Principles a nd Techniques Of Modern Radar System
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Lecture - 49
SAR Processing
(Contd.)

Key Concepts: Mathematical model of the range information in SAR processing, cross range resolution, determination of the phase history of the echo signal

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar Systems. We are discussing SAR system; a basic SAR geometry I have described in the previous class. We have found the range of a moving radar as a function of time and today we will expand that range equation in Taylor series.

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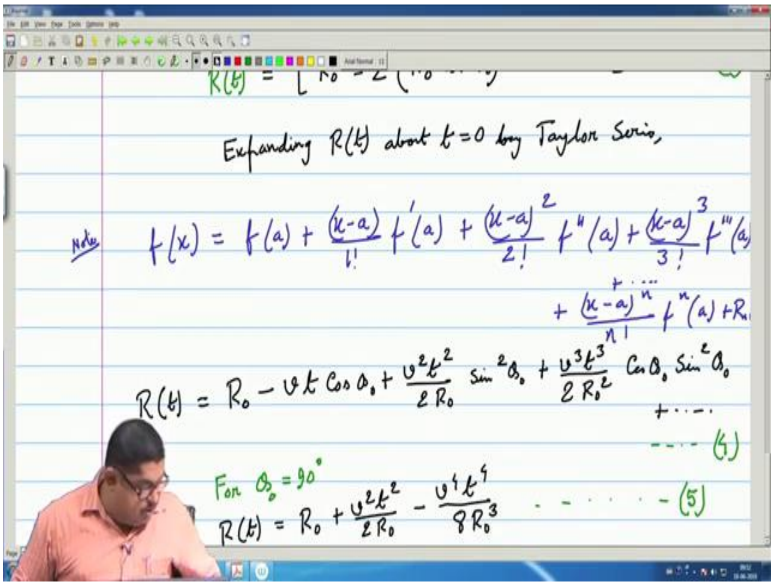

So, what we will do expanding R t about t is equal to 0 by Taylor series expansion. So, generally these we will be able to do. So, I think all of you know. So, this is a note, that what is a Taylor series if I have a function f x and I want to expand it about a point a, then we know this is f a plus x minus a by factorial 1.

The first derivative of x at a point f x plus x minus a whole square by factorial 2, the second derivative then x minus a whole cube by factorial 3 the third derivative etcetera.

And, we know a general term is x minus a whole to the power n factorial the n th derivative and then a remainder ok.

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \cdots + \frac{(x-a)^n}{n!} f^n(a) + R_n$$

So, with this you will be do that one by one I have this expression. So, take some time plus please do it because basically this expansion is a heart of actually Taylor series is very important in engineering science. You will see most of the modern techniques these Taylor series and other various series actually hold that our Newton-Raphson method another method, numerical methods they are based on these Taylor series expansions.

So, do that so, if we do that for our R t I am giving you the thing that retaining please do not approximate from the beginning, go up to third or fourth term, then we can find the real answer. So, R t that becomes the expansion, do it properly all of you will be able to do it, these are basic engineering thing v t cos theta naught plus v square t square by 2 R naught sin square theta naught plus v cube t cube by 2 R 0 square cos theta naught sin square theta naught plus.

$$R(t) = R_0 - vt\cos\theta_0 + \frac{v^2t^2}{2R_0}\sin^2\theta_0 + \frac{v^3t^3}{2R_0^2}\cos\theta_0 \sin^2\theta_0 + \cdots$$

So, this expansion I am calling equation number 4. Now a special case of this is when the squint is 90 degree; that means, the SAR is suppose the radar is moving and the radar antenna is looking 90 degree below; that means, just below to the ground. So, for theta is equal to, 90 theta naught is equal to 90 degree, R t is equal to R naught plus v square t square by 2 R naught minus v 4 t 4 by 8 R naught cube. So, this is also an important formula, so I am naming it 5.

For $\theta_0 = 90^\circ$

$$R(t) = R_0 + \frac{v^2t^2}{2R_0} - \frac{v^4t^4}{8R_0^3}$$

So, you see that for theta 90 degree, but otherwise also that range is varying in a quadratic manner, you see with t square R naught is there plus how much it is increasing as I am progressing in time it is increasing, if I neglect this t 4 term then in a fashion these… quadratic way ok. So, this is an important thing that how range changes, if I have a radar moving uniformly then the range of a point fixed point P that changes quadratically.

Now, if I know these you see immediately I can do one thing that what is the we know that if there is a radar sense a wave plane wave because the ground is at a far distance far field of the antenna. So, it is moving with a plane wave. So, it is going and coming back so, two-way phase shift.

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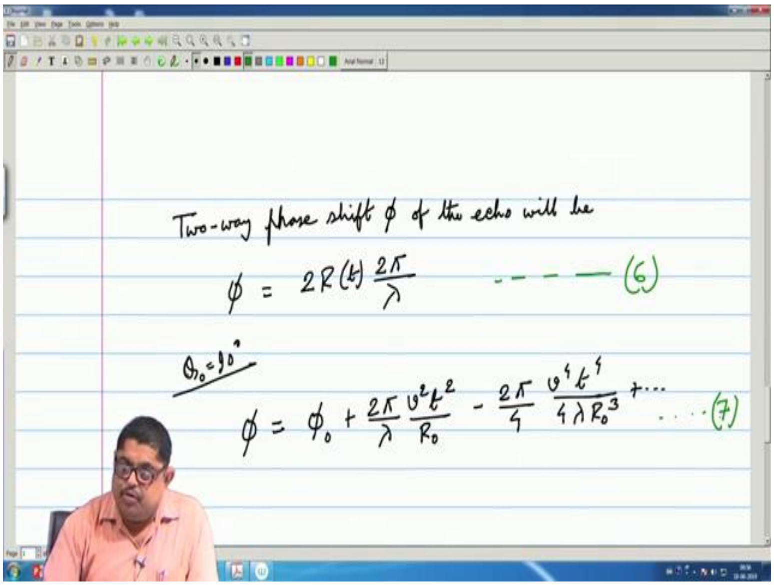

So, I can write two-way phase shift phi of the echo will be, I can easily calculate. So, phi will be this 2 R t, 2 pi by lambda these we know from our plane wave considerations that whatever goes if it moves a distance l then the phase it changes is l into 2 pi by lambda. So, in this case if it goes R t and comes back R t. So, 2 R t the phase will be these.

$$\phi = 2R(t)\frac{2\pi}{\lambda}$$

So, this is again an another important relation I am calling it equation 6. So, here lambda I think all of you know lambda is a carrier signals wavelength. So, now, I can for squint sorry, for theta naught is equal to 90 degree then I can easily write this expression. So, what will be phi? If I put R t expression, I have in equation 5.

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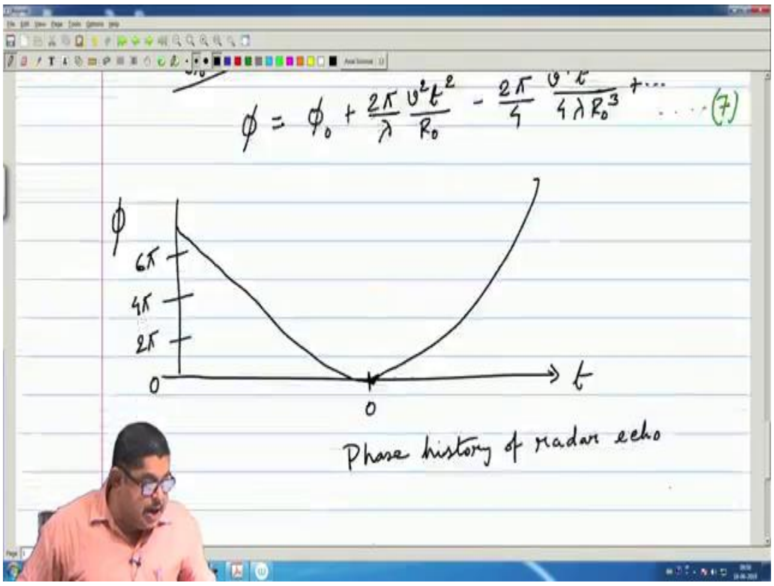

So, if I put it will be immediately phi naught plus 2 pi by lambda v square t square by R naught minus 2 pi by 4 v 4 t 4 by 4 lambda and R naught cube.

$$\phi = \phi_0 + \frac{2\pi}{\lambda}\frac{v^2t^2}{R_0} - \frac{2\pi}{4}\frac{v^4t^4}{4\lambda R_0^3} + \dots$$

So, I have got it that I now know that at various times if what will be the phase shift of the eco signal this is called phase history of the echo. So, I can plot it. So, if I plot it like this, let us say that here I am plotting the phi here I am plotting t. So, let us say t is equal to 0 because I have started from negative time, if you look carefully that 0 is the point when we are at the point A, the midpoint.

So, the shape of the curve will be nothing but a parabola because we have these t square this is well known. So, the shape will be a parabola and this will be phase will be 0, 2 pi, 4 pi, 6 pi etcetera etcetera. So, this is my t is equal to 0, this is called phase history; phase history of radar echo. So, this shows that the range you see if I with respect to these phase history as so the at this point at the point A, the phase is 0, before that it was high as we are approaching the central point the phase is coming down in a parabolic fashion touching 0 then again increasing here.

So, these if I know these so; that means, at which point I have sent the pulses I if I know then I can easily calculate what will be the phase history for that. So, I can as I said in the beginning that I know that at different points I will have different phases. So, with that I can compensate as if I have seen everything together.

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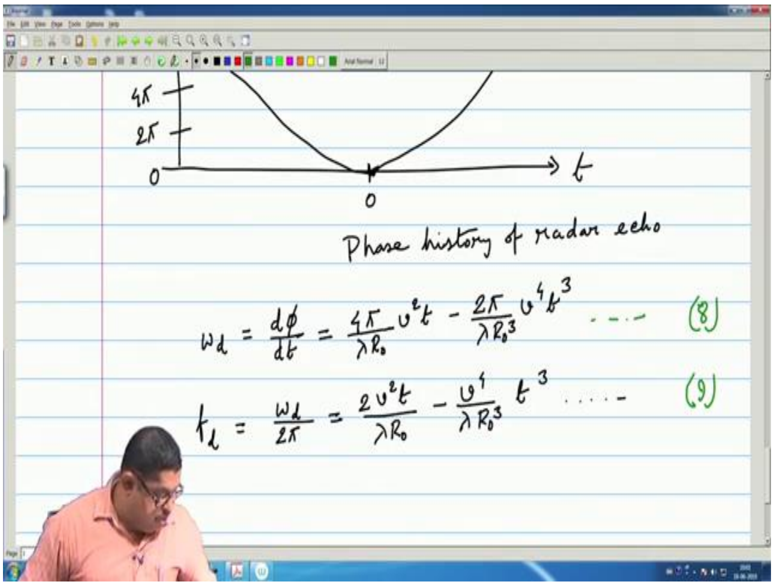

Now, you see I will move further I have these phase history and from there I can find out the Doppler frequency. So, what will be the Doppler frequency for that first let us calculate the omega d Doppler that is nothing, but we know d phi dt I have phi as a function of t I can just differentiate. So, 4 pi by lambda R naught v squared t minus 2 pi by lambda R naught cube v 4 t cube.

$$\omega_d = \frac{d\phi}{dt} = \frac{4\pi}{\lambda R_0} v^2 t - \frac{2\pi}{\lambda R_0^3} v^4 t^3$$

So, this will be my equation 8. And what is Doppler f d? f d is omega d by 2 pi the linear frequency that was angular Doppler. So, this is 2 v square t by lambda R naught minus v fourth by lambda R naught cube t cube. So, this will be equation 9.

$$f_d = \frac{\omega_d}{2\pi} = \frac{2 v^2 t}{\lambda R_0} - \frac{v^4}{\lambda R_0^3} t^3$$

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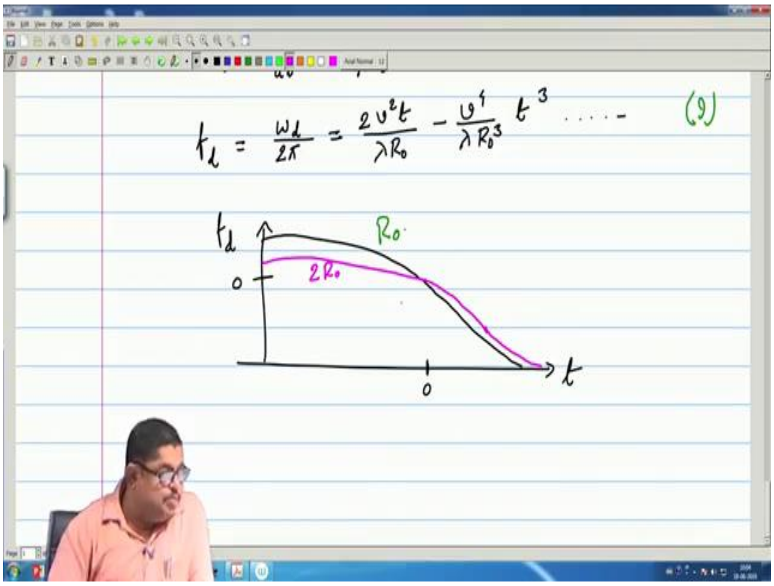

So, I can plot this as well that if I plot f d versus t, let us say that I have a t is equal to 0 here. So, it shows that is the parabolic f d this is again a parabolic thing. So, I can; now here from here you can see that these value when it is touching here these will be also 0 because at t is equal to 0 f d should be 0. Now, if at different ranges suppose R naught is doubled, if R naught is doubled, we will. So, let me suppose this is for use this colour R naught.

Now, if I have double it, R naught double. So, f d will be less, but it will pass through this curve so that means, we can say let us take this colour. So, this is for 2 R naught etcetera. Now, why you see all these f d curves they are passing through f d is equal to 0 at t equal to 0 because look at what is t is equal to 0. In our basic SAR geometry, yes at t is equal to 0 means this point that time this range vector and the velocity vector they are making an angle theta naught.

Now, these curves are all for theta naught is equal to 90 degree. So, they are orthogonal; that means, velocity vector and range vector are orthogonal; we know from our basic Doppler a thing that if the velocity vector and the range vector they are orthogonal then there is no Doppler that is being proved. So, you see that that is why we are getting at t is equal to 0 all the Doppler curves they are passing through 0. Now, let us see what happened to our resolution.

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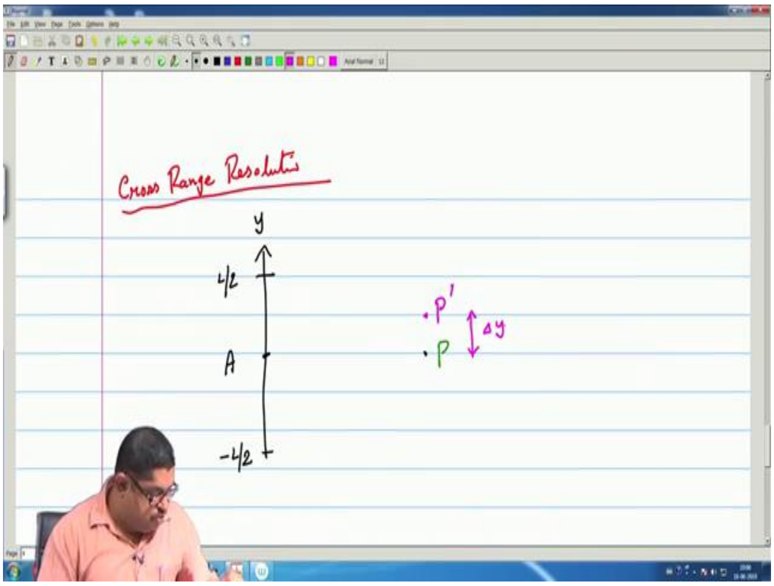

So, that we will see the cross range resolution; cross range resolution. How will you do that? This is our y axis we are observing from minus L by 2 to plus L by 2, this is our centre point A and we are looking at a point P cross range. So, this is radar. So, this is the cross range means along y axis is my cross range another will be along z axis that is let us leave. So, cross range resolution means I will have to have another point which the radar will be able to say that it is a different point; that means, along this y axis I am taking another point P dashed ok, and these separation between them let us call it delta y.

So, if the radar is able to distinguish between P and P dash then I will say that cross range resolution is delta y. My job is to find an expression for delta y in terms of the basic SAR parameters; that means, the observation length L, the observation time t, the velocity v then the range R naught and h naught all these, based on these I will have to tell that what is the value of delta y. So, what we will we will we have already found the f d curves. So, we will utilize that.

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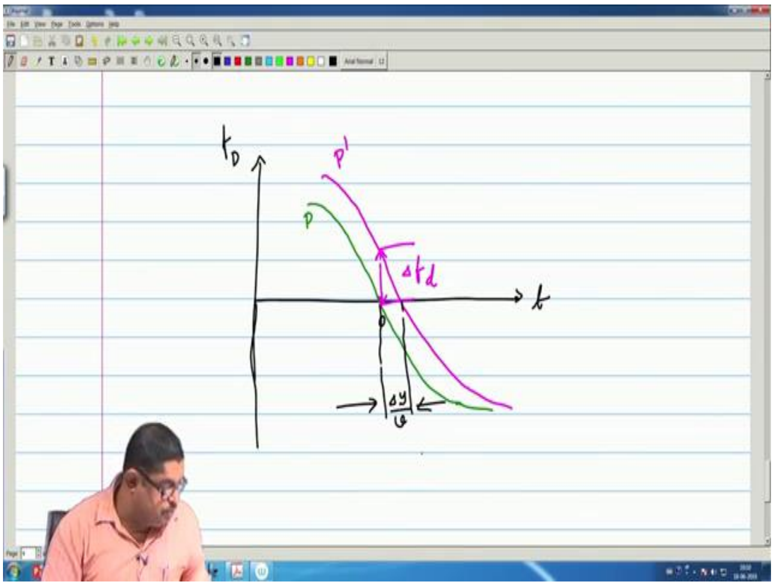

So, we have these curves we have their analytic expressions. So, f D verses t and let us look at 0 point we know at that point it will be at t is equal to 0. So, we are looking at point P and we know what will be the f D curve for that that will be a parabolic nature. So, let me draw that near that it will be something like this the f D expression you can check this is my f D. So, this curve is basically I am drawing here this curve.

So, it will be something like this, now what will happen to the corresponding curve for the point P dashed; obviously, its range is different, but I can say because here you see we have seen that if P and P dashed they are nearby. So, the curves will be just a parallel thing shifted a bit. So, this is for let me write P. So, for P dashed it will be you see P dashed is coming the radar is moving in this direction. So, P dash is coming after P. So, its 0 point will also come a bit later.

So, we know that it will be something like this parallel. So, this is the curve for P dashed ok. And so what is delta y? sorry corresponding to delta y there will be or from this one I can say that if I take this point then these difference is delta f d, because from here, here f d is 0 for P dashed it is not 0. So, I can say that these difference is delta f d and the corresponding thing in here if I take this that along time axis what is the difference that will be. So, this difference I know it will be delta y by v because delta y it will move in how much time delta y by v. So, in time axis there will be these.

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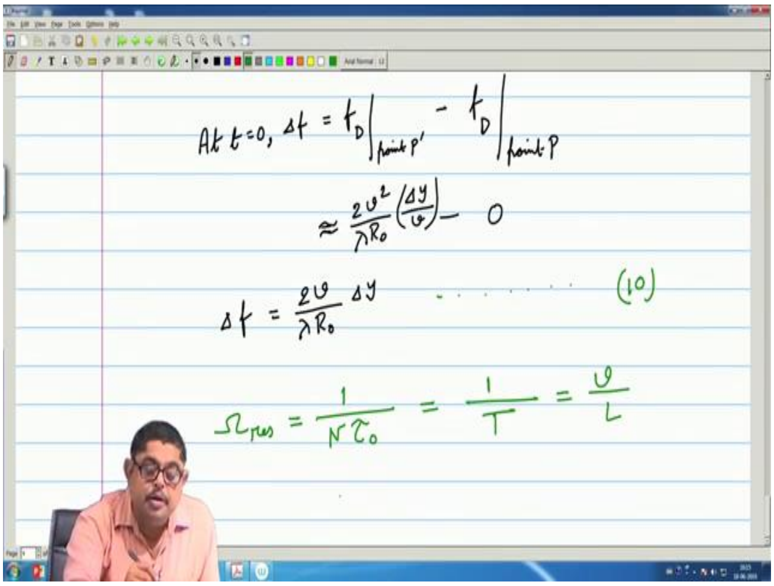

So, now I can easily write that at t is equal to 0, delta f is equal to the f D curve for point P dashed minus f D for point P and now this one I already know that f D at point P that will be 0 because it is crossing 0, but this is non 0 value. So, that value I can find out from this expression f d; f d is 2 v square by lambda R naught t is this thing I can neglect because t q plus t is very small. So, let or if you put also no problem, but we are neglecting. So, taking only the first time I can write it will be so that is why I am writing approximately 2 v square by lambda R naught into the delta t that is delta y by v.

$$\text{At } t=0,\ \Delta f = f_D\Big|_{\text{point } P'} - f_D\Big|_{\text{point } P} \approx \frac{2v^2}{\lambda R_0}\left(\frac{\Delta y}{v}\right) - 0$$

So, delta f is equal to 2 v by lambda R naught delta y this is my equation 10.

$$\Delta f = \frac{2v}{\lambda R_0}\Delta y$$

So, I have written from the my knowledge of delta f and it is something these are all constants because v is the velocity of the radar, lambda is the carriers wavelength in which the pulse is getting transmitted, R naught is the minimum range for point P. So, I have delta y is some distance. Now, I recall that in various previous classes in radar detection and resolution time I have proved that what is the Doppler resolution constant.

I have proved that Doppler resolution constant is for a periodic pulse sending radar, I have shown that if you have n number of pulses integrated then it is 1 by N tau naught. What is capital N that the target is getting illuminated by N number of pulses. So, these we have found. So, this is the we can say the resolution frequency resolution. So, these if I make these equal Doppler frequency resolution. So, if I made this equal then that will be our delta y because actually in frequency Doppler frequency domain radar can distinguish these two, these is the minimum separation of the two Dopplers.

So, if I equate it with these then I will say that this will be that cross-range resolution. So, equating these two [FL]. Now, in our case what is the total signal duration? You see that this N into tau naught is what? That in our case is the total observation time, in our case it is the total time taken to cross this one because by that time P will be illuminated. So, this time is T, capital T. So, in our case this is 1 by T. And what is 1 by T in our case that is v L by, t is equal to L by v our equation 2 I think so, v by L.

$$\Omega_{res} = \frac{1}{N\tau_0} = \frac{1}{T} = \frac{v}{L}$$

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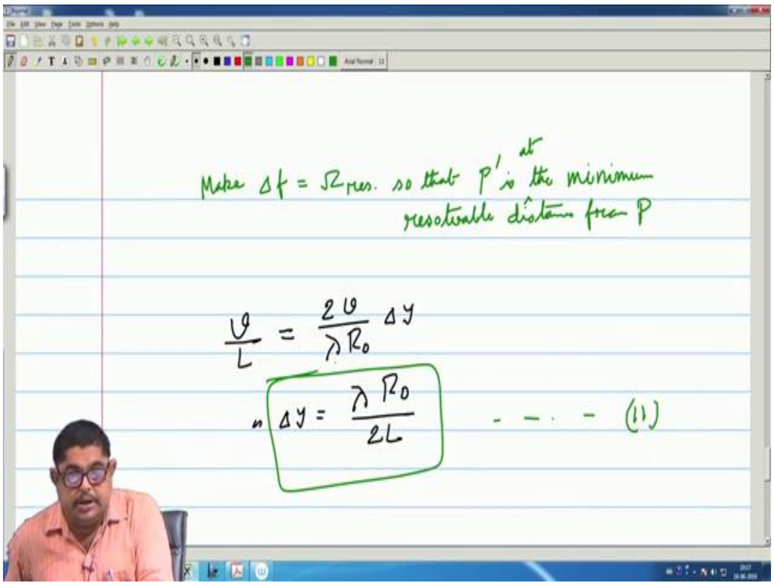

So, now our job is just if I make if I make, make delta f equal to this. So, that P dashed is the; P dashed is at the minimum resolvable distance from P then we can write v by L is equal to 2 v by lambda R naught delta y or delta y is equal to lambda R naught by 2 L.

$$\frac{v}{L} = \frac{2v}{\lambda R_0} \Delta y$$

$$\Delta y = \frac{\lambda R_0}{2L}$$

A very very important thing giving us the cross range resolution that cross range resolution is proportional to lambda, if you move higher and higher in frequency cross range resolution improves. It is proportional to the range minimum range distance.

So, if you look closer because satellites do that satellite has a swath. So, at lower point the resolution is cross-range resolution is better, at longer distance longer R naught the cross range resolution fails. Actually due to these the SAR systems they have various modes bitmap mode, cross SAR mode etcetera. So, just to have this resolution changing with R naught and also it inversely proportional to the total observation length; so, more if I observe then you can improve delta y. But, then you require to have large storage large signal processing for that, but this equation is very important, these are fundamental equation you see from starting from simply writing the range expression from the basic geometry you can derive what is a cross range resolution of SAR.

Now, later we will see the various other things. Now, please remember this expression is the best expression because we have derived it for the theta naught is equal to 90 degree or squint up 90 degree. Now, squint up 90 degrees common for SAR, but if you do not have squint 90 degree you can always derive it because in we have written the expressions. So, just do not make the theta naught is equal to 90 degree whatever value you put from there you can derive this.

So, it will be the similar resolution expression at 90 degrees squint, R naught is minimum. So, resolution is best if you use theta naught generally SARs operate with this, but there are other radars particularly GPR etcetera sometimes they cannot have squint 90 degree. When the GPR also it has a squint of 90 degree because it looks down, but some radars may not have that that time you use that expression.

Now, we will discuss more on these cross range resolution, because this we have found an expression we have not yet found what is the best you can get from a given SAR thing that will see next and there we will have a surprise ready there because something will come that, actually you cannot have go on eating your cake. So, we will see that resolution. So, if I go on increasing L can I really get any delta y any small amount of delta y? The answer will be obviously, no; because physics of the whole thing the various limitations of the practical things they will matter, that we will see in the next class.

Thank you.