Principles and Techniques of Modern Radar Systems
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Lecture – 57
Statistical Detection Theory (Contd.)

Key Concepts: Introduction to Swerling models, classification of RCS fluctuations, classification of propagation environment, fluctuation loss vs probability of detection characteristics, modified radar range equation

Welcome to this NPTEL lecture on Techniques and Principles of Modern Radar Systems. Now, we were discussing detection, realistic detection thing; there we have seen the probability of false alarm and probability of detection based SNR calculation from Albersheim's equation. Now, today we will see that target fluctuation and as I said at the end of the previous class that Peter Swerling has developed models for tackling those fluctuation thing.

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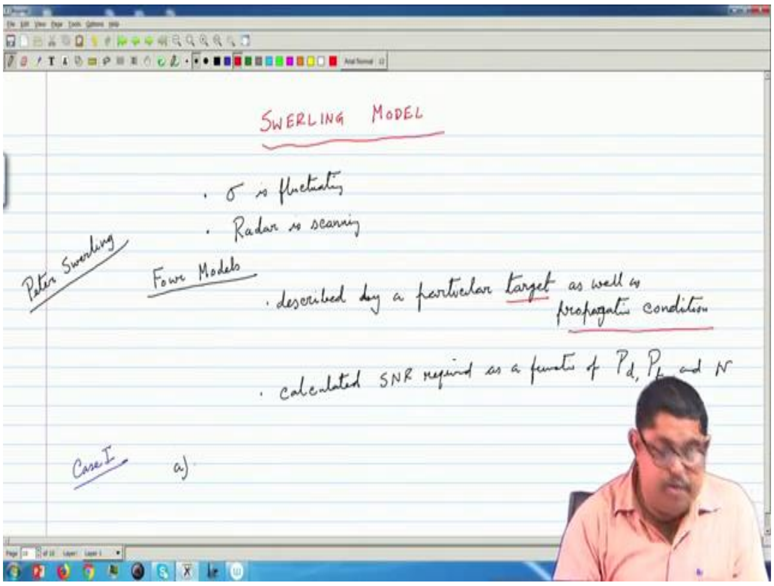

So, I can say that the target cross section RCS is fluctuating number 1. So, also the radar is scanning. So, its antenna beam is moving whether mechanical or electronic steering, but the beam is moving. So, aspect of illumination that is changing so; that means, the beam position is also changing.

So, that will also add to the fluctuation etcetera and so, that in the path propagation path if supposed with this change of beam aspect there may be a new path where a new scatterer may be present. So, that will change the return. So; that means, this is a propagation effect and also there maybe some fading in one of those aspects. So, that time the fading will change the received echo signal etcetera.

So, that is why Peter Swerling considered all these and he described four models, four such basic cases, four models that is a name I should write because these models are now very standard and Peter Swerling. He actually modelled it and for each model he described the model is described by a particular scenario.

And so, four models I should say that described by a particular target as well as propagation condition. So, a target is having two different condition, propagation condition also is having two different convention. So, with permutation you can I sorry with combination you can say there are four models will come actually here four such models.

And finally, what is the outcome of the model? He calculated this models with the help of the model, he calculated SNR required as a function of P d P fa and N. N is the number of integrated pulse because that N is now crucial, because how many Ns you are doing that will affect the whole scenario that whether the scenario is stationary or not. So, let us start with the four models one by one. Case I or Swerling model I, here he says that first let us for the I will underline these two terms target and another is propagation condition.

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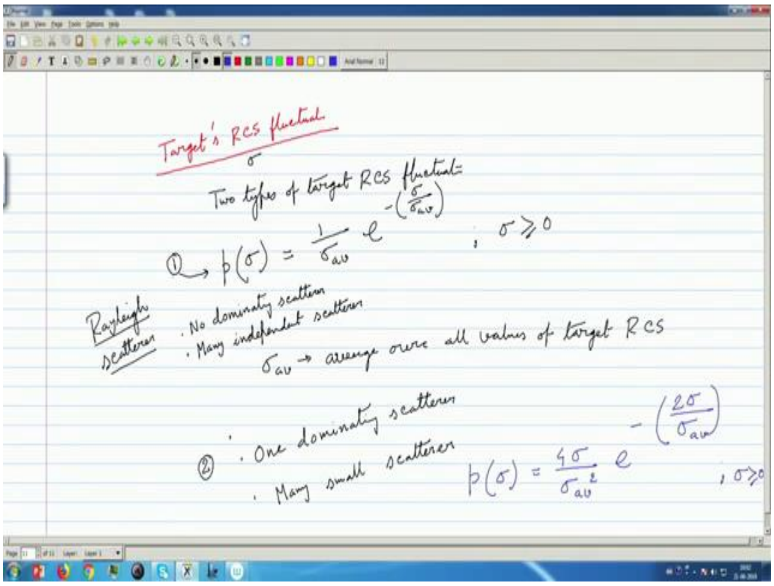

So, what he did or I will before describing the model let me see the target's RCS fluctuation. He actually calculated and gave two types of target; two types of target RCS fluctuation. So, actually here there is a difficulty that RCS is the symbol of RCS is sigma and in probability distribution sigma is the standard deviation. So, these two may get confused, try to understand clearly. So, basically we are finding the probability distribution of target RCS sigma. So, for that the statistical parameter, the standard deviation we will write as sigma average.

So, he wrote that some type of that two types of target RCS fluctuation one is this type, p sigma is equal to sigma average e to the power minus sigma by sigma average sigma greater than equal to 0.

$$p(\sigma) = \frac{1}{\sigma_{av}} e^{-\left(\frac{\sigma}{\sigma_{av}}\right)} ; \quad \sigma \geq 0$$

So, sigma is the RCS, sigma average is the statistical parameter. So, and what is sigma average? Let me define it, it is average over all values of target RCS. The moment I am calling average it is just statistical parameter ok. Now, actually this type of fluctuation Peter Swerling analysed targets and he showed that those target which have many scattering centres, many independent scattering centres, but no dominating scatterer.

So, each one is independent and they affect. So, if there are many such in practice actually, if there are 4-5 such independent scatterers then this the target RCS fluctuation is in this shape, this is called Rayleigh scatterer. So, this one what I said that is for Rayleigh type of scatterer, Rayleigh scatterer. He named it under the famous scientist Rayleigh. Actually you can see that the this is something the counterpart in noise is the Gaussian distribution, that if there are large number of independent scatterers, but no dominating scatterers.

Then by central limit theorem, we say that the individual distributions may be anything, but with large number they become Gaussian, this is similar to that. In case of RCS, if there are no dominant scatterer, but there are many independent scaterers. In practice if there are 4-5 independent scatterers without any dominating one then the target RCS fluctuation distribution is given by this.

Actually, you can see that this is not a Rayleigh pdf, scatterer is Rayleigh, Rayleigh is scatterer. Actually, this is an exponential distribution, actually exponential distribution is the square voltage distribution of Rayleigh. So, this thing, but Peter Swerling namely it after Rayleigh scatterers.

So, we will say that for Rayleigh scatterer this is the thing and another thing I will say number 2 that if there are so, here I can say no dominating scatterer. And, many independent scatterer that makes a Rayleigh scatterer whereas, the another type of targets are that one dominating scatterer and there maybe many small scatterers, but one is clearly dominating. So, that time this fluctuation, the distribution for that will be given by

$$p(\sigma) = \frac{4\sigma}{\sigma_{av}^2} e^{-\left(\frac{2\sigma}{\sigma_{av}}\right)}, \quad \sigma \geq 0$$

So, he said that the targets radar targets they will fall into one of these categories number 1.

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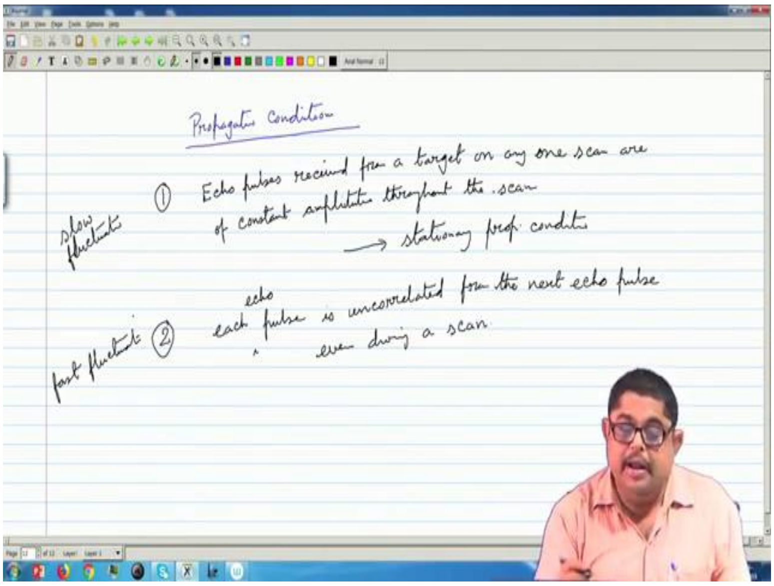

Also the propagation condition that can be two types; one is the echo pulses received from a target on any one scan are of constant amplitude throughout the scan; that means, this is a stationary scenario. Basically, as I am scanning the propagation condition is not changing so, I am getting the same amplitude; obliviously I cannot get the same amplitude because it is going and the thing is coming differently. But, what the meaning is, it is a stationary propagation condition, stationary propagation condition.

Whereas, the second one is that each so, actually the name of this thing is that this is called slow fluctuation case, slow fluctuation and each pulse each echo pulse is uncorrelated from the next echo pulse even during a scan that means the propagation condition is very fast changing. So, one pulse I have sent, then again I am sending a pulse the echo I am receiving these two are totally uncorrelated.

So, this is a fast fluctuation; first fluctuation. So, based on these what Peter Swerling said that you see that RCS for target RCS fluctuation that is two distribution, propagation condition two; he clubbed one; one set from there, one from here and by that he made the model.

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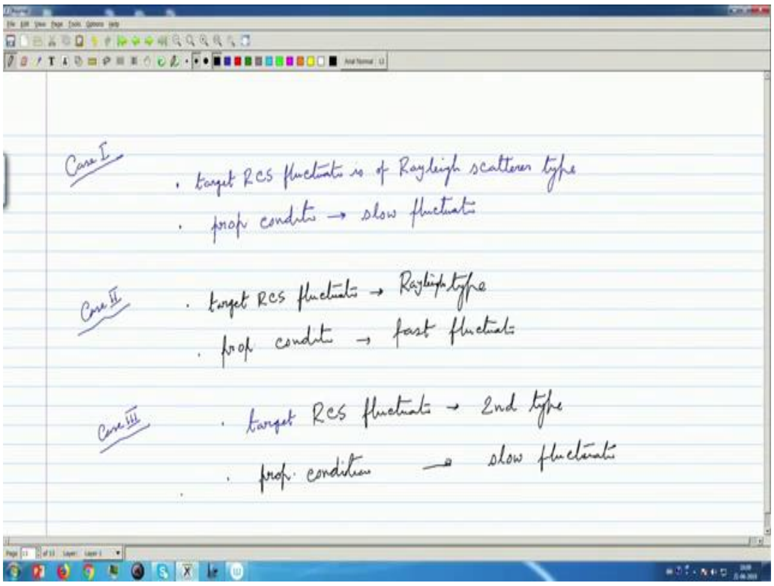

The model so, that model will now discuss case I or Swerling model 1 that it says that the target RCS fluctuation is of Rayleigh scatterer type and propagation condition is slow fluctuation. Case II, the second type he did not give the name or dominating type that one and propagation condition is fast fluctuation.

Case III, target RCS fluctuation oh I made a mistake; this is correct it this is first type Rayleigh type. Target RCS fluctuation here I will say second type and propagation condition case III. So, slow fluctuation or sometimes it is also called scan to scan; that means, in a scan throughout there are no fluctuation, in the next scan there is fluctuations, slow fluctuation.

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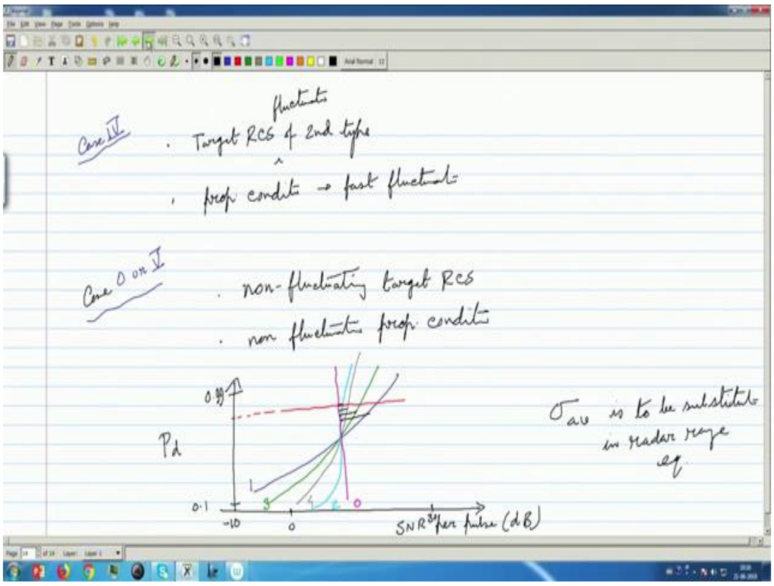

And, case IV is; obviously, the target RCS of 2nd type, target RCS fluctuation and propagation condition is fast fluctuation. So, Swerling model I, Swerling model II so, Swerling model III, Swerling model IV and the no target fluctuation and no propagation condition stationary. So, that is called sometimes case 0, sometimes also called case V. So, that says non-fluctuating. So, actually case V is nothing, but our original radar range equation model, non-fluctuating target RCS non-fluctuating propagation condition.

So, what he did Swerling made that he gave some graphs so, P d versus SNR per pulse in dB and so, and P d he gave from 0.1 to 0.99. And he so, this is the 0 model, then this is 2, this is 4, this is 3 and this is 1. So, he said that his models will be like this and he said that for a given P d you can find SNR from these curves that what is the SNR required. And, also in all four cases the sigma average is to be substituted in radar range equation instead of sigma.

In our original radar range equation we said only sigma, now it should be sigma average with the proper from the distribution you will have to find what is sigma average, that you will have to put and for SNR you will get this. So, you see that from this we can say that for a given P d, for a given P d maximum SNR is demanded by the curve 1. So, case I puts more demand, here you can see here also you can see that case I is always is the most demanding. So, if we do our calculation based on Swerling case I, then we know that we are having the most conservative estimate, but sometimes that is an overkill.

So, that is why people then go in the depth what is the actual model of the target as well as the propagation condition. And, fluctuate if this shows that fluctuating targets will require more SNR than the non-fluctuating case, that you can see from the 0 model. So, always it is for a any given P d you see always some extra SNR is required like here you see for this P d, this is the extra for model 2, this is the extra for model 4, then this is the extra for model 3 and this is the extra for model.

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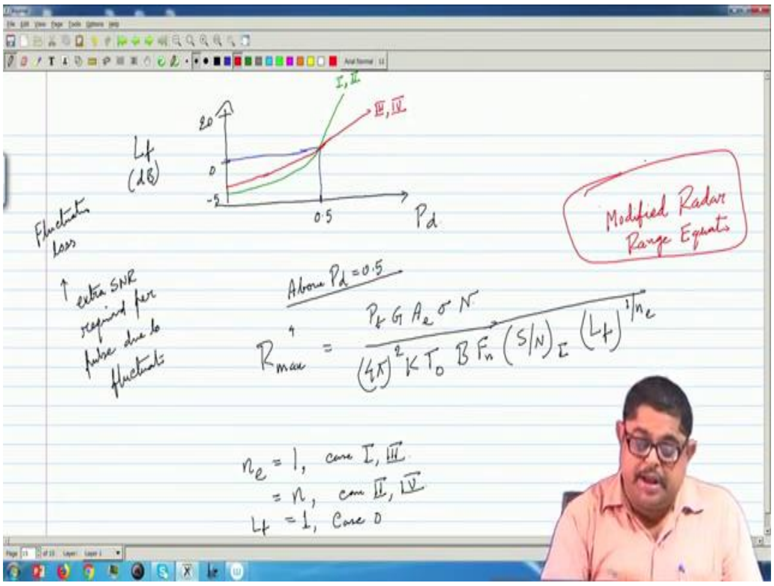

So, now that is why people also do one more thing that that they call the fluctuating loss because, of the target fluctuation and propagation condition fluctuation that loss is coming. So, I can put that as a loss, this is called fluctuating loss fluctuating loss in dB. So, what is its mean? The extra SNR required per pulse due to fluctuation.

So, that people have plotted Lf versus P d. So, here you can see that you can have minus 5 0 20 like that and here actually above 0.5 only these are valid. So, people have shown now the plots are something like this. So, we will see that I and II there both of the Rayleighs scatterer type and III and IV both are of the second type.

So, there Lf is similar and people have from these fluctuating loss they have made an empirical formula that this is valid above P d is equal to 0.5. So, what they have done so, the modified radar range equation is like this P t G Ae sigma, this N is a number of integration. Then 4 pi square K Boltzmann constant T naught B the noise figure then S by N for a single pulse and then Lf whole to the power 1 by ne.

$$R_{max}^{4} = \frac{P_t G A_e \sigma N}{(4\pi)^2 K T_0 B F_n (S/N)_I (L_f)^{1/n_e}}$$

Now, this ne is 1 for case I and case III is equal to n for case II and IV and Lf is equal to 1; if you have K 0 which boils down to the original equation.

So, this is the modified radar range equation after all our a thing. So, you have a extra fluctuating loss and this SNR calculation per pulse; so, this one you can calculate from Albersheim’s equation. So that means, you know now that what is the P fa for a given P fa given P d you are calculating this, then depending on the scenario you are modifying this. So, that completes this is the final thing that modified radar range equation. So, this is the final outcome of your study, now we can find out the range of the radar.

Now, sometimes clutter or nowadays with warfare etcetera deliberate jammer, they maybe many times more than the receiver noise. So, that jammer power etcetera and then if the noise becomes very high noise or jammer jamming thing then your v t will be cross many times; you will get lot of false alarm and that time the probability of detection will fall much below 0.5.

And so, detection is meaningless there, now in automatic radar thing then there is a need to overcome that; because if it is seen that lot of false alarms coming; that means, definitely the noise condition has changed. So, that noise condition can be sensed actually the so; that means, the clutter, clutter or jammer or that power that is sensed automatically.

So, what they do? They now take the echo’s and find their correlation and from that they can actually they can put two tap delay lines in both sides of a particular echo. And, they are compared with real targets video signal.

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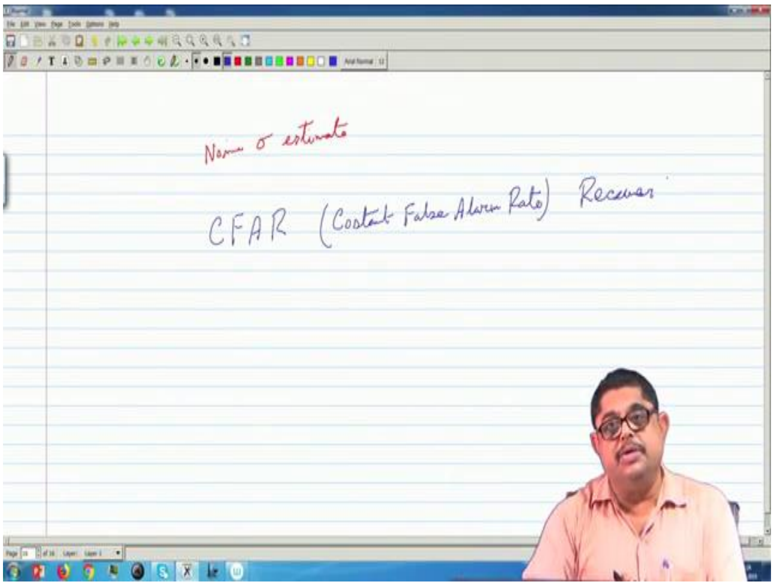

And, accordingly an estimate of the noise sigma, noise sigma estimate is found and accordingly the v t is adjusted. So, the receiver which does that is called CFAR, that is an advanced topic just I am telling that it is Constant False Alarm Rate receiver CFAR. So, how to make it automatically without any operator, that is a very good work of radar studies. So, basically the basic idea is same that constant if you want to maintain constant false alarm rate, then you will have to always have any estimation, dynamic estimation of the environment that is done by estimating the what is the power of the noise.

So, that gives you an estimate of the sigma of the whole scenario, the this sigma is the standard deviation; that means, statistical parameter of the noise or jamming or clutter thing. So, based on that again from that basic equation you can calculate to maintain constant false alarm rate what should be my v t, it is an exponential variation. So, you can have that thing and then you put that v t, you will see that automatically the false alarm rate will come down and it will be maintained that thing.

So, with that I conclude this detection thing, this statistical detection; now you know that if I am making at any measurement when you make we have a probability of our success, that is in this case probability of detection. And with a given probability of false alarm; that means, with the given noise statistics we can always say that this will be the radar range. So, with all these things that model is the modified radar range model we have described that, with that you can analyse almost all the cases that you will encounter in your professional life.

Thank you.