross coupling between them. If I need to determine these the sigma and frequency affects it; similarly if I need to do this the sigma dashed tan omega affects it. So, that will make the matter complicated because my more number of readings I need to take and I need to decouple them so, that is the challenge etcetera. Now finally, to cross check whether our we are saying that it is not Ohm's law abiding.

So, to cross check again to crosscheck let us find the current density. So, what is J? J we know is J c plus J d. So, what is J c? I can write sigma E, but this sigma is a complex sigma plus j omega epsilon E. So, these I will write as sigma dashed plus j sigma double dashed E plus j omega then epsilon dashed minus j epsilon double prime E. So, that will give me j omega epsilon dashed plus sigma double dashed by omega minus j epsilon double prime plus sigma by sigma dashed by omega E.

$$\begin{aligned}\tilde{J} &= \tilde{J}_c + \tilde{J}_d = \sigma \tilde{E} + j\omega \epsilon \tilde{E} \\ &= (\sigma' + j\sigma'') \tilde{E} + j\omega (\epsilon' - j\epsilon'') \tilde{E} \\ &= j\omega \left[ \left( \epsilon' + \frac{\sigma''}{\omega} \right) - j \left( \epsilon'' + \frac{\sigma'}{\omega} \right) \right] \tilde{E}\end{aligned}$$

(Refer Slide Time: 28:37)



So, I can now write that  $\tilde{J}$  is  $j\omega \epsilon_{eff} \tilde{E}$  as before.

$$\tilde{J} = j\omega \epsilon_{eff} \tilde{E}$$

But remember that this due to this presence of this  $\tilde{J}$  is not in phase with  $\tilde{E}$ ; as was demanded by the Ohm's law. So, this is the only theoretically new thing other things are various techniques. So, we will start with that technique in the next lecture that GPR how it measures those quantities. Basically it needs to measure those force now still it is an area of active research that how to determine the conductivities thing, various people are trying actually various formulas various ways are doing.

But no clear cut direction has been found that how to measure conductivity. Permittivity people can measure to certain good extent but conductivity. And then we will see that once you have these information then you can not only detect an object you can even image that object like an optical camera. So, though microwave camera has not been invented yet that is also another active research area of active research; that we will talk about in our next lecture.

Thank you.