Principles and Techniques of Modern Radar Systems

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Lecture - 53
Statistical Detection Theory: Introduction

Key Concepts: Revisiting the radar range equation, practical problems of the radar range equation, modified block diagram of a radar receiver, probabilistic approach for characterization of the envelope of the received signal

Welcome to this NPTEL lecture on Techniques and Principles of Modern Radar systems. We have seen various technological aspects of radar. Now, I take you to the initial classes, where we said that to have any engineering study of any system like radar, we need a model that model we that time introduced as radar range equation.

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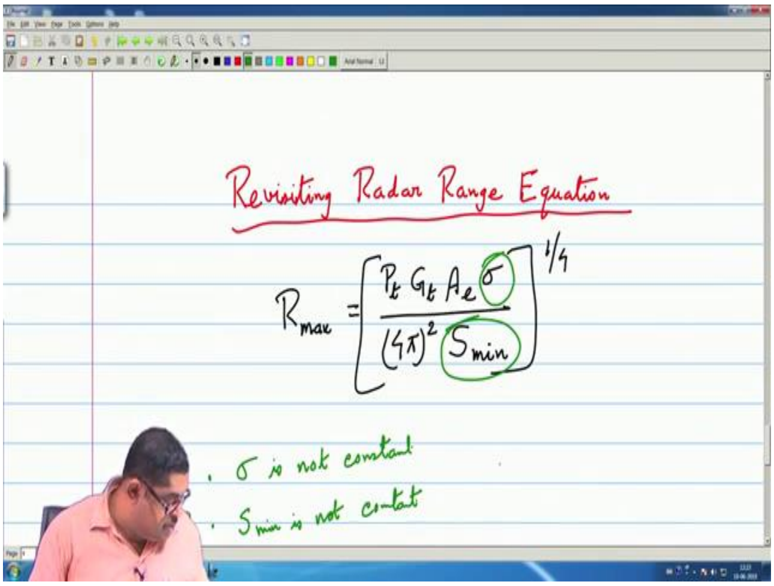

So, this you can see is the radar range equation.

$$R_{max} = \left[\frac{P_t G_t A_e \sigma}{(4\pi)^2 S_{min}}\right]^{1/4}$$

We will revisit that to have that model more realistic. Because that is the engineering way we have started with a simplified model. We have described all the techniques with respect to the model. Now, I will say that if you calculate the maximum range from these parameters of a radar system.

Many times we do not get the correct result, why? Because you see there are several points here one thing is the RCS; RCS of a target is not a fixed quantity actually its a fluctuating things. Because actually as I when we studied RCS we saw that it embodies the scattering by the target then that thing in the environment that signal echo signal gets transmitted.

So, environment also models that. So, various times atmosphere is different. Suppose in the rainy season or in cloudy weather or in severe storm the atmospherics thing is not same. So, the RCS of any target that will also have that fluctuation, but here we are keeping a that is fixed so, that is one part. So, I will write that what are the problems with this model that sigma is not constant this is one thing.

Then another part is this the receiver sensitivity S min. Now, receiver sensitivity is also dynamically changing because now, suppose your receiver has been heated it is operating for some time. So, the noise will pick up, so that time receiver sensitivity will go down. So, so let me write that S min is not constant ok.

Then also you see that in this whole expression we are assuming losses there are no losses. Apart from the spreading loss that all those r square r 4 that term, but apart from that there are no losses in the system but every electronic system will have at least the thermal noise. So, there will be some losses due to that etcetera.

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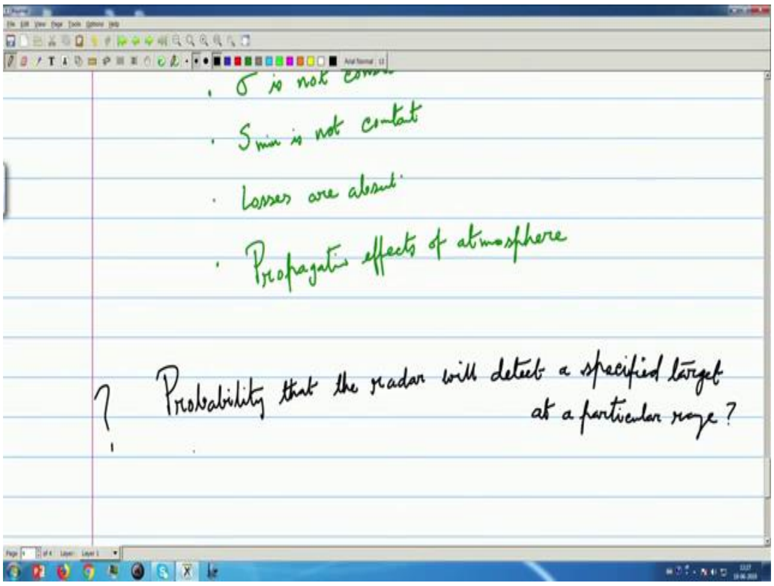

So, I will say losses are absent losses are absent in the model. Then I already said that radar signal transmission reception everything depends on propagation effects propagation effects of atmosphere etcetera. So, all these actually influences and our model should account for that. That is why we want to revisit their model. Also I want to say one thing that this sigma not a constant s min not a constant etcetera.

So, actually every time the whole thing is changing. So, we can say that these are something like a statistical thing also noise is there which is a statistical phenomena. We know that noise cannot be represented by any analytic function or by any tabular function. I do not know what is this value etcetera for deterministic signal also I do not know the value.

But the thing is deterministic signal I have the expression it will always fit there etcetera. But for noise or this random processes statistical processes this is not true. So, some amount of statistical a we want to need to find. And also we want to say that whether a target will be detected or not that we should have a statistical measure for that.

Because the noise etcetera they are statistically changing. So, my answer that whether there is a target or not that should also be a statistical it should depends on statistical techniques. So that means, when I am saying that radar has detected an target I may fail also. So, there will be some cases when I will fail also when there is a target I may miss that.

So, all those says that there should be some concept of probability. So, I should have a concept of probabilistic thing what is the probability? What is the probability that the radar will detect a specified target at a particular range?

So, I will have to say it probabilistically I cannot say always I will be able to give you the correct answer. But it should be such that the probability should be high then only I will say that radar is a good sensor for detection this is one thing.

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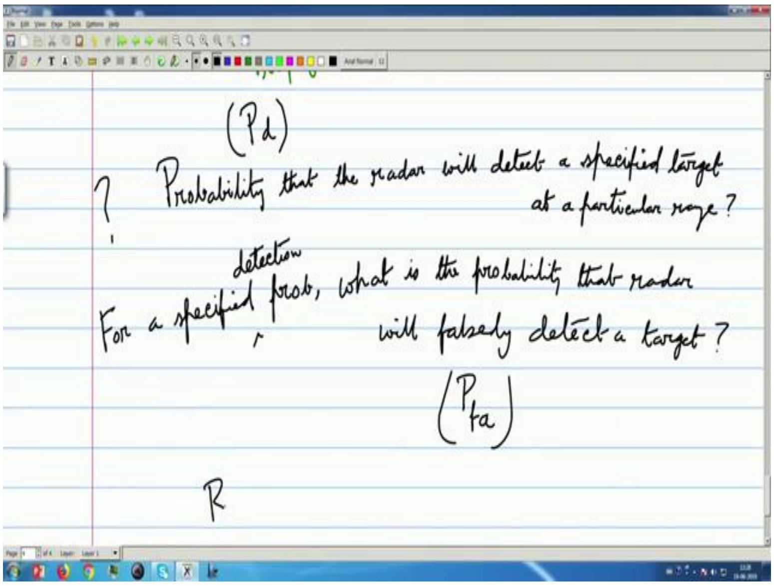

Another thing is that for a specified probability what is the for a specified. That for a specified detection probability; that means, suppose I have specified these detection probability, what is the probability that radar will falsely detect a target? That means, what is the probability of false alarm. So, these two the first one the probability that the radar will detect this is given a symbol P d probability of detection and this one probability of false alarm P f a.

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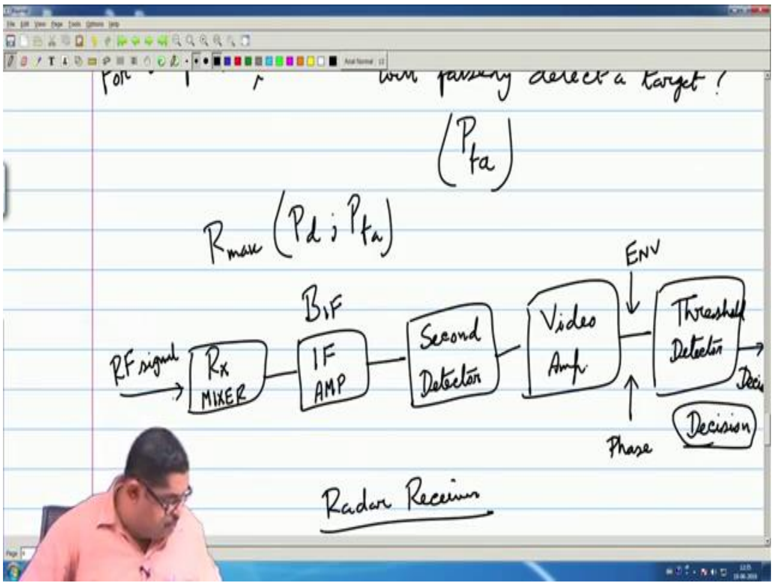

So, our model that R max see this model R max. That should be specified in terms of these two probabilities P d and P f a. That means what is the probability of detection, what is the probability of false alarm? Based on that what is your maximum range what is your all those parameters that what is your unambiguous range what is your receiver sensitivity what is your RCS etcetera.

So, these we will have to now do that we will do this is called the actual statistical detection theory this part I will now cover now. To do that actually I need to develop the basics of these. So, for that what I will do now I will make that a again a simplified block diagram of the receiver. Because this decision whether there is a target or not that is taken at the detector in the receiver.

So, for that is why I will now give you a model that I have an R F receiver mixer because the R F signal is coming. So, I can say that R F signal. Then mixer after that we know that after mixing it has come to I F. I am assuming it may be two stage three stage, but I am assuming one stage only.

So, I F there will be an amplifier then after that. We know there is a second detector actually these are all radar terminology. I will explain these second detector. And then there is a video amplifier. Then there is a threshold detector, who takes the decision sorry, let me call this decision.

So, this is our now point that how this decision is taken. Now, here actually these video amplifier output here I may have envelope detector. So, I am not specifying or I can have phase detector based on that the decision is taken that if it is above a threshold we say ok. Something is there if it is below the threshold we say nothing is there.

Even though the actual case may be opposite that suppose when I am saying there is a target actually may be noise is such high that I am falsely saying that there is a target. Similarly suppose a target it is there, but it is for some time it is embedded in noise. So, I was observing noise. So, I will think no it is noise there is no target. So, I can miss that that is why now here.

So, this is my radar receiver block diagram radar receiver. Based on this I will build up the theory. So, now this second detector actually this is also a actually diode stage; because after IF amplifier you have this diode in generally all the thing. Now, this mixer this also employs diode that is why it is called second detector or second diode that is why the name came.

And now video amplifier you know that this is the bandwidth falls in the video bandwidth that is why the name video amplifier. So, this is an amplifier and output of the video amplifier there you are getting either the envelope or phase and based on that. If you have an envelope detector, that is this second detector doing envelope is getting detected. If you have a phase then you have another mixer part etcetera.

So, and we assume that throughout this all these blocks the maximum bandwidth that is BIF. So, I will say that this I am assuming is the bandwidth of the whole receiver; that means, none of this is crossing these bandwidth so, next page.

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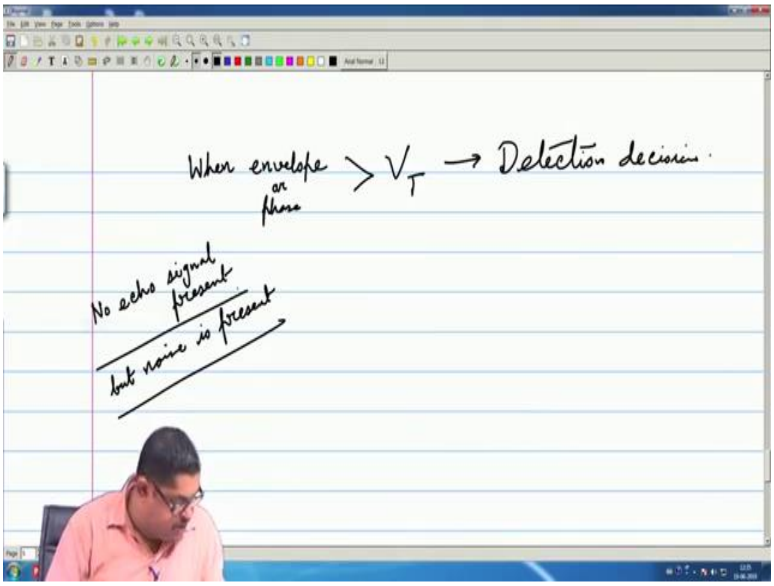

So, I can say that when envelope or phase is above a threshold, I am writing V T. In case of phase it will be some phase a thing so, V T. So, the detection decision made. So, these are statistical decision as I explained ok. Now, to have a answer to our that probability that what is the probability that I will detect correctly or I will give you a false picture for to understand that.

Let us assume that a case that this RF signal is absent, but we know that if signal is not there even if the signal is not there since it is coming through the free space. So, there are noises. So, noise has come to the antenna. So, there will be always some signal and something here that will be noise. Actually in general this signal plus noise comes now first I will take a case where no echo signal present.

So, what is present? No echo signal present, but noise is present. So, that noise is getting passed through all this block diagram. Now, noise is we assume a white Gaussian noise infinite bandwidth. But the moment is comes through this receiver of bandwidth BIF its bandwidth also becomes the same BIF it is not an infinite bandwidth anymore. So, its a band pass noise because this is an RF receiver. So, I have band pass noise.

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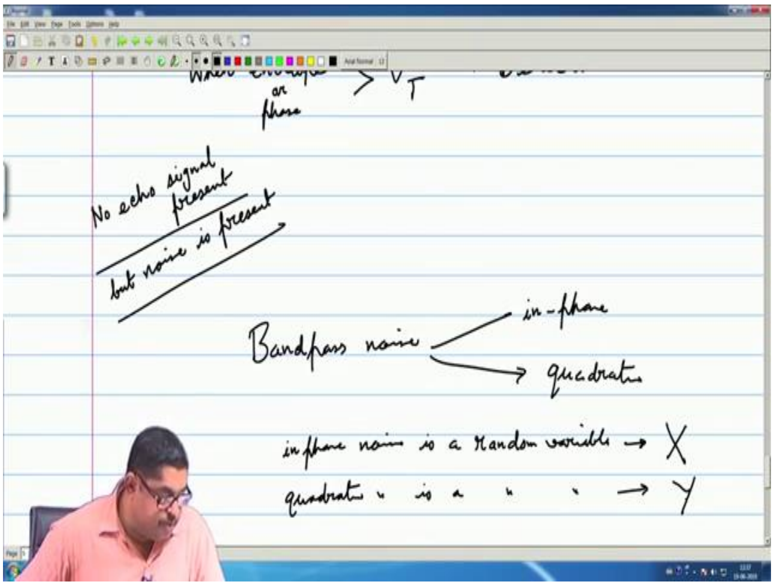

Now, we know that any band pass signal or this noise can be broken into two parts, in phase and quadrature two orthogonal parts any noise can be written. So, we will assume that this in phase noise is a random variable. Let me call that capital X is the random variable and quadrature noise is also a random variable let me call it capital Y.

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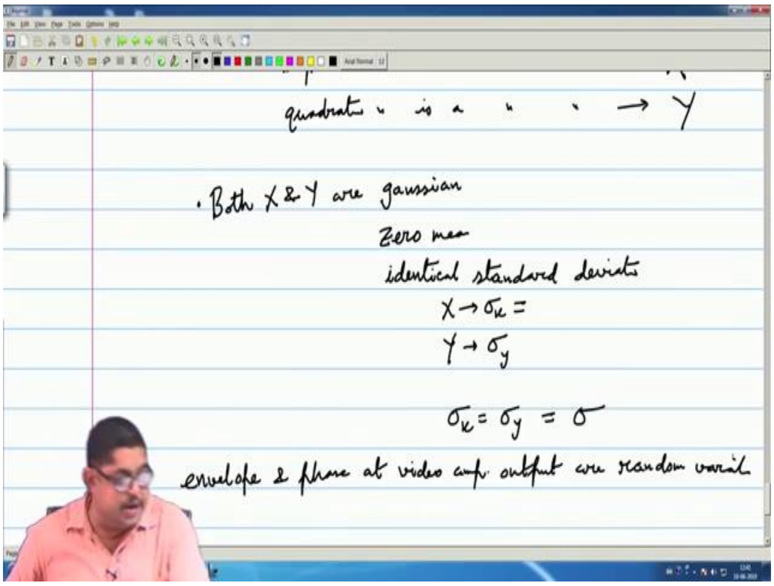

So, they are random variable capital Y and we assume both X and Y are Gaussian zero mean and identical standard deviation. Because any noise or any random variable gets characterized by mean and standard deviation. So, let us call that their standard deviation sigma x for X is having sigma x. Y its standard y random variable its standard deviation.

Now, due to this identical standard deviation I can say sigma x is equal to sigma y and let me call it sigma let me drop the subscript x and y.

$$\sigma_x = \sigma_y = \sigma$$

So, question is that suppose assume that in my this block diagram I have doing an envelope detection. Now, no signal is present noise has entered this type of noise I have characterized that now what will be the envelope detect.

Envelope that envelope of these will also be a random variable what is the probability distribution of that envelope? That means, what will be the distribution at the output of the video amplifier will get? This was a fundamental question radar people solved actually this thing was solved by radar people Rice actually solved that and.

So, now, I can I will do that theory these are the statistical part of the communication theory you have probably done it in your communication classes. But I will revisit because based on that actually we will have to modify our radar range equation. So, I can say that both envelope and phase at video amplifier output are random variable. Because they are envelope of x and y to in phase and quadrature noise also the phase with respect to the phase of quadrature with respect to the in phase that also is a random variable. So, if their random variable their character one of the character characterization is their p d f.

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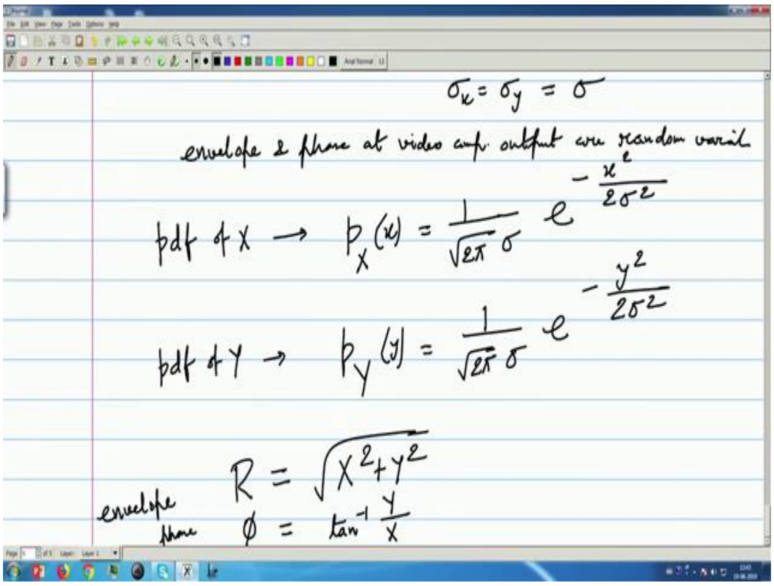

So, what is the p d f probability density function of X? Since we have assumed that they are Gaussian we can easily write it. So, that will be p X x is equal to 1 by root over 2 pi sigma out of this bracket e to the power minus x square 0 mean. So, nothing there 2 sigma square this I know.

$$p_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

Similarly, I can write p d f of Y; that will be p capital Y small y 1 by root over 2 pi sigma e to the power minus y square again 0 mean.

$$p_Y(y) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{y^2}{2\sigma^2}}$$

x may take any value from minus infinity to plus infinity y may take any value from minus infinity to plus infinity this is their probability distribution. Now, what is the envelope of these?

So, I can write R is the envelope that will be X square plus Y square square-root this is the envelope. So, I can write envelope R and phase phi that will be tan inverse Y by X.

$$R = \sqrt{X^2 + Y^2}$$
$$\phi = \tan^{-1}\frac{Y}{X}$$

So, X and Y are random variable. So, R and phi are also random variable what we need to do given these I know the p d f of X, I know the p d f of Y what will be the p d f of R what will be the p d f of phi this is the problem.

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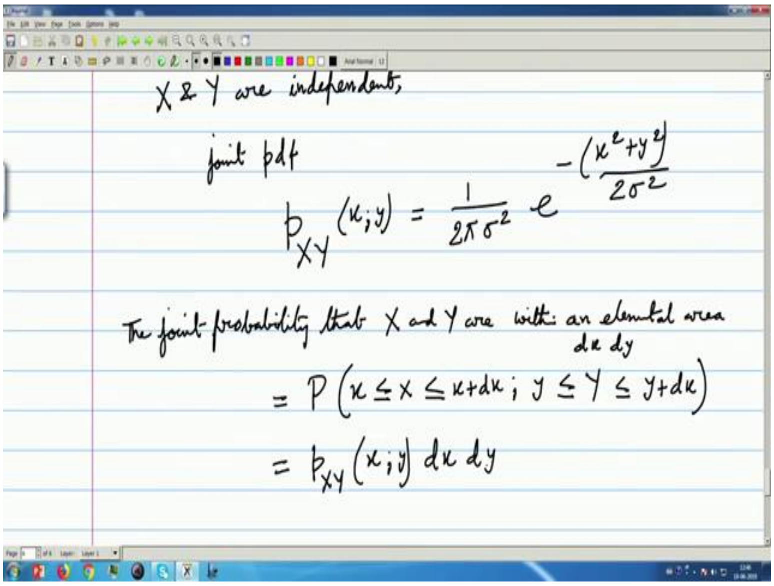

So, I can easily do this, because x and y are in quadrature. So, they are also independent. So, they are joint p d f I can write independent. So, their joint p d f is nothing but p X Y; x y is equal to 1 by product of those two 2 pi is not it. Two independent variable their joint p d f will be 1 by 2 root over 2 pi root over 2 pi that is this, e to the power minus x square plus y square by 2 sigma square.

$$p_{XY}(x;y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

So, this is their p d f. What is the joint probability? The joint probability that X and Y are within an elemental area d x d y. That will be probability that X that x is capital X, the random variable is having a value between x and x plus d x. Similarly capital Y is having a value from y to y plus d x this is probability.

And if I know the joint p d f I can write this probability. So, that will be p X Y x y into x is varying from x plus d x. So, it is d x d y these this we know from our elemental knowledge of probability ok.

$$P\left(x \leq X \leq x+dx;\ y \leq Y \leq y+dx\right)$$
$$= p_{XY}(x;y)\, dx\, dy$$

Now, you see that we know that R and phi they are dependent on X and Y. R is equal to square root of plus square root of X square plus Y square and phi is equal to tan inverse Y by X.

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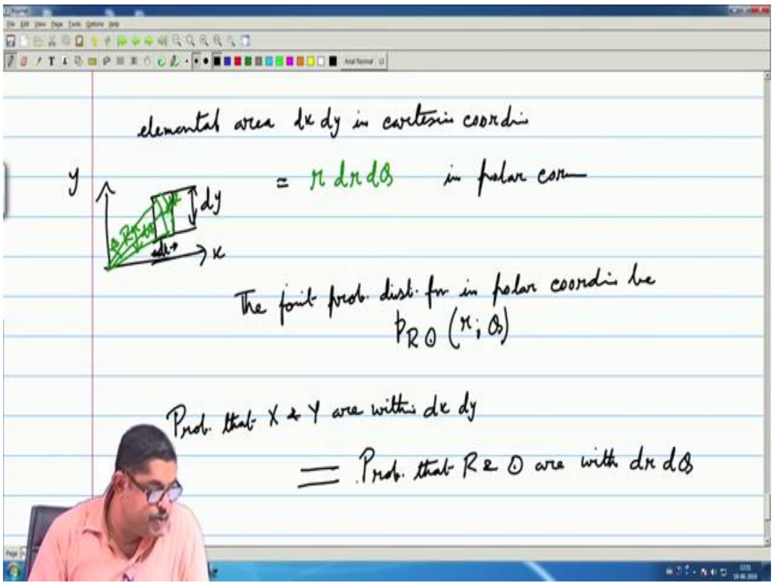

Now, generally R phi we represent in 2 D polar coordinate. So, question is this elemental area elemental area d x d y in Cartesian coordinate is equal to what elemental area? In polar coordinate; that means, suppose in the Cartesian coordinate I do not require this 2 dimensional is enough.

So, there I have suppose this is x axis this is y axis. So, I have an elemental area this; that means, this is my d y and this is my d x and this is my R this is my d R and this is my theta and this is my d theta. So, what is the same elemental area in polar coordinate can I not say that this is r d r d theta in polar coordinate. So, these two are same elemental area d x d y in Cartesian coordinate.

And r d r d theta this we know from our basic a thing. So, I can say that the joint probability, joint probability in polar coordinate. Let the joint probability distribution function in polar coordinate be p R theta r theta ok. Now, the probability that x is within x plus x d x and y is from y to y plus d y that will be then same as probability that r is from r to r plus d r.

And theta is from theta to theta plus d theta; that means I can write that this is joint probability distribution now and probability will be same; because if the elemental areas are same then probability is same. So, probability that X and Y are within d x d y is same as probability that R and phi are within d r d theta same elemental area.

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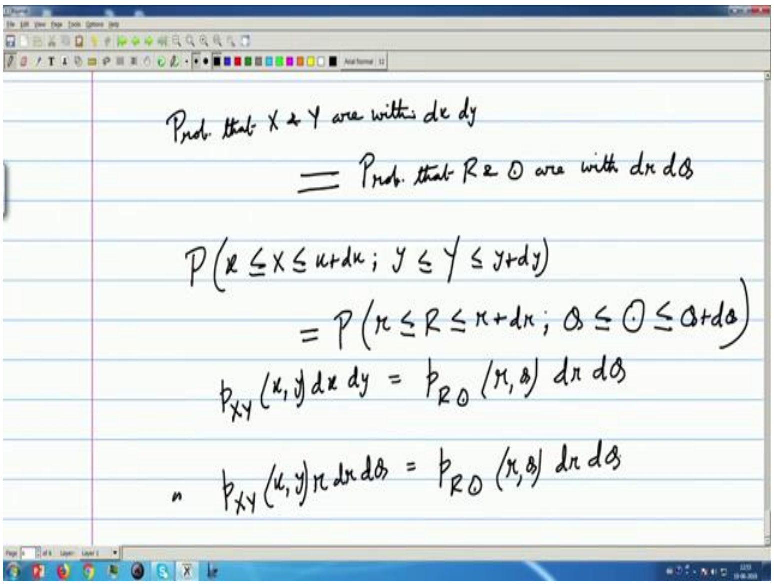
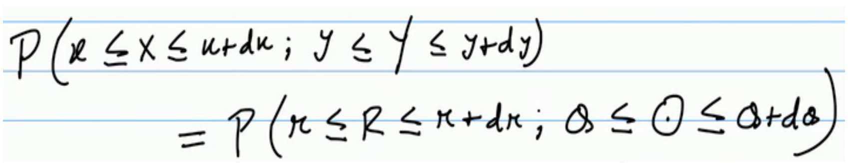


So, I can write that P x X plus d x; y capital Y y plus d y. It should be equal to P envelope is r plus d r and theta this should be same. So, these two probability should be same because the areas are same elemental areas.



So, then in terms of joint probability distribution I can write that p X Y x y d x d y should be equal to p R theta r theta d r d theta this is the outcome of these. Now only thing is what is d x d y in polar coordinate let me put that value. p X Y x y sorry I know this area is r; d r d theta is equal to p d theta the.

$$p_{XY}(x,y)\,dx\,dy = p_{R\Theta}(r,\theta)\,dr\,d\theta$$

$$p_{XY}(x,y)\,r\,dr\,d\theta = p_{R\Theta}(r,\theta)\,dr\,d\theta$$

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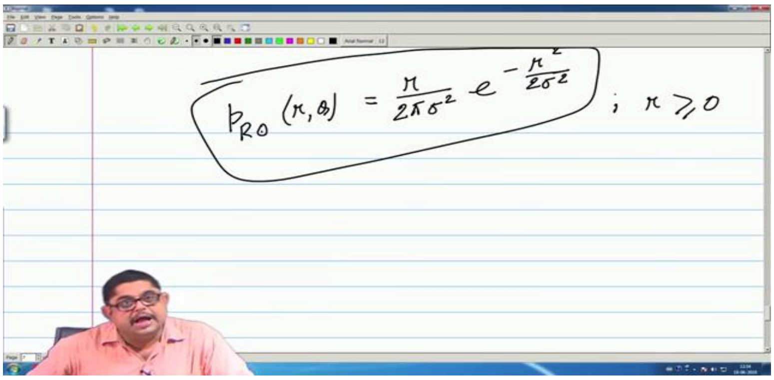

So, I have got the joint distribution r theta is nothing but I have already written that. So, r by 2 pi sigma square e to the power minus r square by 2 sigma square. And only one thing envelope cannot be negative. So, envelope is greater than equal to 0; envelope cannot be negative.

$$p_{R\Theta}(r,\theta) = \frac{r}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \quad ; \quad r \geq 0$$

So, minus infinity to plus infinity it cannot vary it can vary from r is ……..

This is the joint probability distribution just you see from the elemental knowledge we can relate this to and this is an. Note that this joint distribution is independent of the phase variable theta that shows that the envelope and theta they are completely independent. And so, we can find out either the envelope distribution or the phase distribution from this joint distribution by properly integrating. That we will see in the next class that will solve our problem.

Thank you.