Principles And Techniques Of Modern Radar Systems

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Lecture – 50
SAR Processing (Contd.)

Key Concepts: Factors influencing the scan length, determination of maximum value of the cross range resolution, link budget calculation in SAR

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar Systems, we were discussing SAR systems, synthetic aperture.

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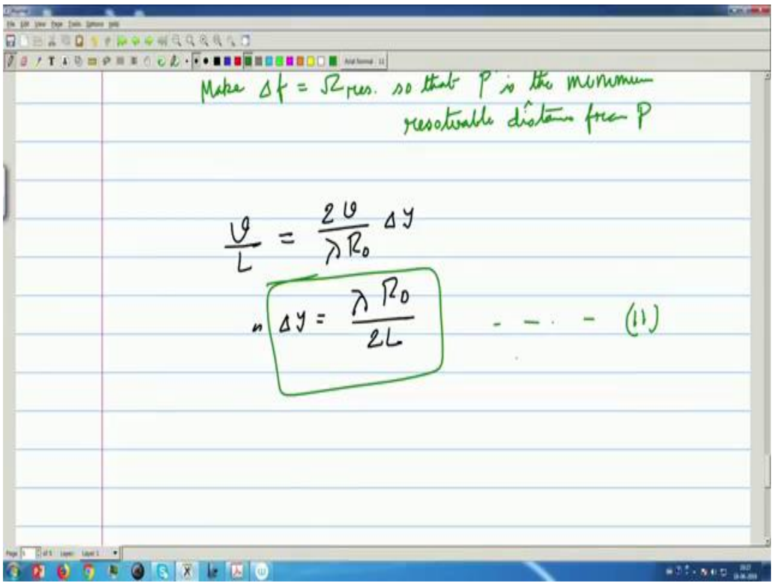

Make $\Delta f = \Omega_{res}$. so that P' is the minimum resolvable distance from P

$$\frac{v}{L} = \frac{2v}{\lambda R_0}\Delta y$$

$$\boxed{\Delta y = \frac{\lambda R_0}{2L}} \quad \text{- - . . -} \quad (11)$$

Now, we have found the cross range resolution and we have seen the doppler shift of the signal due to the SAR motion. Remember that doppler frequency of each signal is almost linearly varying near t is equal to 0; near t is equal to 0 that quadratic variation becomes almost linear.

So, the radar echo signal is not a constant frequency signal. The doppler filter in the receiver that will have to differentiate these two linearly varying frequencies close together, but with present days' electronics that is not a tough problem with digital filters. In case of constant frequency signal the will see that SARs phase compensation is a linear function of time. In SAR filter this phase compensation is a quadratic function that we have already seen, that phase is quadratic. Different Doppler filters are required for different ranges and if R 0 is maximum the slope of the f D versus t curve is less, ok.

Now, let us see that what is the limit to this cross range resolution it is seen that the cross range resolution is inversely related to L so; obviously, increasing L improves the resolution, but there are two limiting factors of L.

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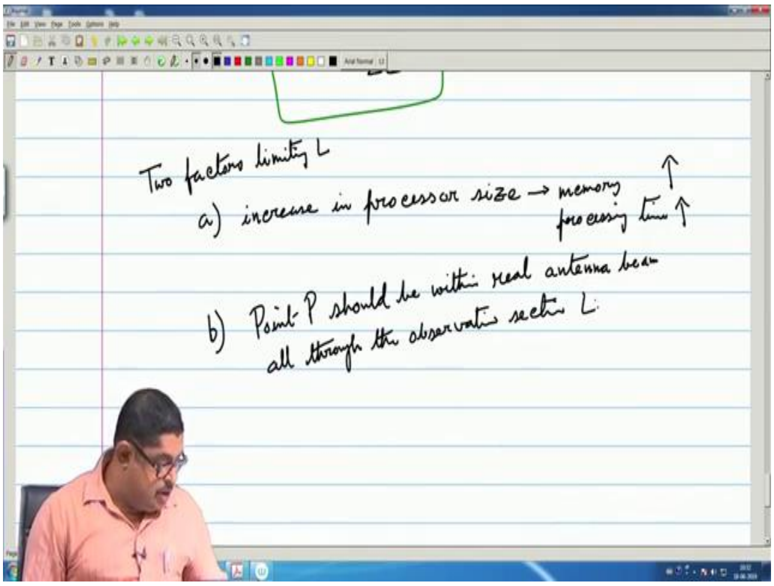

Two factors limiting L

a) increase in processor size → memory ↑
processing time ↑

b) Point P should be within real antenna beam all through the observation section L.

So, let me write that two factors, limiting L, the first one is increase in processor size; why? Please see what is L. This L is so during L many pulses are being sent and they need to be kept all of them so; that means, the processor will have to have access to a good storage. So, if I increase L the storage will increase and then it will have to compensate phase compensate each of those echoes. So, computation will increase also the; so, we will need to say that because these are generally satellite systems etcetera or aircraft systems. So, on board you will have to have that. So, increase in processer size means memory and processing time increases.

So, I can say this is increasing this is also increasing. You remember processing time increase means to generate an image you will take longer time etcetera, but that I will say that not the main thing, second thing is a geometric limitation that point P should be within real antenna beam all through the observation section L. Now, this point we will discuss now.

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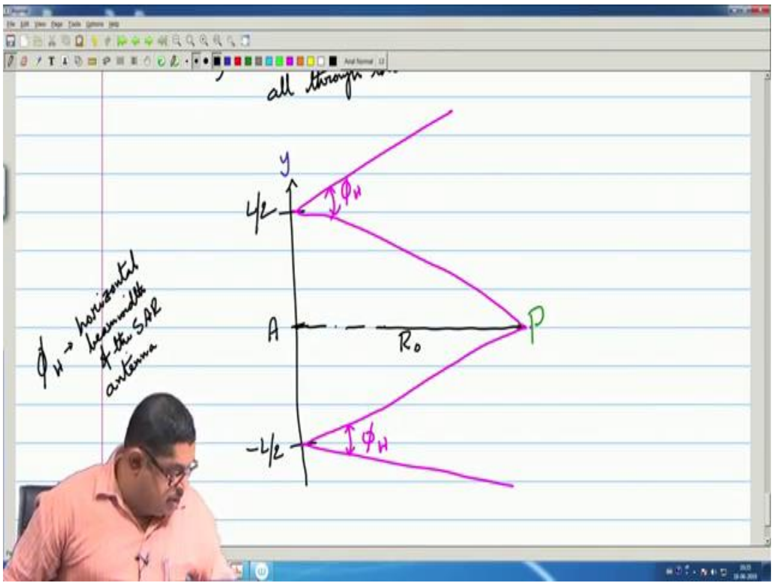

Actually you see that, this is my let us say L by 2, this is plus L by 2, the point 1 2 3; 1 2 3. So, the point P is here ok, this is my centre point A this is my R naught and this is our y-axis. So, at this point at when the radar is at minus L by 2, I should have the beam at least like this; that means, beam is the upper portion of the beam is; so, this is the beam, the upper portion of the beam is illuminating point P and let me call the horizontal because this is at the ground. So, horizontal beam width let us call it phi H and that the upper portion of the beam should touch P. Similarly, from here I can say that lower portion of the beam should touch this. So, these are the two extreme points.

So, let me call with this what is my phi H, this is the horizontal beam width of the SAR antenna.

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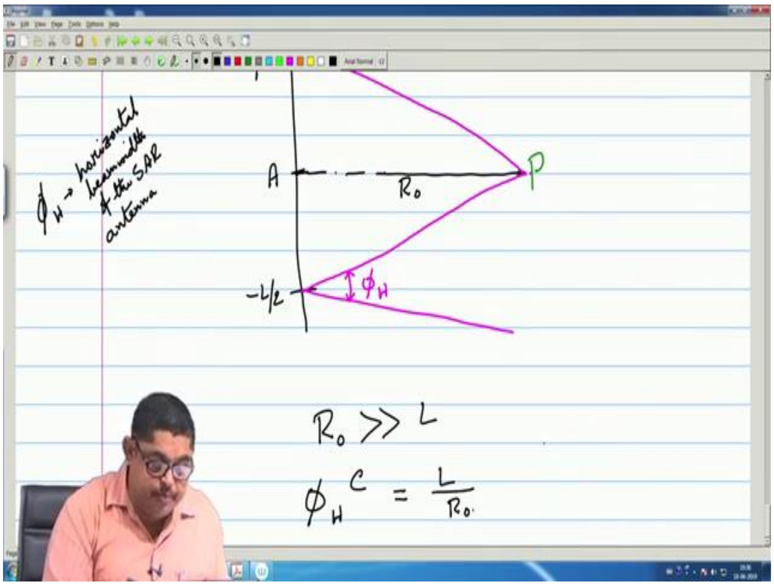

$\phi_H \rightarrow$ horizontal beamwidth of the SAR antenna

$$R_0 >> L$$

$$\phi_H^{C} = \frac{L}{R_0}.$$

Here, I will have to make one realistic assumption that R naught which is the range you see these are all aircraft based system or satellite based system. So, R naught is much much greater than L. So, we can say not theta we have used different thing. So, phi H in radian these will be. So, from this geometry easily I can say L by R naught, this if R naught is much greater than L then this is L by R naught.

$$R_0 >> L$$

$$\phi_H^{C} = \frac{L}{R_0}.$$

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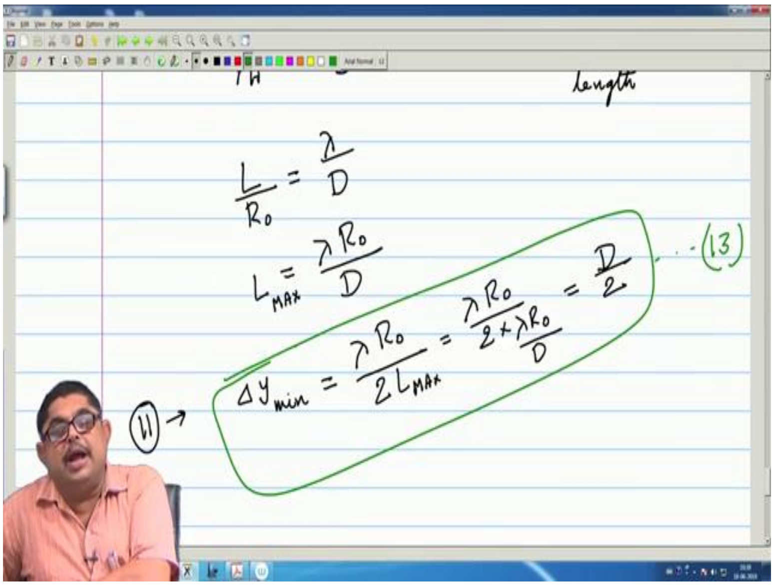


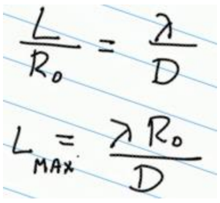
$$\frac{L}{R_0} = \frac{\lambda}{D}$$

$$L_{MAX} = \frac{\lambda R_0}{D}$$

$$\boxed{\Delta y_{min} = \frac{\lambda R_0}{2 L_{MAX}} = \frac{\lambda R_0}{2 \times \frac{\lambda R_0}{D}} = \frac{D}{2}} \quad \cdots (13)$$

(11) →

Also we know for an antenna the beam width is given by lambda by D, where D is the real aperture length I am saying we generally call it dimension the largest dimension of the antenna. So, in this case real aperture length; so, I can say that L by R naught is equal to lambda by D. So, L is equal to lambda R naught by D. Now, this is the maximum that we can have, maximum L should be like this. If, L is more than this then this point P cannot be illuminated by these. So, basically, I can say that this is the condition for L MAX, the maximum L can be these.



$$\frac{L}{R_0} = \frac{\lambda}{D}$$

$$L_{MAX} = \frac{\lambda R_0}{D}$$

So, the second factor of this point P should be within L antenna beam also the observations section L tells me that physically you cannot have a SAR antenna length greater than that value.

So, if I put that into this equation 11, I get the minimum possible resolution value. So, I can say from 11, equation 11 I get delta y min because this is maximum and this is inversely related. So, delta y min will be lambda R naught by 2 L MAX and if I do that naught by 2 into lambda R naught by D it becomes a surprise, that it is D by 2.

$$\Delta y_{min} = \frac{\lambda R_0}{2 L_{MAX}} = \frac{\lambda R_0}{2 \times \frac{\lambda R_0}{D}} = \frac{D}{2}$$

I think we should give it a number, but before that I will say that this also generally is an important thing because from this approximation this came and with that this will become 13 or this that the cross range resolution, the best you can have is half of the length of the antenna. So, if you have a 1-meter dish you cannot have a cross range resolution better than 50 centimetre like that.

So, this is actually you see the, I will say the, you cannot have everything so, you cannot go on reducing L and whatever the real actually we synthesized the antenna we wanted a larger antenna, but resolution we cannot beat that. So, resolution will be half of the size of the antenna, but still that is a very very good value and people are using it etcetera. Now, here one question can be asked that ok, I will make my antenna the real antenna smaller that will improve my resolution, can we do that? I think no SAR engineer will recommend that. Why because if you make your antenna smaller actually the gain of the antenna that will fall.

So, you will have to have large amount of power to for the same thing. So, receives signal strength will decrease if the gain is less because it is a two way gain for radar because it is sending and again receiving. So, both the gains will decrease. So, that will make your transmitter more powerful you need to make etcetera.

So, actually there is a cross check conflict between these that what will be the optimum size of the antenna. To answer that question actually you we need to find the this resolution and what is the power of the transmitter that we need to have an expression that we will derive. But I am saying actually you see these conflict that if you want to reduce the size of the antenna the power requirement gets more, the same this conflict was there in downrange also, that pulse when we discuss pulse compression, actually we saw that if you have one to have better range resolution you should have short pulse, but short pulse will not be able to carry power.

So, that is why pulse compression was invented that you have long pulse, you carry power, but at the processing time you get the benefit of the small pulse. Similarly, here in the SAR that you synthesize the antenna etcetera, but up to a limit that limit is this D by 2.

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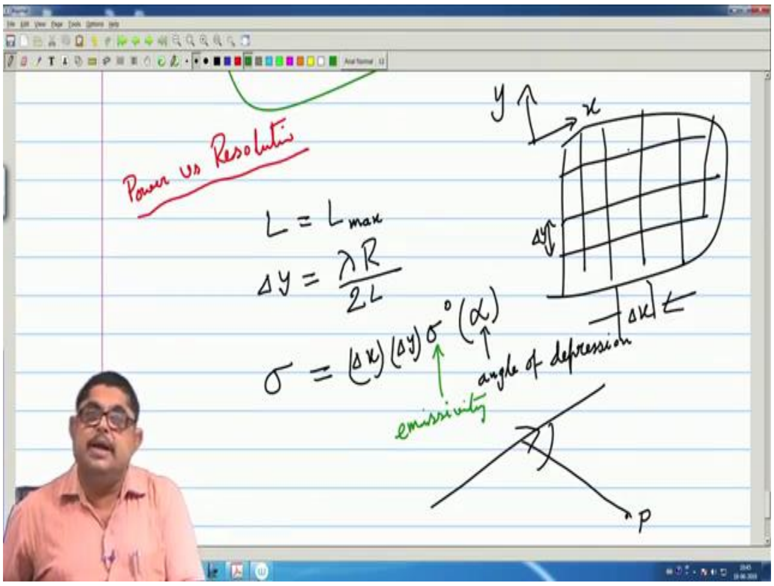


Power vs Resolution

$$L = L_{max}$$

$$\Delta y = \frac{\lambda R}{2L}$$

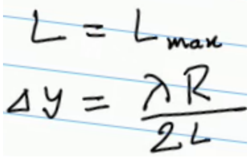
$$\sigma = (\Delta x)(\Delta y)\sigma^{\circ}(\alpha)$$

emissivity

angle of depression

Now, let us come to the point that power versus resolution expression. So, here again we will start that L we are taking as L max and we cross range resolution I am again writing for a reason. We have already found that expression delta y where is delta y, delta y lambda R naught by 2 L, but you see this here it is R naught; R naught is at t is equal to 0 you get this resolution, but at other points it is not R naught it is R.

So, that is why I rewrite that the delta y is lambda R by 2 L.



$$L = L_{max}$$

$$\Delta y = \frac{\lambda R}{2L}$$

And also here I say that for ground actually people have found out the something called emissivity which is related to the RCS of the ground by this thing that, if you suppose you have this ground and let us say that this is your y direction, this is your x direction. So, these are your x direction.

So, let us say that my resolution in x direction is delta x and here it is delta y, then delta x delta y into the emissivity as a function of alpha this is angle of depression.

$$\sigma = (\Delta x)(\Delta y)\sigma^{0}(\alpha)$$

That means, the way the radar is looking at the ground, the radar is flying like this and point P is here so, it is looking at the angle of depression. So, that so something like squint angle. So, this term is called, so this sigma naught this is called emissivity, people have found for the various regions the emissivity values are known.

So, we will use that because radar people use that thing so; that means, for an unit area what is the emissivity that this is the relation between. So, I will say this is RCS and this is emissivity.

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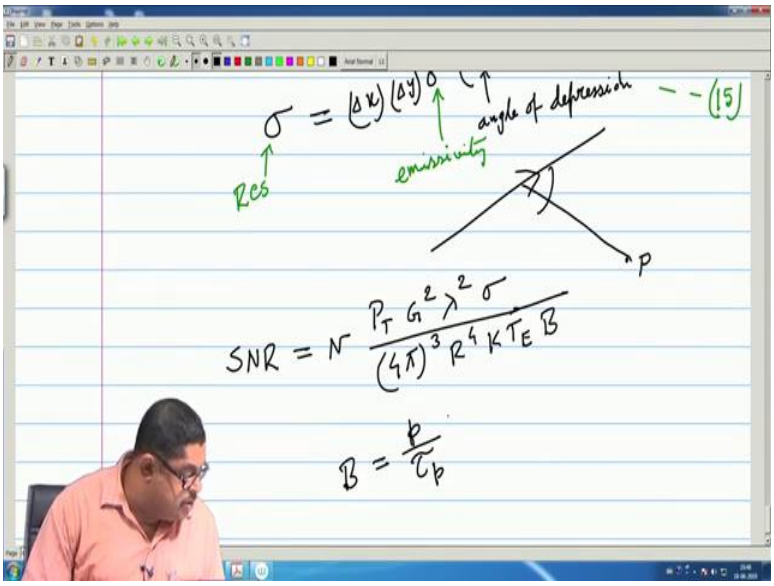

$$\sigma = (\Delta x)(\Delta y)\sigma^{0} (\quad --(15)$$

$$SNR = N \frac{P_T G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 K T_E B}$$

$$B = \frac{p}{\tau_p}$$

So, let me call this is an important relation. So, this is let me call equation 15. And we have already found earlier in the basic classes the SNR expression for radar. So, if we have N number of pulses integrated the SNR improves by N. So, that is why I am writing like this, P T G square lambda square sigma by 4 pi whole cube R to the power 4 K T E B.

$$SNR = N \frac{P_T G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 K T_E B}.$$

Here all terms have their usual significant, T is the effective temperature of the receiver, B is the receiver bandwidth.

And here for the band width again B we can relate to the pulse width and we have in the resolution classes seen that it is not exactly B is equal to 1 by pulse width.

$$B = \frac{p}{\tau_p}$$

It will be some factor so that I can write….. So, let us say some P is a constant it may be two-third it may be something so, T P is the pulse width.

Now, what is the value of this N that for SAR we can find out because, SAR is moving the radar is moving with the constant velocity and it has a PRF so, immediately we can find what is the value of N. So, N will be the PRF is f R and time L by v is the time. So, in observation time t how much if the f R is the PRF. So, f R into t will be the number of pulses.

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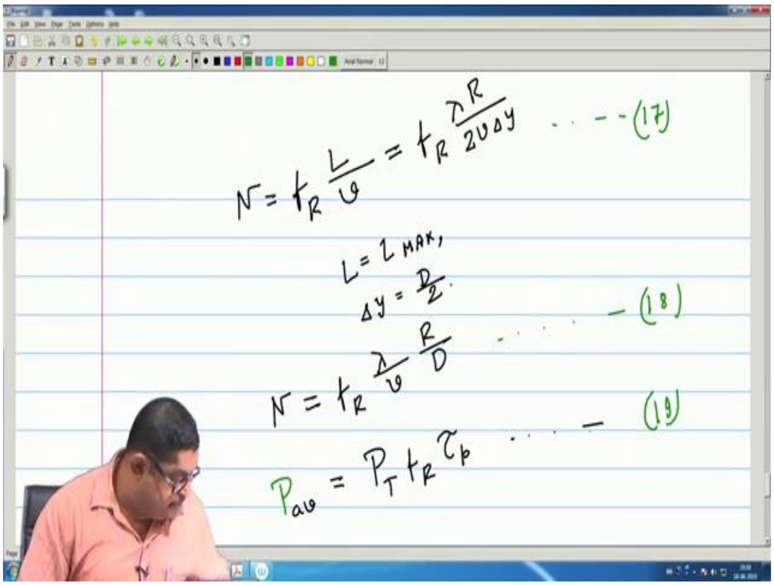

$$N = f_R \frac{L}{v} = f_R \frac{\lambda R}{2 v \Delta y} \quad \ldots -(17)$$

$$L = L_{MAX},$$

$$\Delta y = \frac{D}{2}.$$

$$N = f_R \frac{\lambda}{v} \frac{R}{D} \quad \ldots - (18)$$

$$P_{av} = P_T f_R \tau_p \quad \ldots - (19)$$

So, this we can put f R and L again is L max. So, that value we can put lambda R by 2 v delta y actually by this we have brought this delta y the cross range resolution.

$$N = f_R \frac{L}{v} = f_R \frac{\lambda R}{2 v \Delta y}$$

So, using equation 11, we have said this L is equal to L is L max and delta y will be again D by 2.

$$L = L_{MAX},$$
$$\Delta y = \frac{D}{2}.$$

So, doing all these you will get that N will be f R lambda by v R by D.

$$N = f_R \frac{\lambda}{v} \frac{R}{D}$$

So, I think these will give a number 17, these will give a number 18 and also the oh, sorry, time average power of the transmitter that we have already found out P T into f R into tau p.

$$P_{av} = P_T f_R \tau_p$$

So, these will call 19.

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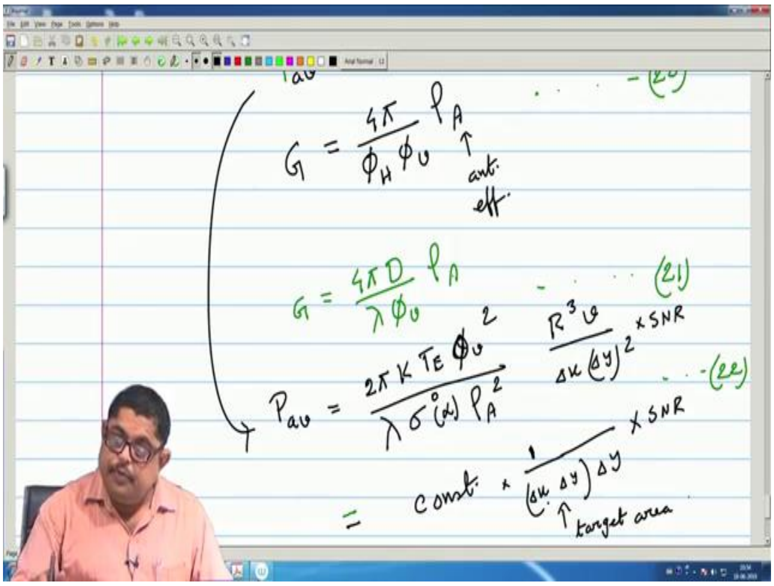

$$P_{av} \quad \cdots - (20)$$

$$G = \frac{4\pi}{\phi_H \phi_v} \underset{\substack{\uparrow \\ \text{ant.} \\ \text{eff.}}}{\rho_A}$$

$$G = \frac{4\pi D}{\lambda \phi_v} \rho_A \quad \cdots (21)$$

$$P_{av} = \frac{2\pi K T_E \phi_v^2}{\lambda \sigma^{\circ}(\alpha) \rho_A^2} \frac{R^3 v}{\Delta x (\Delta y)^2} \times SNR \quad \cdots (22)$$

$$= \text{const.} \times \frac{1}{\underset{\substack{\uparrow \\ \text{target area}}}{(\Delta x \, \Delta y)} \Delta y} \times SNR$$

And also for an antenna, we can say that for a generally a horn type of antenna etcetera aperture antenna we can say 4 pi by phi H phi v in radians into the aperture efficiency, antenna efficiency, this is antenna efficiency. These are the bandwidth in the horizontal and e plane this is coming directly from antenna knowledge.

So, putting this value of phi H, we can write G will become 4 pi by D by lambda phi v rho A that we will make it equation 21.

$$G = \frac{4\pi}{\phi_H \phi_v} \underset{\substack{\uparrow \\ \text{ant.} \\ \text{eff}}}{\rho_A}$$

$$G = \frac{4\pi D}{\lambda \phi_v} \rho_A$$

And so putting all these this P average, this can be written as if I put all these things it can be written as 2 pi K T E phi v square by lambda emissivity rho A square in to R cube v by delta x delta y square in to SNR, these we can call equation 22.

$$P_{av} = \frac{2\pi K T_E \phi_v^2}{\lambda \sigma^\circ(\alpha) \rho_A^2} \frac{R^3 v}{\Delta x (\Delta y)^2} \times SNR$$

So, we can say that it is these whole things are constant, you see all these things are constants into R cube is also we can assume because this is the range for a fix range because we have seeing power versus resolution. So, R cube v will also go finally we are left with delta x delta y or I will make them together delta x delta y into delta y into SNR.

$$P_{av} = \frac{2\pi K T_E \phi_v^2}{\lambda \sigma^\circ(\alpha) \rho_A^2} \frac{R^3 v}{\Delta x (\Delta y)^2} \times SNR$$

$$= \text{const.} \times \frac{1}{(\Delta x \cdot \Delta y)\Delta y} \times SNR$$

Now, delta x delta y I can call target area. So, this is the target area; so, what we are seeing is the power smaller the target more transmitted power required to obtain necessary SNR. Actually this additional inverse dependence of delta y is coming from SAR cross range processing.

So, if delta y is decreased further more PT is required to keep a fixed SNR or conversely we can say SNR will degrade if PT is kept fixed. So, you see that this is the problem that you cannot go on decreasing delta y further and further because then your PT requirement will be more and all transmitters has a limitation on final how much PT it can give. So, the real antenna aperture cannot be decreased beyond a point found by PT.

So, cross range resolution even after SAR processing is dependent on radar receiver detectable SNR, also it suggests that one cannot go on reducing antenna size to improve cross range resolution, please mind it. So, you will have to now within these as an engineer's choice we will have to make a value that what will be your resolution.

So, do not expect that cross range resolution will be as fine as millimetre etcetera for ground mapping or earth mapping that is also not required. But nowadays it is getting required for earthquake prediction etcetera ISRO and NASA they are having a joint project that they will be they are trying to predict whether any earthquake is possible in any place on earth. So, for that millimeter type of resolution etcetera maybe required and that is SAR system is being developed also. So, for that they are also using some other means, the polarization diversity etcetera interferometry that those concepts they are using.

But it is good that actually what we are saying that it is you cannot go beyond that, but now those limits are getting challenged and people are trying to find out cross range resolution of the order of millimetre also. Hopefully one day someone will teach that that in a SAR processing you are having a with a real antenna also you can have so less power, that small antenna small power, but with processing you will get all the imagery possible, ok.

So, with this I conclude the SAR, one thing is remaining that what is that structure of SAR processor the architecture of a SAR processor that I will do in the next class. So, what we have seen that how from the basic movement of the SAR you can find out the phase history, you can find out cross range resolution expression, then what is the limit on that resolution so that turns out to be half of the real aperture length. Now, the question is can we use small antennas, the question is ultimately you are governed by the power handling capability of the transmitter. So, if you have infinite power, definitely you can go on reducing the antenna, but with a limited power and capable and transmitters you cannot go on reducing D.

So, you fix from an engineer’s perspective in the application at hand you fix a particular antenna diameter and then you diameter or antenna length maximum dimension of the antenna and then your cross range resolution cannot to be better than half of that. So, with that some basic SAR thing we have seen how what is the resolution concept in cross range.

In the next class the SAR processors we will do, there also we will the same that phase history part that at various points; that means, if in the movement of the SAR at various point I transmit a pulse and get the echo I know what is the phase of that in terms of a reference. And then we will see how the SAR processor does that phase compensation and focuses the image to a particular point. That is will be the topic SAR processor in the next class.

Thank you.