**Principles and Techniques of Modern Radar Systems**
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**Lecture – 58**
**Tutorial**

**Key Concepts:** Tutorial-8

Welcome to the NPTEL lecture on Principles and Techniques of Modern Radar Systems.

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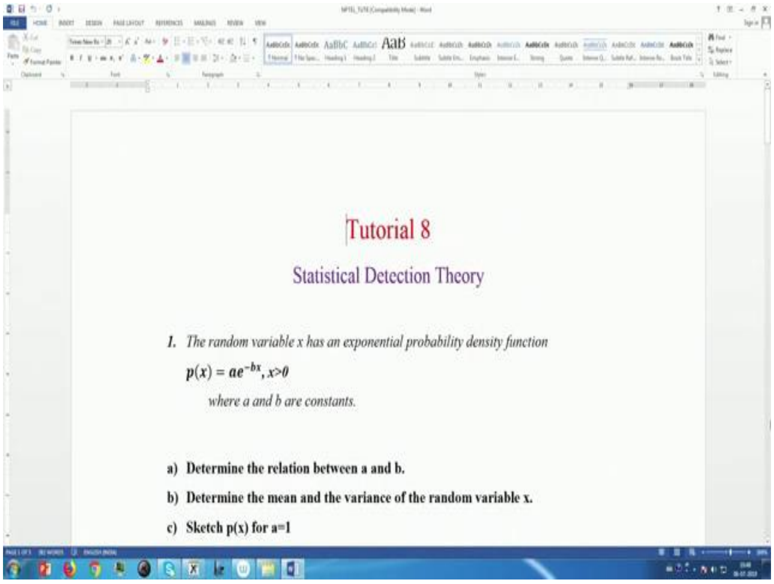


In last few classes, we have seen statistical detection theory; we have applied statistical principles for the detection, because there are lot of fluctuations present. So, today we will see some problems on that thing. The first thing is a recapitulation of your communication, in communication or in this your intermediate classes in statistics. You have seen that a random variable gets completely specified by the if the probability density function can be specified, so that is an example that.

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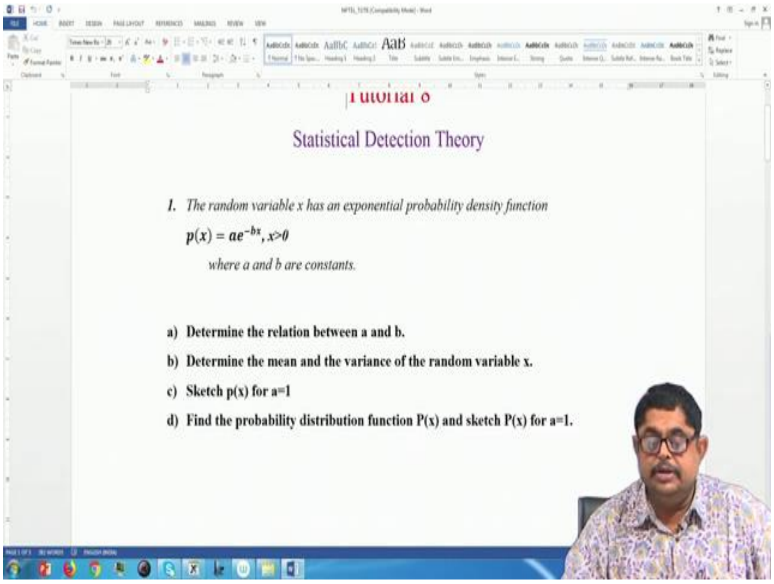


And x has an exponential probability distribution, please remember it is not Gaussian not Rayleigh Rice, it is an exponential probability distribution. This PDF is given by p x is equal to a e to the power minus b x, x is greater than 0, and a and b are constants. Determine the relation between a and b; a and b are two constant. Are there any relation? Then b is determine the mean and variance of the random variable x, then sketch p x for a is equal to 1.

And finally, find that probability distribution function which is a cumulative thing of that at any time what is the probability. This is remember probability density function, if you integrate that up to certain point, you get the probability distribution function. So, this is the problem, let us solve it.

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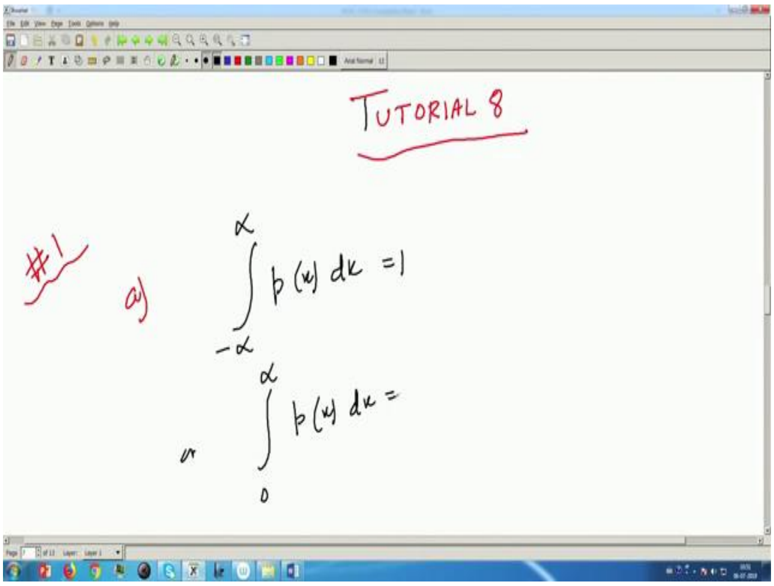


So, that we are calling tutorial 8, problem 1. So, how to find the relation the, first thing we know that the area under the any PDF curve should be always one. So, we can say that minus infinity to infinity p x dx should be always 1. Now, in this case, it is already said that the p x exist only for x greater than 0. So, we can say it is 0 to infinity p x dx is equal to 1 or 0 to infinity.

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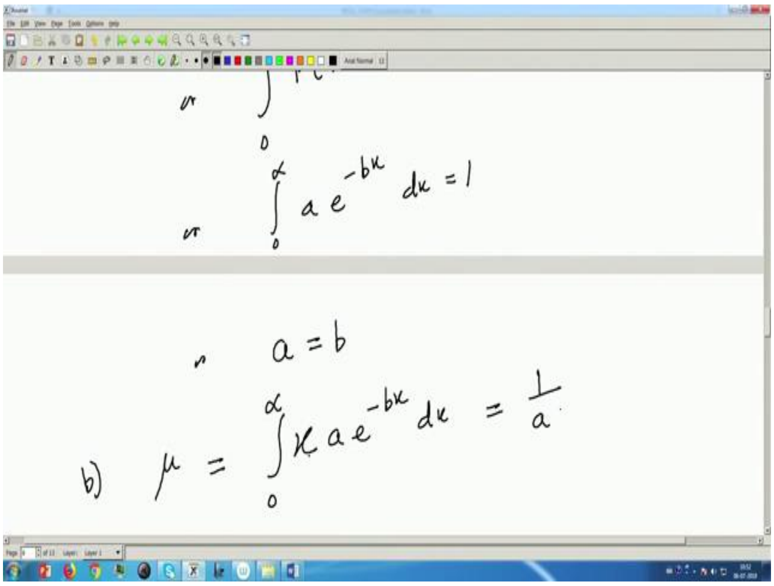


What is the p x? a e to the power minus b x dx that should be 1. So, you can easily do this integration and so that will give you that a is equal to b. So, relation between a and b we were asked that is a is equal to b. The second part what is the mean. So, mean is we know 0 to infinity I am writing I am not writing minus infinity to infinity and what is it, mean is always the first moment, so x into a e to the power minus b x dx.

So, you will have to integrate this again the instead of e to power minus b x if this is x e to the power minus dx. So, integration problem you could do this integration simple integration, it will come as 1 by a. And what is the second is what is asked the mean and variance.

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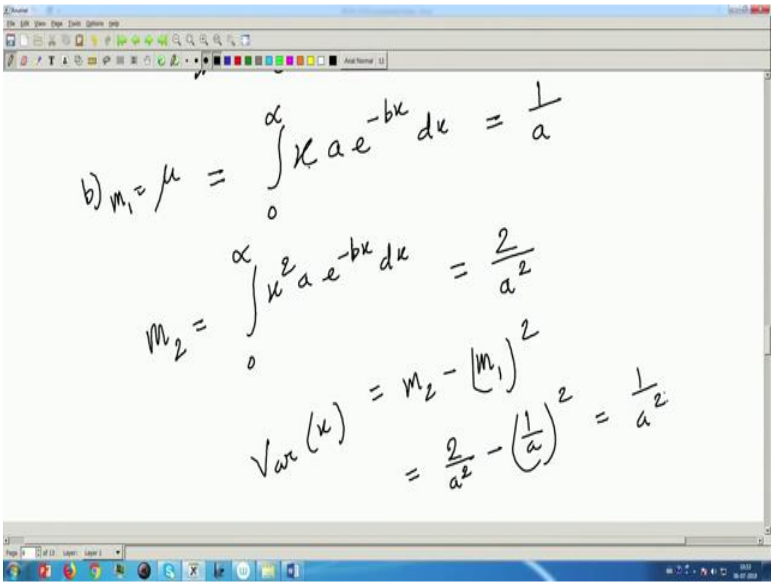

So to find variance mean is already defined, you find the second moment. Second moment is m 2, m 2 is 0 to infinity x square a e to the power minus b x dx. And again you can do it these are all doable very easily, so that will give you 2 by a square. So, what is variance, remember because in Gaussian cases generally we assume 0 mean. So, people generally say m 2 and variants same, but they are not same.

Variance is m 2 minus the first moment square m 1 or mu square actually I should have written this is m 1, m 1 square. So, this is 2 by a square minus 1 by a whole square, so that gives you 1 by a square. So, this is the variance. Standard deviation is square root of that ok, but that was not asked to let us not discuss that.

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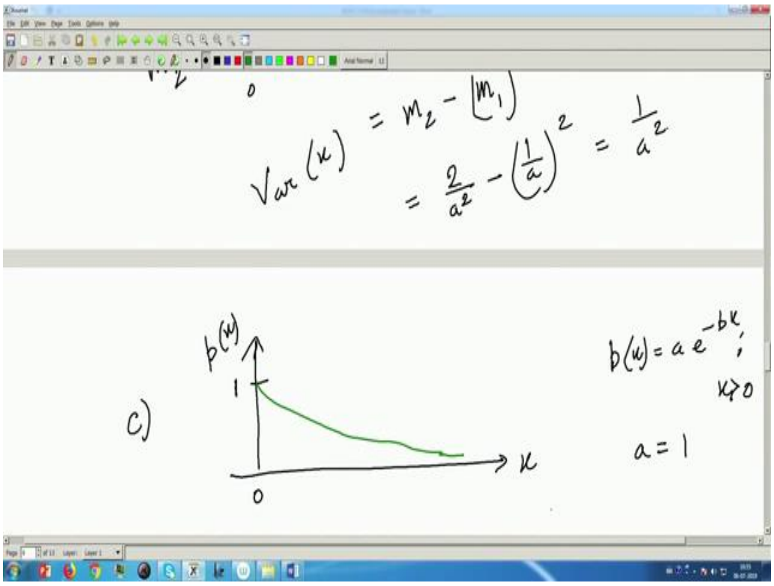

Then part c is sketch, you see that it should be apt to sketch any given function. So, p x what was p x, remember p x, p x was a e to the power minus b x x greater than 0. So, if I asked you to do that p x versus x, so it should start from 0. And at 0 what will be its value that will be 1 for a is equal to 1, it is given that otherwise it will depend on a. So, we are putting a is equal to 1, so that is why value; otherwise it would have been a, so 1.

And then it is e to the power minus and a and b are same, so e to the power minus x. So, that means, the shape will be something like this never it will go to 0, but it is exponentially falling down as simple as that.

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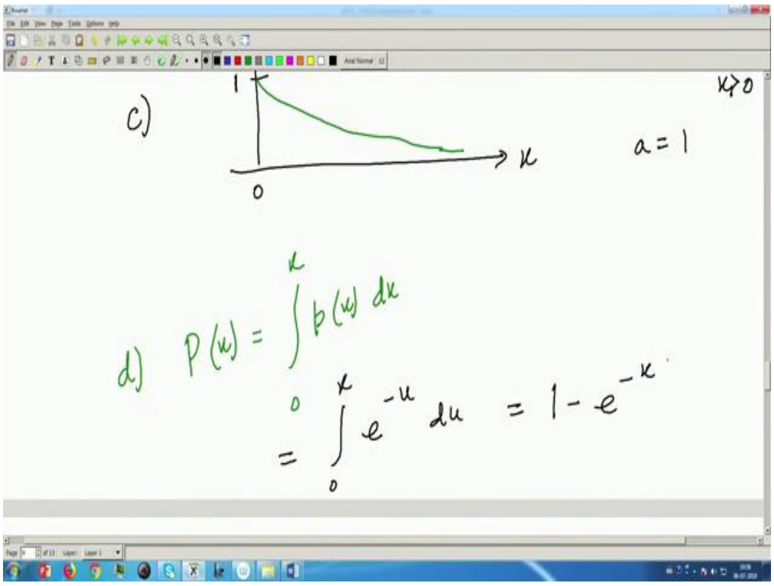


Then for d is the probability distribution function. So, what is probability distribution function, it is 0 to x p x dx. So, you can find out 0 to x, so then you change these variable for a is equal to 1 e to the power minus u d u; so, that if you do, it will come as 1 minus e to the power minus x.

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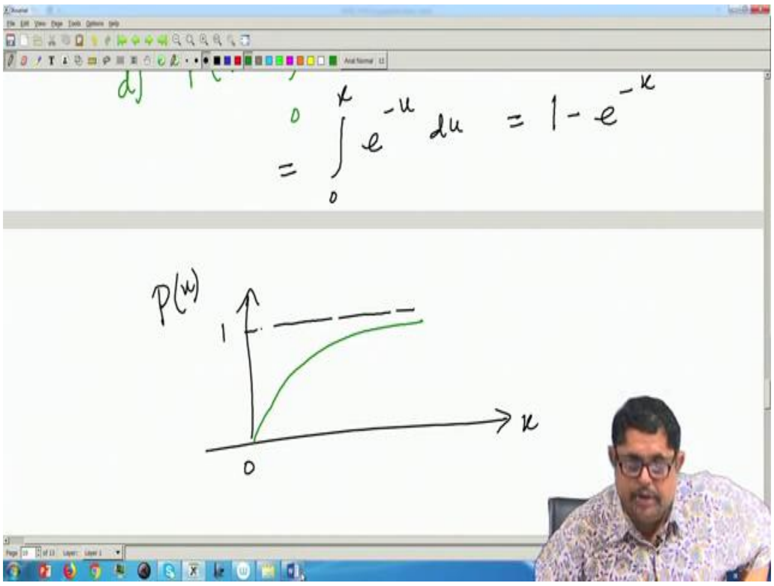


So, you can again sketch this. This is your probability distribution function. This is again x 0. So, this is the one point. Now, here at x is equal to 0, the value is 0. So, the sketch will be something like this. It is approaching this value okay that is all. Then let us go to, so this is just recapitulation that you should be familiar with these things.

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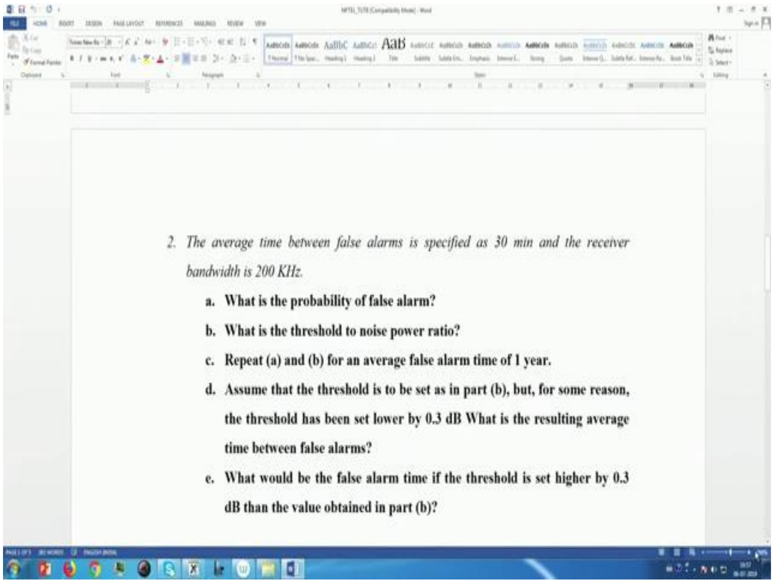


Then let us come to our actual radar detection theory. Second problem is that the average time between false alarms is specified as 30 minute. So, every half an hour, there is a false alarm coming and receiver bandwidth is so and so. So, what is the probability of false alarm ok. So, let us solve this problem.

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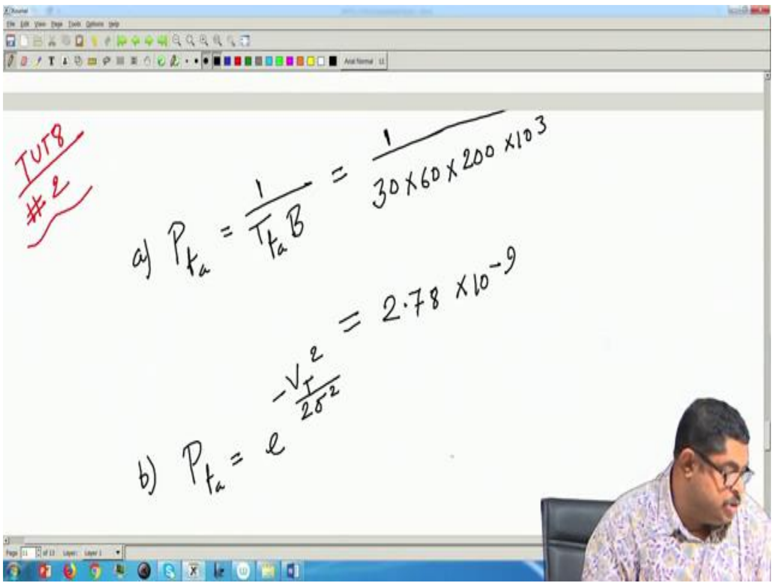

So, I can say tutorial 8, problem 2 so, part a. So, I will have to find probability, we have already found out this in the theory class that T f a into B. T f a is given, bandwidth given. So, T f a is 30 minutes that that means, 30 into 60 so and so seconds, and bandwidth is 200 kilo Hertz. So, that will give you 2.78 into 10 to the power minus 9 quite good P f a. So, so that operator will be able to handle it.

Now, what is the question what is the threshold noise power ratio, that means, where you all you put the threshold I know the noise statistics we are assuming Gaussian noise. So, I know sigma then how to calculate what will be the V T to achieve this P f a or T f a. So, we know that the relationship between P f a and V T that we have found out from that Rayleigh distribution and all those. So, V T square by 2 sigma square. So, P f a is known. Now, I will have to find V T square by 2 sigma square.

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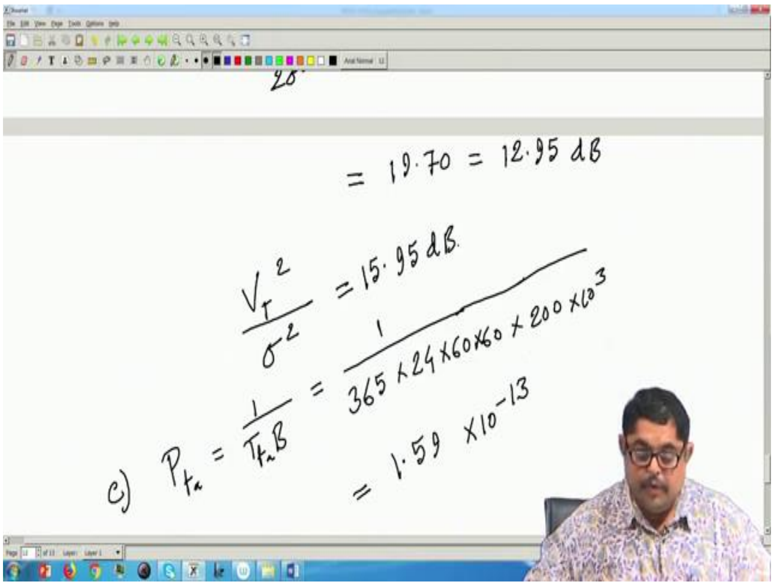

So, V T square by 2 sigma square is nothing but minus l n P f a. So, you can put that P f a value we know, so it will come out to be 19.70 or in dB it will be 12.95 dB, power ratios so 10 log no problem. So, basically that was V T square by 2, but actually we should give what is the voltage level compared to the a threshold square divided by the these will be V T square by sigma square, so that will be it will 3 dB more. So, it will be 15 point…. 12, so another say 3 dB more 15.95 dB okay.

Then part c, what is part c, because part c repeat a and b, for an average false alarm time of one year. Suppose we want to make false alarm time 1 year that means, each year only

1 income, so that will be very good. So, let us calculate that so P f a again we will say T f a into B; and T f a will be 1 year, so 365 days into 24 hour into 60 into 60 so and so second into B again, 200 into 10 to the power 3 if we do that, you will get 1.59 in to 10 to the power minus 13. In previous case it was 10 to the power minus 9, now it is 10 to the power minus 13.

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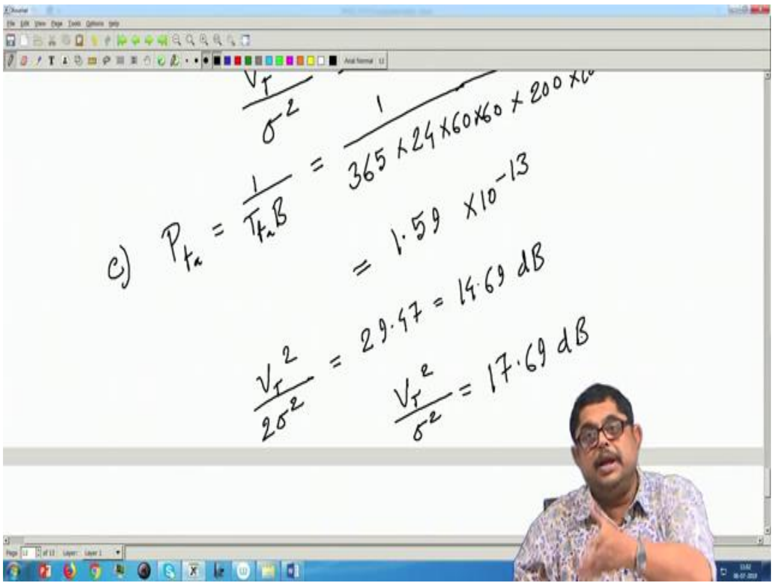

So, accordingly we can find the V T square by 2 sigma square so that, will be 29.47 seven or that is 14.69 dB. So, V T square by sigma square will be 3 dB more than this that means 17.69 dB. So, if we instead of 15.95 dB if we give the V T square above sigma square by 17 that means 16, so 1.7 dB only if we increase, we will get it. But remember that this will also make that various actual targets, you would not be able to detect that, so we do not do it. But if we want that you should not have false alarm, we can always do that by proper thresholding, ok.

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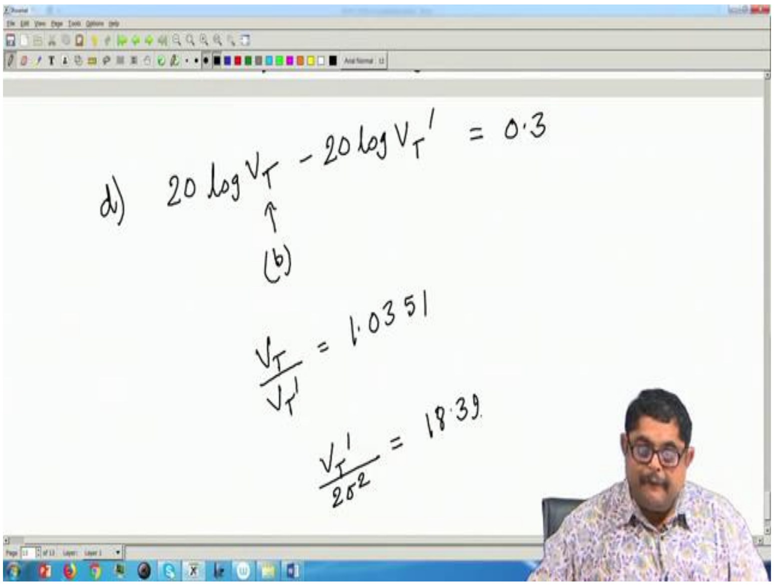


Then next is let us see what is part d, assume that the threshold is to be set as in part b that means, the first case. But for some reason that threshold has been set lower by 0.3 dB. What is the resulting average time between false alarm, good. So, by mistake or by something, suppose you have done a 0.3 dB different.

So, what is said is 20 you see, V T is voltage threshold voltage, so that is why I am using 20 log. So, these V T is as in this is the answer given in part b minus the new V T that I am calling let us say V T dashed, so that is lower by 0.3 dB, so that is why 0.3. So, from these I can solve for V T V T dash, V T by V T dashed is 1.03, it will come as this if we just solve this equation. So, V T dashed by 2 sigma square assuming that the sigma is same in both the cases, it will come as 18.39.

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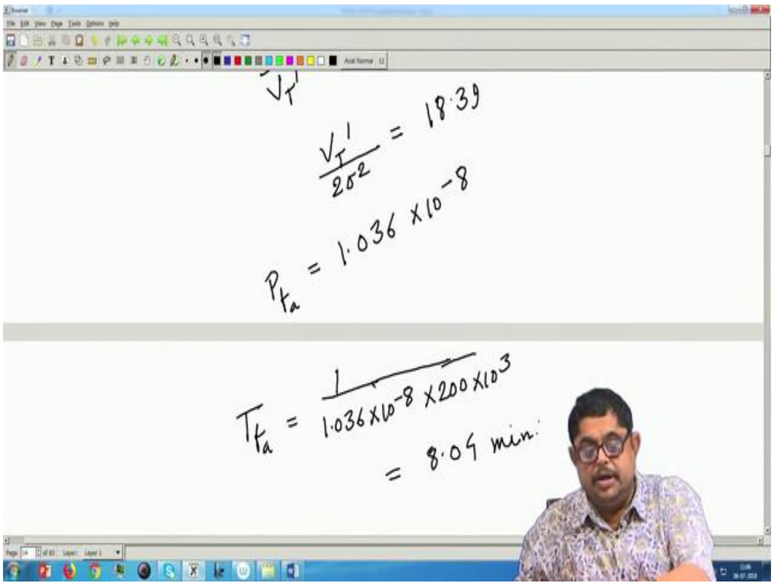


And so the moment you get V T dash by 2 sigma square, you can get P f a as 1.036 into 10 to the power minus 8. But more you see meaningful is so P f a we got from here we can easily find T f a T f a will be 1 by 1.036 into 10 to the power minus 8 into bandwidth is the same 200 kilo Hertz, so that will give you 8.04 minute.

So, you see that instead of 30 minute if I set the by error or something I put the threshold just by 0.3 dB lower, but drastically the false alarm time gets reduced to 8 minima, 8 minute. Similarly, what is the next question is what would be the false alarm if the threshold is set higher by 0.3 dB so that does the opposite.

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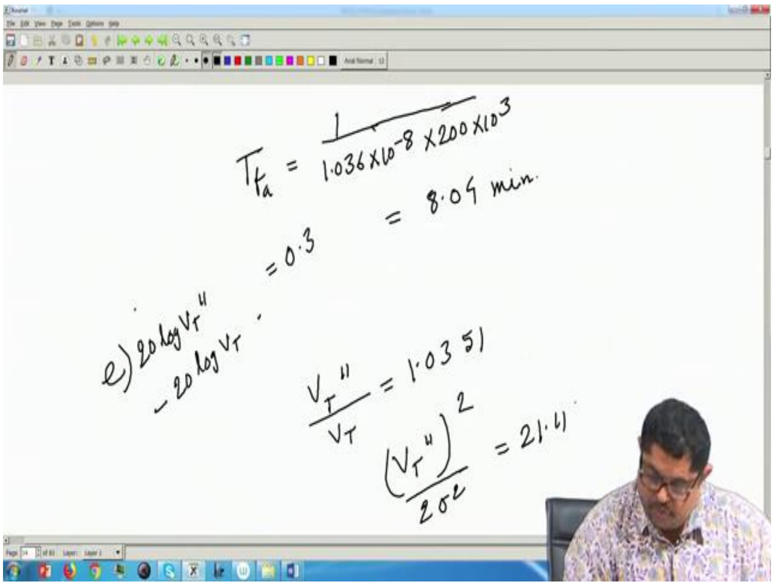


So, threshold T f a should increase from 30 minute in this case. So, now in this case let me call the threshold V T double prime. So, V T dash minus V T 0.3 sorry 20 log that 20 log of V T dash sorry 20 log V T dash 20 log V T double dashed minus 20 log V T is 0.3. So, for that you can solve that V T double dashed by V T will be 1.0351 and so we will get V T double dash square by 2 sigma square as 21.11.

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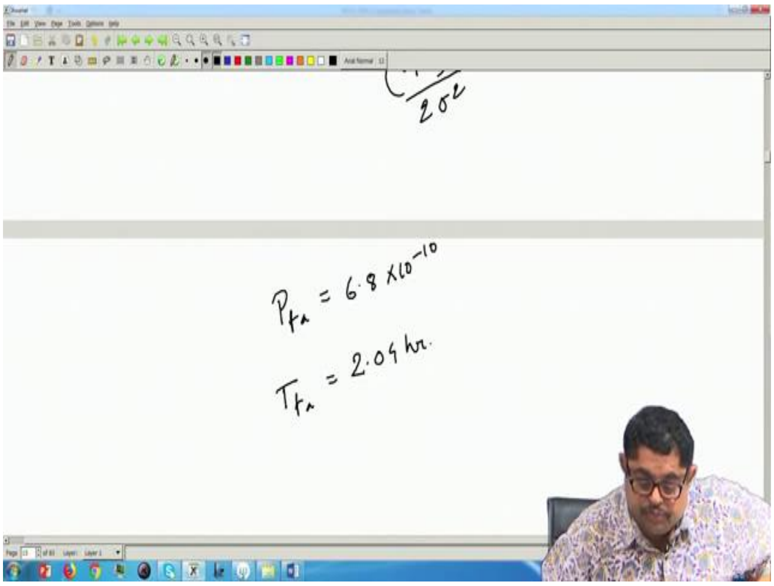

And that will give you a P f a of 6.8 into 10 to the power minus 10 that will give you a T f a of 2.04 hour. So, instead of 30 minute, you are getting almost four-fold change. So okay. That is the meaning of exponential variation.

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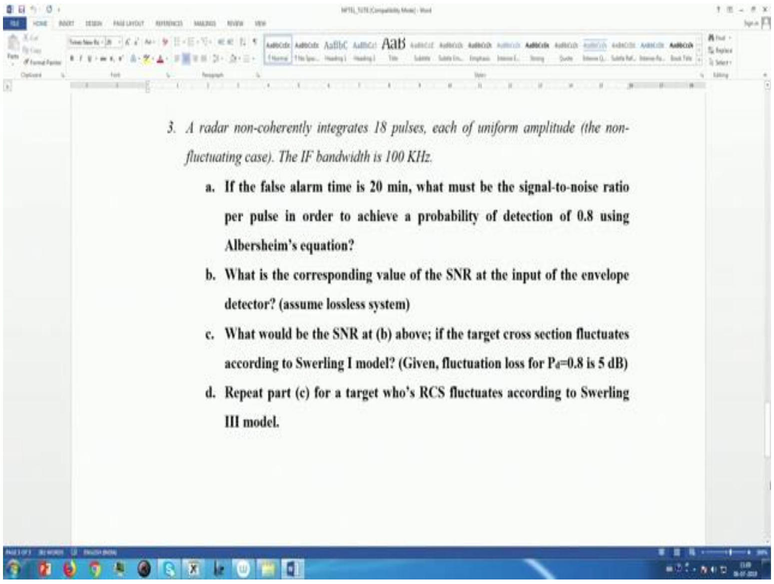


Let us see the next problem. So, this is on Albersheim's equation and Swerling model etcetera. A radar non-coherently integrates 18 pulses each of uniform amplitude the non fluctuating case, that means, our original case, so Swerling's 0 case. The IF bandwidth is a 100 kilo Hertz if the false alarm time is 20 minute what must be the SNR per pulse in order to achieve a probability of detection of 0.8 using Albersheim's equation, so okay.

So, P f a false alarm time given means P f a given and probability of detection P d given so from Albersheim's equation…. Part b, what is the corresponding value of the SNR at the input of the envelop detector assume lossless system that means, now you are considering the Swerling case. So, you will have to tell that also there is integration of pulses, so how SNR will be there.

Then part c is what would be the SNR at b above if the target cross section fluctuates according to Swerling I model. Now, if the fluctuation is Swerling I model then also given the fluctuation loss for P d 0.8 is 5 dB. And part d is repeat part c for a target whose RCS fluctuate according to Swerling 3 models. So, all the models are getting covered here let us solve this it is a good problem practical things.

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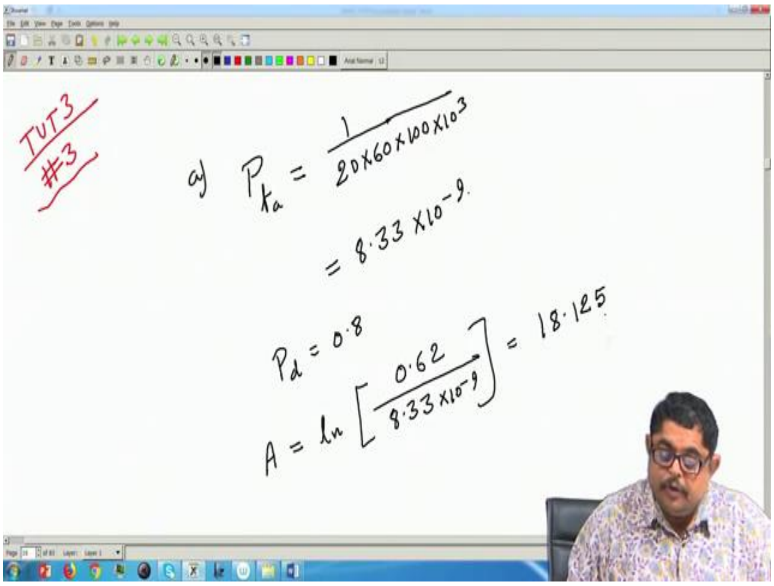


So, I will say that this is tutorial 3, problem number 3. So, part a is first I will have to find the false alarm time given; that means, T f a given 20 minutes. So, P f a I require because Albersheim’s equation is in terms of P f a and P d. So, P f a will be 1 by 20 minutes. So, 20 into 60 into 100 kilo Hertz, so that is giving 8.33 in to 10 to the power minus 9, and P d is given as 0.8. So, you can find those A and B’s of Albersheim’s equation ln 0.62 by eight point P f a A depends on P f a. So, this is 18.125 ok.

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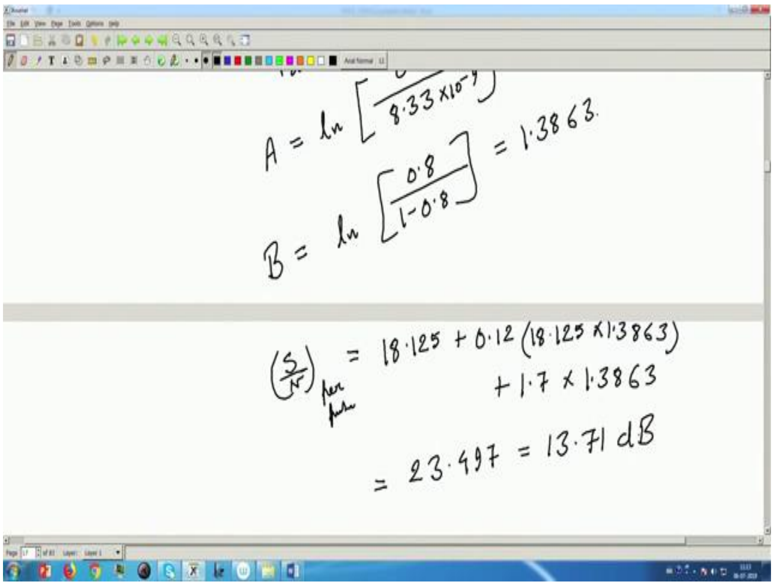

Then B, B depends on P d l n 0.8 by 1 minus 0.8, so that will be 1.3863. So, SNR per pulse per pulse will be Albersheim's equation that 18.125 plus 0.12 into 18.125 into 1.3863 plus 1.7 into 1.3863. So, so that will give SNR 23.497, which is 13.71 dB that is all.

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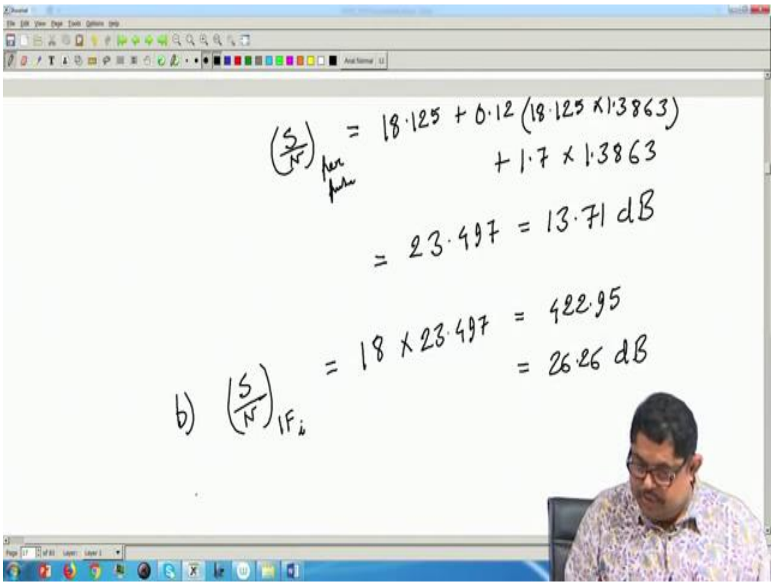


Now, now let us see part b, what is the corresponding value of the SNR at the input of the envelop detector assume lossless system, so that is okay; so, lossless system that means, just 18 pulses are integrated. So, you add 18 pulses, so it will be 18 in to 23.497, so that will be 422.95 that is 26.26 dB ok.

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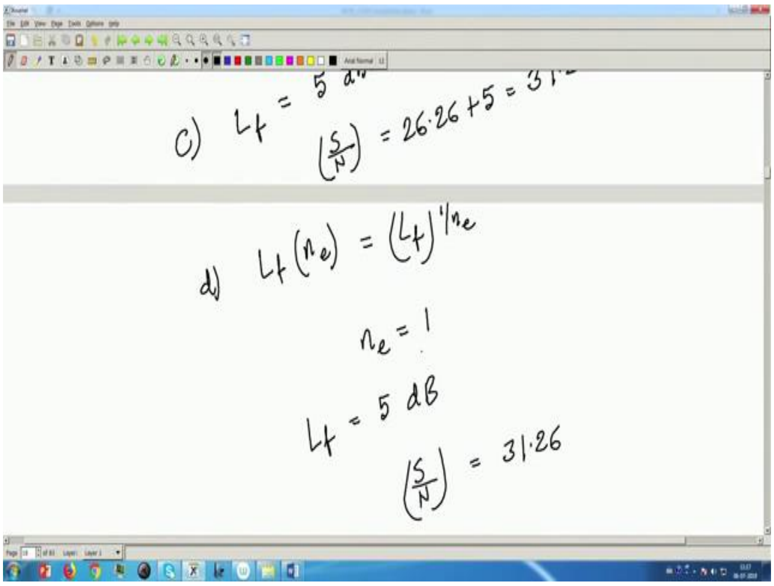


Then we should see part c part c, I think the Swerling model is getting changed. What would be the SNR at b above, if the target cross section fluctuates according to Swerling 1 model. So, Swerling one model given fluctuation loss is 5 dB. For this P d, so that is good, so that means, in this case, we will have to add that fluctuation loss fluctuation is 5 dB. So, SNR will be 26.26 plus 5 that is 31.26 dB.

And I think the next one is Swerling 3 model according to Swerling 3 model. So, Swerling 3 has a different L f calculation. So, L f will be based on these ne. So, that is L f whole to the power 1 by n e and n e value you can take generally 1. So, L f again comes to be 5 dB now actually if you take it generally n e is varying from 1 to 2, so you can take various values. So, if you take a thing it will be actually it will be a bit lower, but if you take n e is equal to 1 that is a conservative estimate. So, here also S N is 31.26 dB in the safe side.

But in many practical cases people take something n e is 1.2, 1.4 etcetera. So, in that time it will be a bit lower than the SNR will be required that is all. You see that by this you can solve lot of practical problems by applying this Swerling models and Albersheim's equation. We have tried to show you how to do that. In assignments, we will get more such questions.

Thank you.