Principles and Techniques of Modern Radar Systems
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Lecture - 56
Statistical Detection Theory
(Contd.)

Key Concepts: Mathematical expression for false alarm time, analysis of false alarm time: with pulse compression, noise voltage threshold, probability of detection, Albersheim's formula

Welcome to this NPTEL course on Techniques and Principles of Modern Radar Systems.

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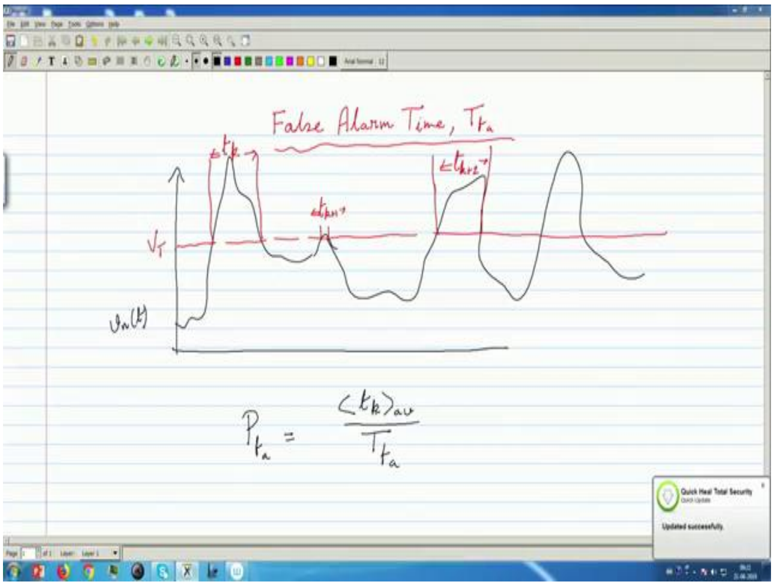

So, we were discussing the false alarm time T f a and you can refer to your graph I have drawn it again the same thing. So, we were we ended here that probability of false alarm is equal to the time that the noise voltage stays above the threshold average of that by T f a.

$$P_{fa} = \frac{\langle t_k \rangle_{av}}{T_{fa}}$$

Now what is this t k average? Now consider that I have a receiver, so receiver with a bandwidth B. Now so if I increase that bandwidth more will be noise power because more noise will come that we know.

So, then you see that if noise power is suddenly increased then there will be many such noise crossings and so I can say that average time it stays will decrease. Please understand these that if a event occurs much more time than the average time it is above the threshold that will decrease whereas, if the noise power is less this only a few crossings will be there but those crossings will be for larger duration.

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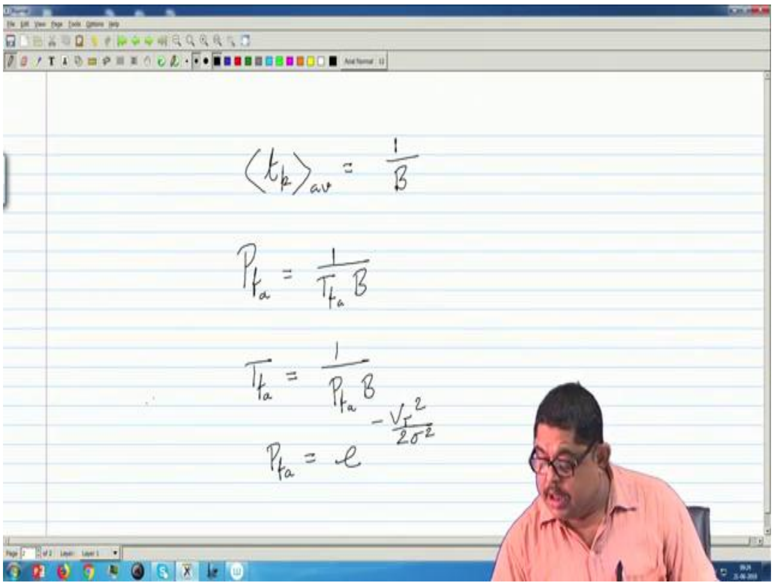

So, from here we can take an important decision that what will be our t k average that we can say it is inversely proportional to the bandwidth.

$$\langle t_k \rangle_{av} = \frac{1}{B}$$

You see that if I increase more bandwidth more noise power. So, average time that it stays over that will decrease. So, that we have got our answer. So, P fa is nothing, but 1 by T f a into B.

$$P_{fa} = \frac{1}{T_{fa} B}$$

So, what we were finding that instead of specifying probability of false alarm if we can tell what is the average time interval after which a false alarm will come. So, that T fa is nothing, but 1 by T fa is 1 by P fa into B and we have already found a expression for P fa. We know that from that Rayleigh statistics we have found that probability of false alarm means e to the power minus V T square by 2 sigma square.

$$P_{fa} = e^{-\frac{V_T^2}{2\sigma^2}}$$

So, if we put that here that is the thing let us go.

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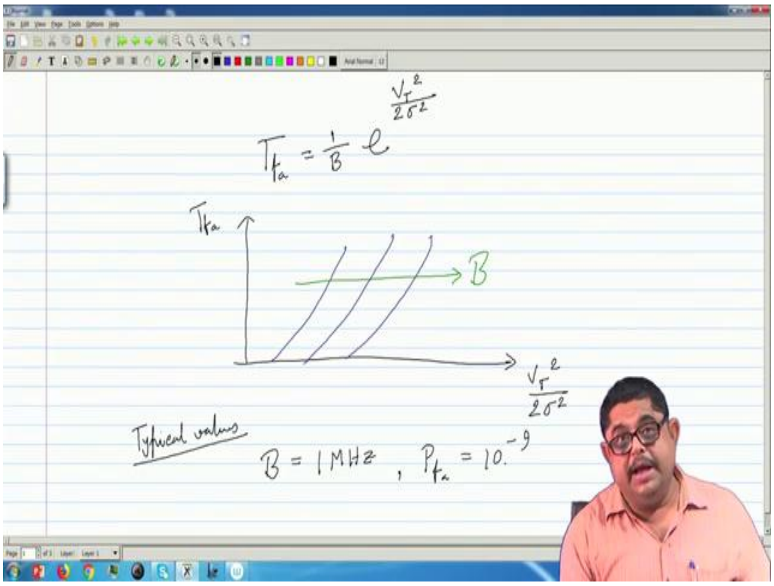

If we put that we can get that T f a is 1 by B e to the power plus V T square by 2 sigma square.

$$T_{fa} = \frac{1}{B} e^{\frac{V_T^2}{2\sigma^2}}$$

So, this is the time. So, the now if for a given threshold and a given noise statistics; that means, given sigma and a receiver bandwidth if I know I can always calculate T fa actually people have plotted T fa. So, T fa versus V T square by 2 sigma square if you do it is and exponential behavior.

So, I can easily draw them that they will be this exponential curves and I can say that the parameter here is B. So, with increasing B, I will have graphs like this. So, let us take some typical value suppose typically. So, suppose radar receiver. So, let us say the bandwidth is 1 megahertz and P fa generally this is a good P fa 10 to the power minus 9; that means every 10 to the power 9 second there will be it.

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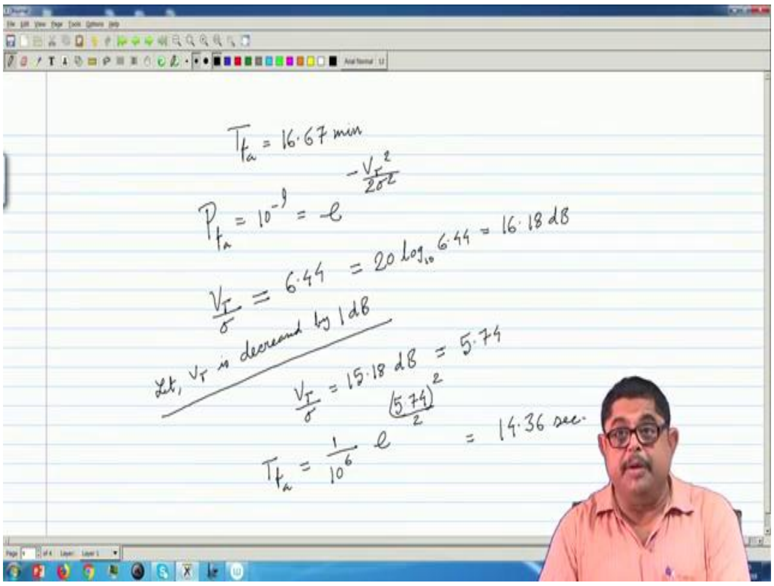

And, so with this values if you calculate from that formula T fa will come to be 16.67 minute, so every 17 minute it is expected a alarm. Now, if suppose this is comfortable actually what happens when a false alarm comes then it track is created with that target and after sometime track means it is a locus of the positions of the target.

So, after sometime because noise is the random things after sometime it will go and so in the next one it will be seen that there is nothing. So, immediately operator understand that was a false alarm problem is it too many false alarms comes one by one then he will have to open many tracks and keeping track of that is difficult or he becomes strainous that is the only problem.

So, if a after a sufficient interval if this false alarm comes then for an trained radar operator it is not a problem he can easily distinguish that. Now, to have this T fa of approximately 17 minute where one should to fix the threshold V T. So, we know that the expression so we can put that P f a is 10 to the power minus 9 and that P fa is e to the power minus V T square by 2 sigma square.

So, we can solve for this that V T by sigma from this will come to 6.44 that means the threshold should be 6.44 times the standard deviation of the noise. And in log terms if you want to convert it to log remember V T is a voltage. So, I will have to take 20 log base 10 6.44 and that comes to 16.18 dB. Now you see that if we look at this sorry if I look at this curve that T f a varies exponentially with V T by sigma so; that means, with a small decrease in V T; I will get a large change in T fa. So, let us say that say let V T is decreased from this value by 1 dB.

So, let us see what happens to the T f a. So, if V T is decreased by 1 d B; that means, V T by sigma is now 15.18 dB. So; that means, in absolute terms if you calculate it will be 5.74. So, if you put that in the T f a expression so T f a is 1 by V; that means, 1 by 10 to the power 6 e to the power plus this 5.74 square by 2. If you calculate it comes to 14.36 second.

So, you see that just we have decrease the V T by 1 dB, but T f a came down from 17 minutes roughly to 14 second. So, this is problematic that if V T is not properly fixed every 15 second if a false alarm starts coming so too many tracks will come and the operator won't able to handle it.

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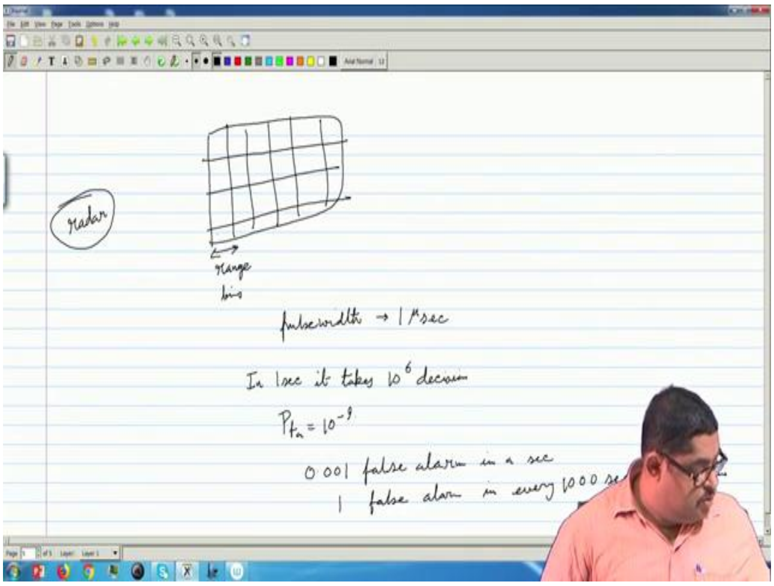

Now, let us come to the detection thing of radar, a radar makes a decision when we have seen that there are range resolution cells. So, if it is an imaging radar so suppose this is a radar it is not moving sorry that was a such thing, but let us say that it is taking a decision. So, radar is here and it is the whole zone has been divided, so these are one range bins.

So, every range bin it will have to take a decision. So, every range resolution cell the radar will have to take a decision. So, roughly I can say if a radar has a pulse width let us say 1 microsecond pulse width of 1 microsecond and suppose for start that it is not having pulse compression. So, I can say that roughly this is the resolution cell actually two third of this. So, roughly every 1 microsecond the radar will have to take a decision that whether there is a target or not.

So, I can say in one second it takes 10 to the power 6 decisions. Now for a P fa of again the same that 10 to the power minus 9 it means what that point 0.001 false alarm in a second, or you can say that 1 false alarm in every 1000 second every 1000 second. So, roughly 16; 17 minute that same thing that we have seen that time, so 17 minute this is comfortable.

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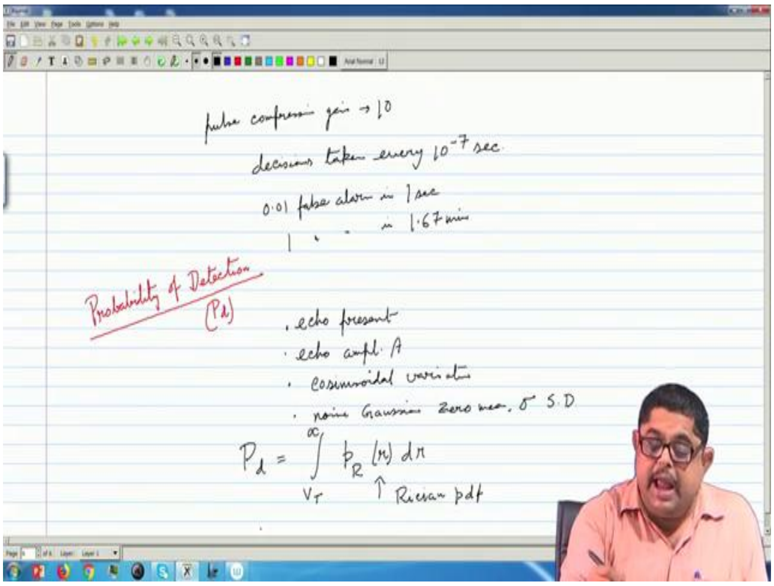

Now if over these there is a pulse compression ratio. Let us say that pulse compression gain of 20 a 10, let us say 10 pulse compression gain of 10. So; that means, decisions taken every 10 to the power minus 7 second. So, 0.01 false alarm in 1 second, or 1 false alarm in 1.67 minute. So, you see with pulse compression more false alarm will come that is obvious because you are making your pulse width effectively small. So, more decisions you will have to take; so, with a fixed P fa you will increase and so okay.

So, we have got that probability of false alarm or time of false alarm both are two sides of the same coin that thing one. Now the detection probability that given a all these things that given a noise statistics given a threshold etcetera. What is the probability that I will have a successful detection?

So, that will be our next topic that probability probability of detection. Previously it was probability of failure, now it is probability of detection p d. So, this is the case means when echo is present so; that means, this is the case when echo present echo's amplitude is A; cosinusoidal variation these are all my assumptions cosinusoidal variation. Then noise is Gaussian zero mean, sigma standard deviation.

So, I know that P d will so this statistics envelope distribution at the output of the video amplifier that will be now Rician. So, I will have to take the help of Rician distribution and my P d. I can write as that from V T to infinity any value if the envelope goes I will say ok there is a target.

So, this is p R r d r where this one is Rician PDF; Rician PDF.

$$P_d = \int_{V_T}^{\infty} p_R(r)\, dr \quad \uparrow \text{Rician pdf}$$

Now I can I know the Rician PDF I can put the value, but this integration is not so easy and so it does not have any closed form solution. So, what people have done people have plotted this things graphically or nowadays with numerical integration people have solve this problem.

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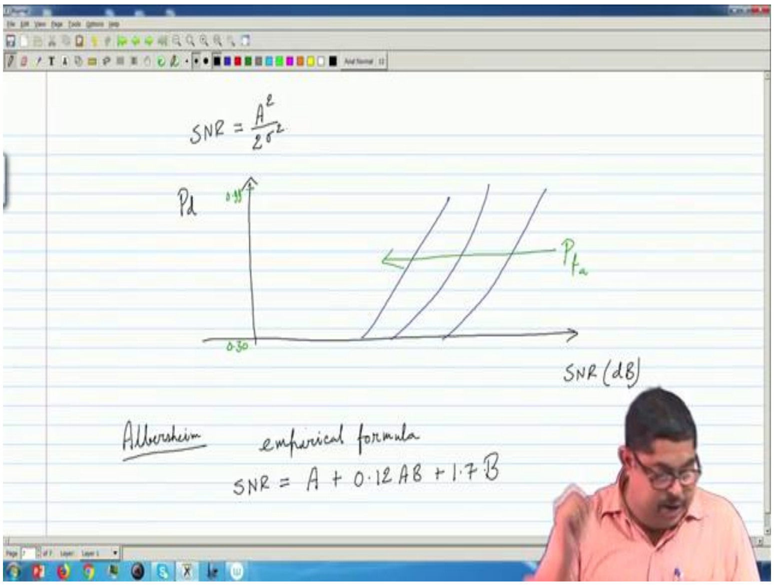

So, there actually they noted that what is the SNR, note that what is a SNR. SNR is we have seen when we discuss the Rician factor basically that A square by 2 sigma square.

$$SNR = \frac{A^2}{2\sigma^2}$$

So, people have drawn the graphs of P d versus this SNR in dB. So, again those graphs are exponential and the parameter here is P fa. If you change P fa; obviously, this thing but the shape is same. So, P d generally actually for good detection we want P d to vary from 0.3 to 0.99 etcetera and so these are the graph. So, if you have this graph generated then you can find out what is P d? There was a scientist called Albersheim; scientist Albersheim he actually studies this graphs and developed a simple empirical formula.

So, that is called Albersheim formula empirical formula which actually gives us the answer actually this formula gives us the SNR. It has expressed in terms of P d and P fa actually this is a thing we were looking for you seen when we started this topic that we want to modify the radar range equation to have more realistic thing. So, there we needed this so that was done by Albersheim is formula is I am writing SNR is equal to sum constant A plus 0.12 AB plus 1.7 B.

$$SNR = A + 0.12AB + 1.7B$$

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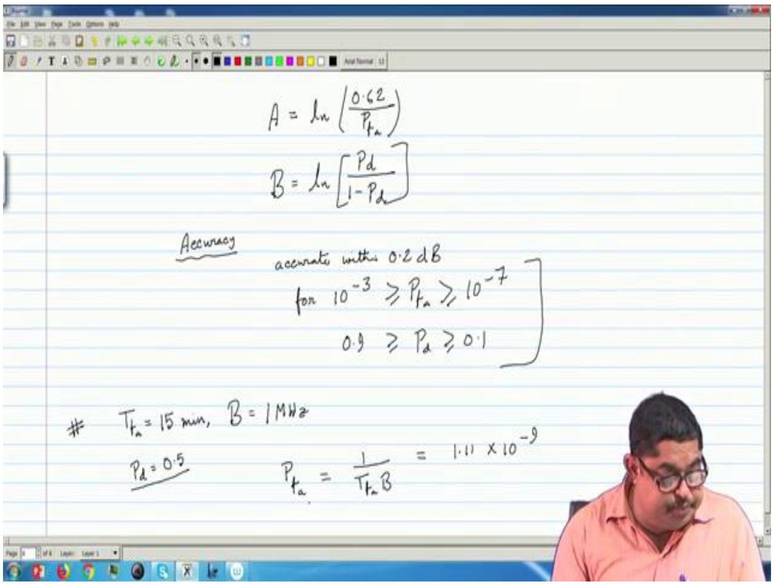

Now, here A and B are some constant basically A is a function of P fa 0.62 by P f a and B is a function of P d l n P d by 1 minus P d.

$$A = \ln\left(\frac{0.62}{P_{fa}}\right)$$

$$B = \ln\left[\frac{P_d}{1-P_d}\right]$$

So, and one thing I should say that please note that these SNR again I am writing not in d B this is in absolute value though this graph is d B, but actually while developing the formula he converted it to the absolute values and put it.

So, this actually relates everything this formula a what is the accuracy of this formula. Because every empirical formula should be always stated the accuracy. So, this formula is accurate within 0.2 dB in SNR so that is quite ok. But for what values?

So, your these values P fa and P d they should be within a range then only it is accurate to certain extent. P fa is 10 to the power minus 3 to 10 to the power minus 7. If it is varies in this zone then it is so you see that previously we have taken 10 to the power 9 minus 9 etcetera that it is not covering and P d is range is what you say 0.9 to P d 0.1.

So, these two if you satisfy then this formula is accurate. So, let us see a problem that suppose T fa. Let us take 15 minute and B typical radar receiver in VHF-UHF band 1 megahertz okay. So, now take various T fa given means actually P fa is given they are the same thing.

So, P d you will have to take so we will do P d for three cases you see that first let us take P d is equal to 0.5; that means, probability of detection is half. So, in half of the cases I will be able to detect on an average half of the cases I will not be able to detect. So, it is not a very good detection. So, let us start with the that what is the SNR; that means, what is a echo signals SNR required for this.

So, you put it the formula first will have to calculate the P fa; because these formulas A B are in terms of P fa. So, P fa if T fa is given B is given we can find P fa P fa is 1 by T fa into B and that if you put those values it will come to 1.11 into 10 to the power minus 9 ok. Then for P d is equal to 0.5, I can calculate A.

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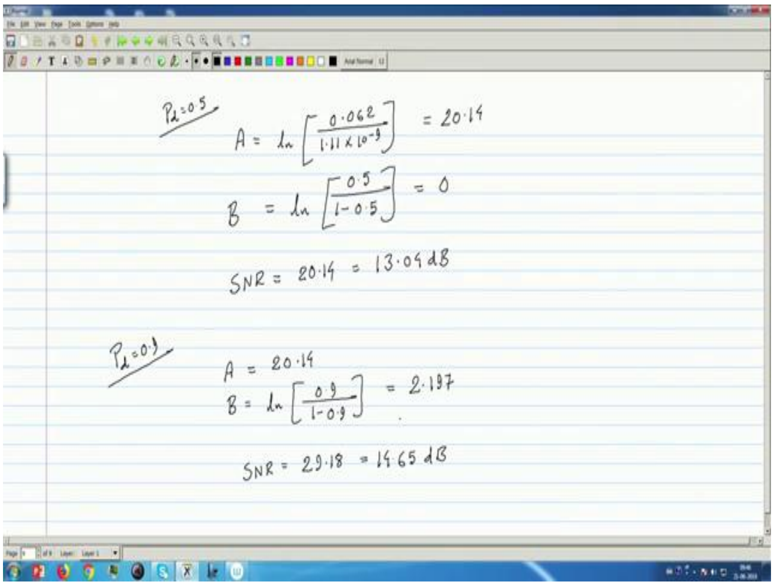

So, I am writing P d 0.5; A will be l n of that 0.062 divided by that P f a 1.11 into 10 to the power minus 9. So, this will come to be 20.14; B is l n 0.5 by 1 minus 0.5 that is 0. So, SNR will be in absolute value SNR is 20.14 because B is 0. So, this is actually so this SNR; SNR always is a power ratio. So, 10 log of these that will give you 13.04 dB. So, if I want to have this type of detection that T fa is every 15 minute I am I can get a false alarm and probability of detection 0.5 13 dB, SNR is required for that case. Now let us do for P d is equal to 0.9.

So, again you see A will be same because P fa you are not changing, but B will change B will be ln 0.9 by 1 minus 0.9 and that will give you 2.197 and so SNR you can put in that value it will come out to be 29.18 which in d B term is 14.65 dB. You see that from 13 dB I have change to 14.6 or 14.7. So, 1.7 d B the SNR increase the detection from a 0.5 became a quite good P d so that is the point that exponential variation. So, these all was same equation we can ok. So, we have got that this will incorporate in the SNR thing or radar range equation. But before going there there is another problem I am saying that we have not still addressed that is the.

So, this thing is addressed that probabilistically we are able to say what is the SNR requirement. In the simple radar range equation that was not possible. Now we have made that possible will do that, but another thing is remaining that is the targets fluctuation and the whole propagation affects. So, that we have not touched now we will next touch that and then finally, will modify the radar range equation. So, to do that we will see that actually one thing you should remember that any pulse radar it does not take a decision just by getting a single echo.

Actually it waits because it does integration without integration actually the radar receiver sensitivity is such that you cannot detect. Just a single echo telling it that there is a target and then finding its parameters that is not easy. So, for integration it waits for several pulses to come from the same target and they need to take the decision. The inherent assumption in the original radar range equation was the whole propagation scenario the target’s fluctuation everything is stationary during this time. But that is not true because the target may have some targets are very fast fluctuating things, some targets are slow fluctuating similarly propagation condition also sometimes fast sometime slow. So, based on that if we want to incorporate that we need to model those fluctuations that was done by Peter Swerling actually that is why they are called Swerling models.

So, we will see those Swerling models and based on that again we will find out that there is a extra term extra SNR will have to pump for those fluctuating cases. We will calculate that and then we will put these thing and that new the target fluctuation and propagation effect thing in the radar range equation and modify that and that will be more realistic model for many practical cases that we will see in the next class. Thank you.