Principles And Techniques Of Modern Radar Systems
Prof. Amitabha Bhattacharya
Department of E & ECE
Indian Institute of Technology, Kharagpur

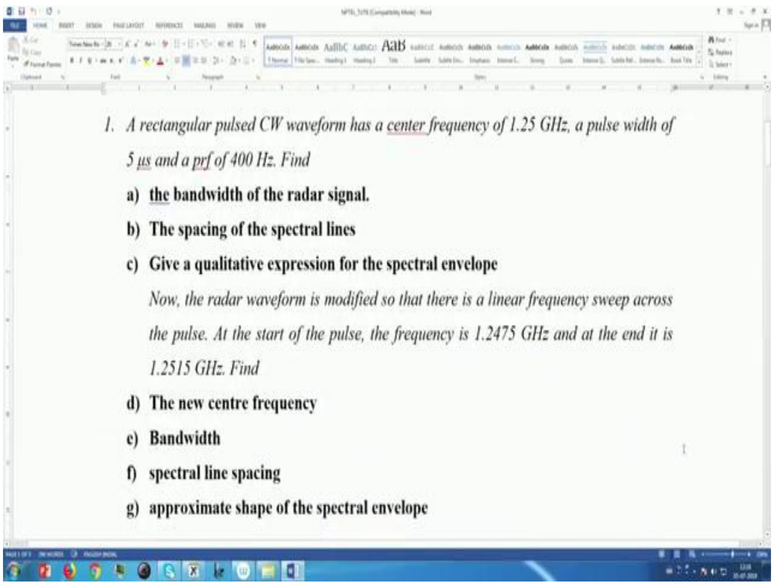
Lecture - 47
Tutorial Problems on Detection in Radar Receiver

Key Concepts: Tutorial 6

Welcome to the NPTEL lectures on Techniques and Principles of Modern Radar Systems. So, we have in previous several lectures, we have seen match filter then the various correlation properties etcetera. And, we have discussed about resolutions then finally, we have seen how to get improve the range resolution pulse compression etcetera. So, all those are detection in radar receiver.

So, today we will see some tutorials on that.

(Refer Slide Time: 01:01)

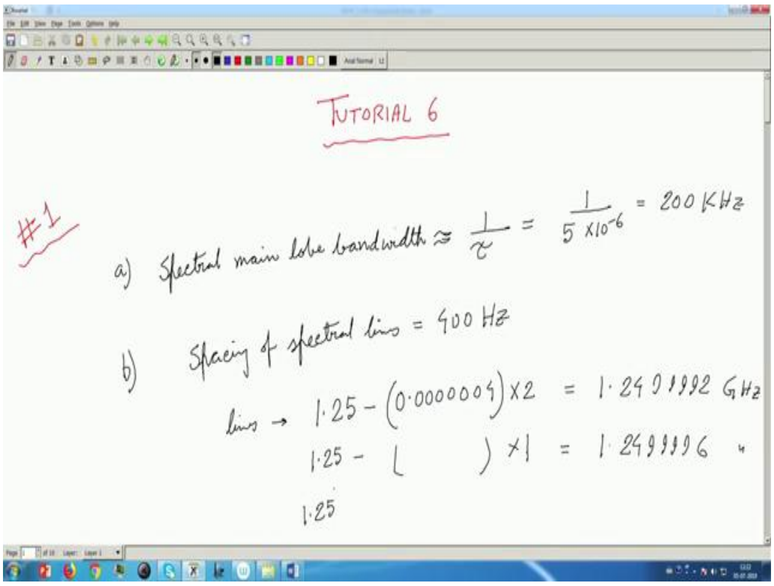


First problem is this detect. A rectangular pulsed CW waveform has a center frequency of 1.25 gigahertz a pulse width of 5 micro second and the prf of 400 hertz. Find the bandwidth of the radar signal, the spacing of the spectral lines, give a qualitative expression for the spectral envelop. Now, the radar waveform is modified.

So, that there is a linear frequency sweep across the pulse. At the start of the pulse frequency is given and at the end it is so and so find the new centre frequency, bandwidth, spectral line spacing, and approximate shape of the spectral envelope.

So, simple problem just you will have to know the basic definitions and that will be seen.

(Refer Slide Time: 01:59)



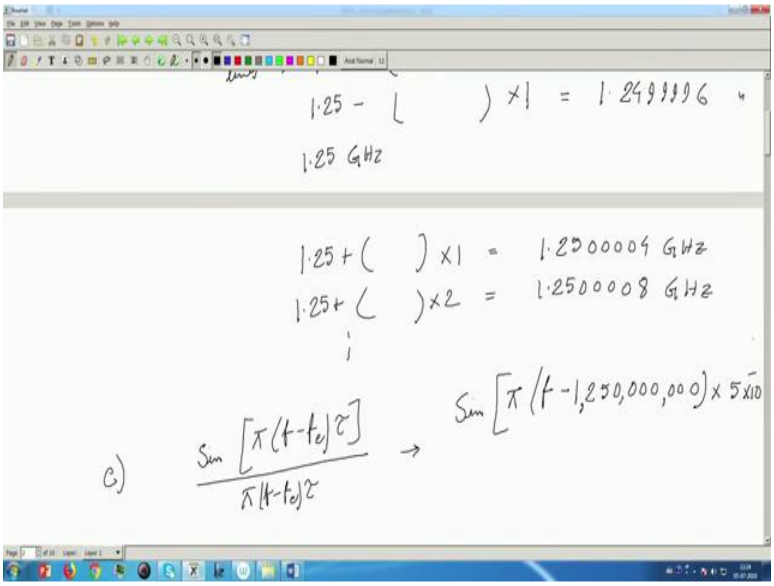
So, this is the first problem. So, what is the spectral main lobe bandwidth, spectral main lobe bandwidth, that we know roughly this is not the resolution type of a thing. So, roughly I can say it is 1 by tau it is a constant frequency pulse so, 1 by so, tau value is given 5 microsecond o, it is 200 kilo Hertz.

Now, then the question is what is the, what is the spacing of the spectral lines?. So, spacing of the spectral lines will be; obviously, in a pulse radar if you have pulse repetition. So, at every PRF things you will have a spectral line. So, we can say that spacing of spectral lines, they will be 400 Hertz.

So, every 400 Hertz you will have a spectral line. So, we have lines at you can say first is 1.25 center frequencies of 1.25 minus that 400 Hertz.

So, 0.0000004 into 2 so, that will be 1.24 99992 Gigahertz, then there will be 1.25 minus these thing into 3 or you can say that this is 2 lower side if we see it will be, you can say into 1. So, that will be something 1.2499996 gigahertz.

(Refer Slide Time: 04:57)

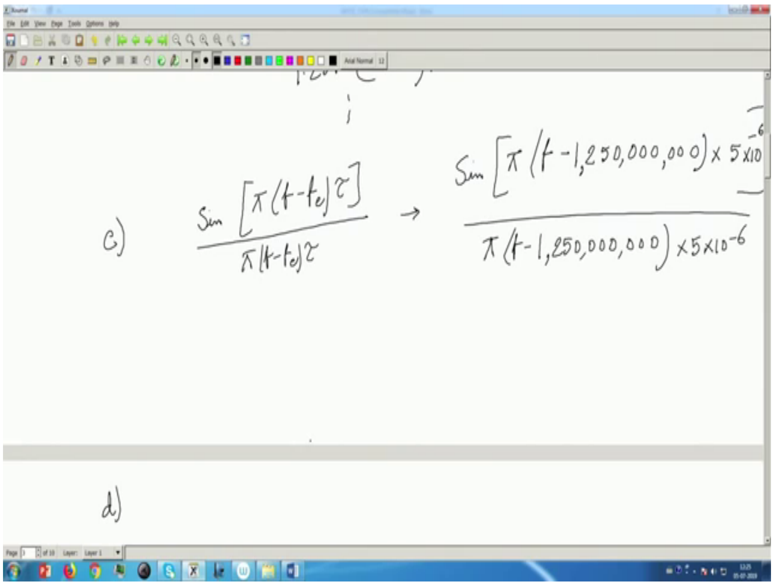


Obviously, 1 and 2 1.25 Gigahertz, then there will be plus 1, so, 1.25 plus that into 1.

So, that will be 1.2500004 Gigahertz then these into 2. So, 1.2500008 Gigahertz like that this side, this side also. So, there will be spectral lines like this. Then come to 3, what is the, what is the question here? That the give a qualitative expression for the spectral envelop we know, it will be a sinc function.

So, spectral envelope will be something like sin pi f minus fc tau by pi f minus fc into tau. So; that means, in our case this will be sin pi f minus 1,250,000,000 Gigahertz into 5 into 10 to the power minus 10 to the power minus 6.

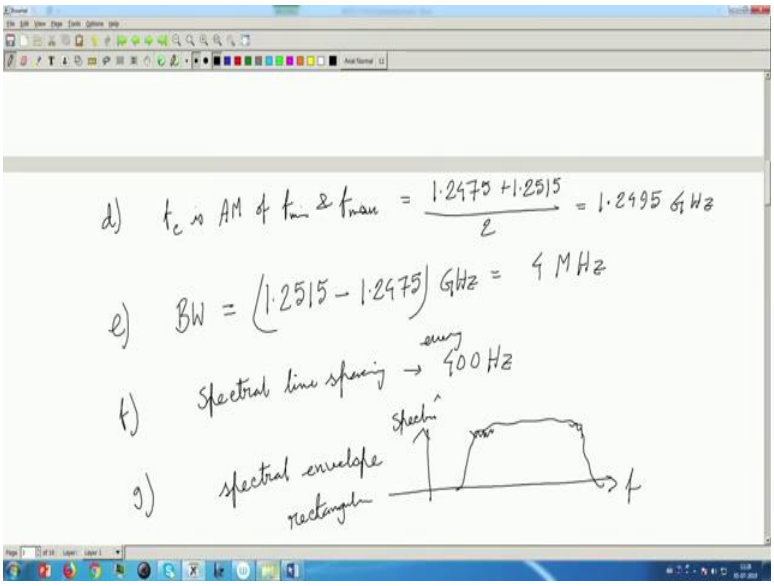
(Refer Slide Time: 06:51)



Divided by pi into f minus 1,250,000,000 into 5 into 10 to the so, this will be the shape then let us come to part d that what was the problem.

Now, the linear modulation is given. So, what is the new center frequency, what is new center frequency?

(Refer Slide Time: 07:41)



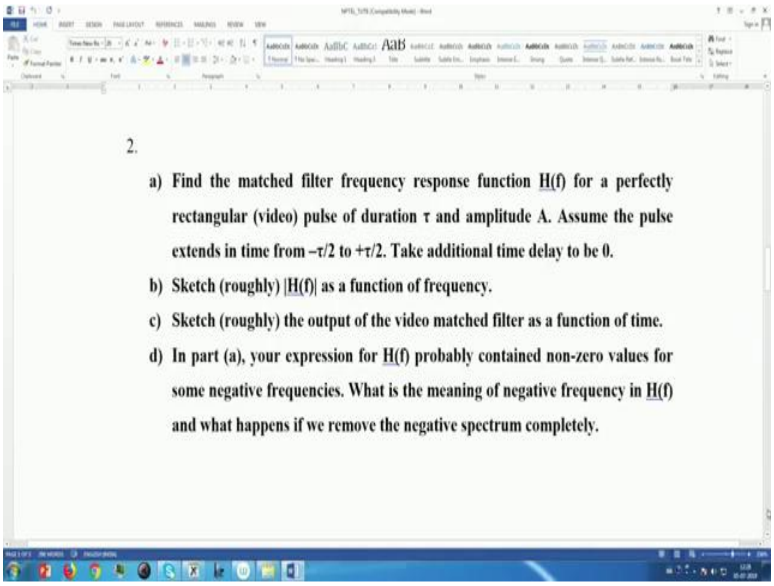
So, we can say f c is A M of f mean and f max the new center frequency is something like this. So, it is 1.2475 plus 1.2515 by 2. So, that is 1.2495 Gigahertz that is all. So, part d is over. Then what we have to do part e.

What is part e the bandwidth ok? Bandwidth is now independent of pulse width that is the new thing that LFM that makes it is independent of pulse width.

So, what will be the bandwidth? You see previously it was dependent on the pulse width, but now it is independent of pulse width. So, it is 1.2515 minus 1.2475 Gigahertz. So, that comes to 4 Megahertz. So, previously it was 200 kilo Hertz, now it has increased, but it is not dependent on the pulse width.

And f spectral line spacing so spectral line spacing in the new case. So, that will be same as PRF, because PRF will make that every 400 Hertz, it will have its signature. And, spectral envelop we have seen that LFM makes roughly this rectangular. So, it will have something like this. So, you can say spectral envelop is roughly a rectangular there will be some ringings that we have seen that this is spectrum ok. So, this problem was easy.

(Refer Slide Time: 10:37)

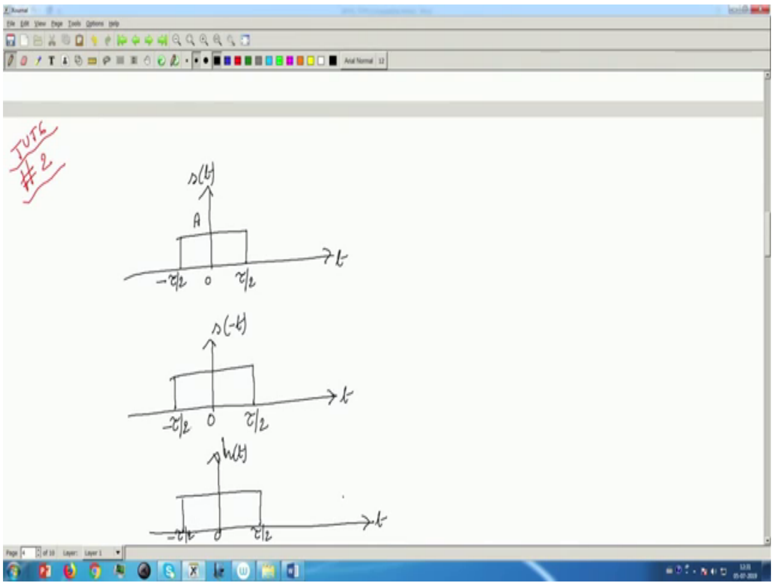


Now, let us go to the second problem, this is a classical problem. Find the matched filter frequency response function, for a perfect rectangular video pulse of duration tau and amplitude A. A ssume the pulse exchanging time from minus tau by t 2 plus tau by 2. Take additional time delay a matched filter requires a time delay, but assume that is to be 0.

Then sketch roughly the magnitude portion magnitude frequency response, sketch and magnitude this transfer function of the match filter. In part c sketch roughly the output of the video match filter as a function of time; that means, the match filter output.

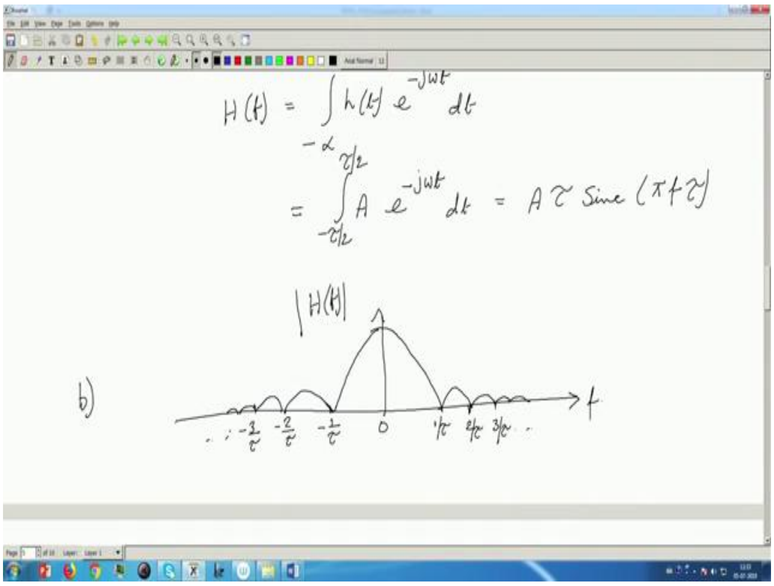
Now, you in d there is some your expression for H f probably contained nonzero values for some negative frequencies, what is the meaning of negative frequency in H f. And, what happens if we remove the negative spectrum completely so, good conceptual question. So, we will see that I think this is now problem 2 tutorial 6.

(Refer Slide Time: 11:53)



So, we know let us say the signal s t is this is 0, this is A, this is minus tau by 2, this is plus tau by 2. So, I can easily draw what will be s of minus t 0. So, for a rectangular function it is good it is not changing. So, I can say that my ht the impulse response of the match filter, that will be also like this that 0 tau by 2 minus tau by 2.

(Refer Slide Time: 13:25)



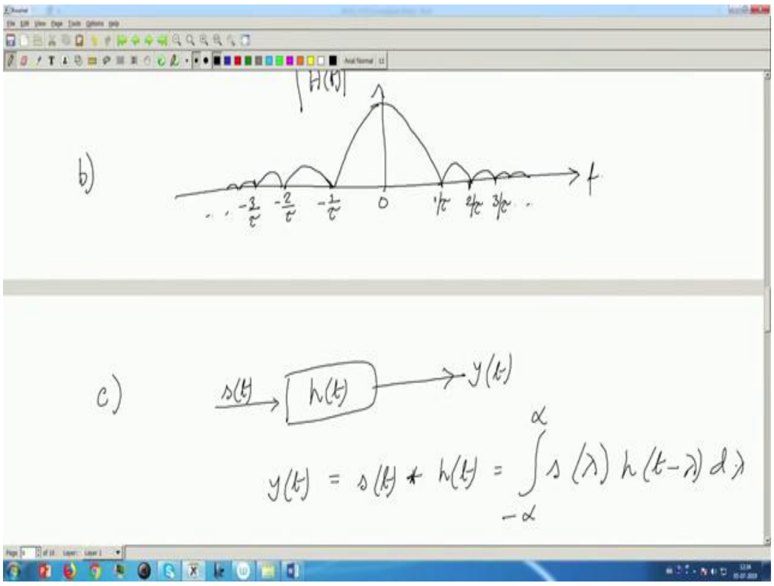
So, I will have to now find the H f. So, H f is minus Fourier transform h t e to the power minus j omega t dt. So, this is it exists from minus tau by 2 to plus tau by 2 ht's value is A. So, it is A e to the power minus j omega t dt. So, this we know simple to integrate A tau sinc pi f tau so, as easy as these.

So, in part b we are asked to find the magnitude part; obviously, this is the thing. So, magnitude will be I can write H f magnitude in general this is a complex quantity, but in this case it is not a complex quantity. So, magnitude will be sinc function since we are putting the magnitude.

So, there won't be negative signs negative portions like that your. So, this is H f this is f this is 0 and we know these points are 1 by tau this point is 2 by tau this is 3 by tau etcetera. Similarly, minus 1 by tau minus 2 by tau minus 3 by tau etcetera.

So, this is a sketching of the magnitude portion.

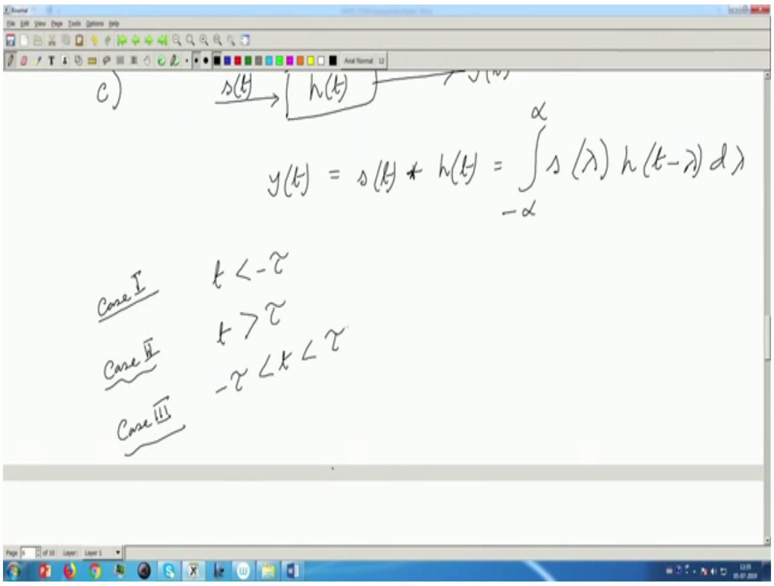
(Refer Slide Time: 15:39)



Then, then let me again check what was the so, sketch finish. So, sketch roughly the output of the video matched filter as a function of time that is a next part. So, this will require this is also you know simple, but you will have to meticulously do it. So, s t is coming h t now we know we will have to find y t.

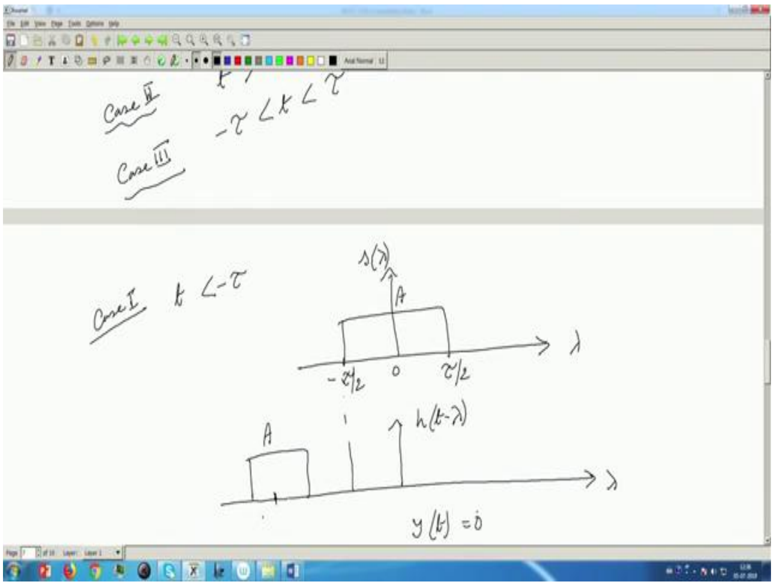
So, what is y t, y t is s t convolution h t we know what is convolution minus infinity to infinity s make a new running variable, s lambda h t minus lambda d lambda.

(Refer Slide Time: 16:45)



So, now there will be several cases first case I is t is less than minus tau, we will take case II. As, t is greater than plus tau and case 3 is minus tau less than t less than tau ok.

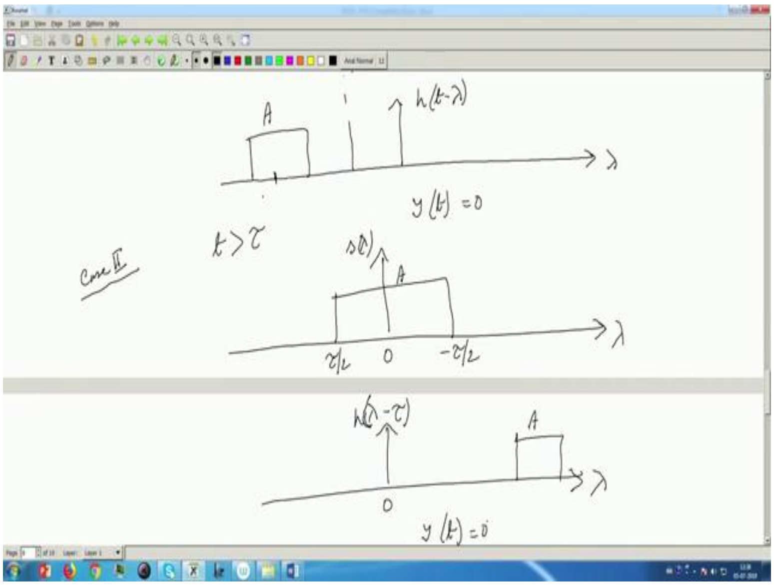
(Refer Slide Time: 17:19)



So, the total integration will be so, first see let us see what is case I, t is less than minus tau. So; that means, I have s lambda this is my lambda minus tau by 2 tau by 2 0 A. And, what is my h t minus t minus lambda, this is lambda.

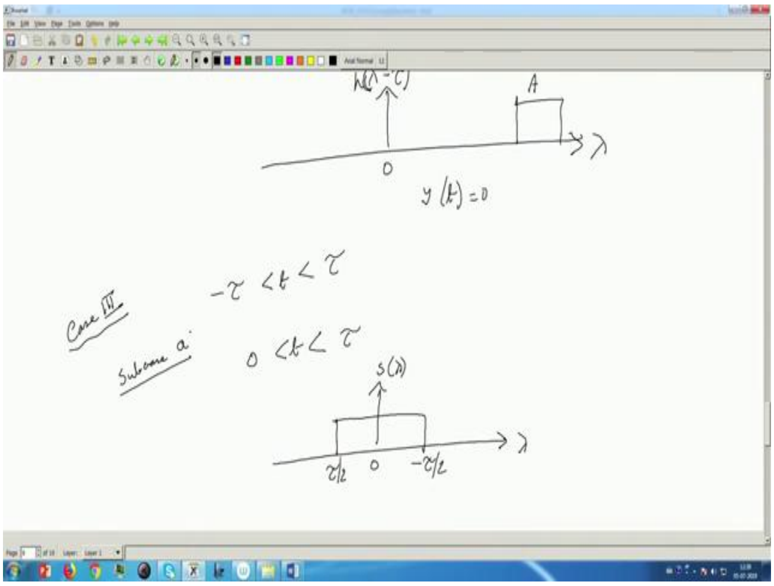
So, t is less than these; that means I have something here. So, this is my here. So, you see that roughly this is minus tau by 2. So, it is here. So, we can say that if I multiply y t will be 0 here. So, case I does not g ive me anything.

(Refer Slide Time: 18:43)



Similarly, case II I have t is greater than tau. So, again I can s t is as before not s t, s lambda, it is running variable has changed. So, this is also lambda 0 minus tau by 2 to tau by 2 A. And, what will be h t minus tau, this is h lambda that lambda minus tau this is lambda. So, it will be somewhere here this is A. So, again I can say y t is equal to 0.

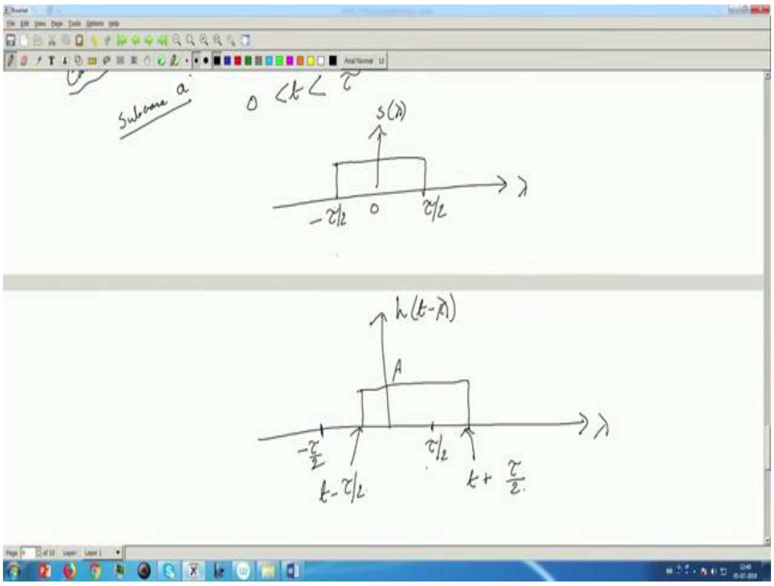
(Refer Slide Time: 20:19)



So, case I and case II are out of my… Case III. This will give me, something what is case III minus tau less than t then plus tau. So, there are 2 sub cases, sub I will have to break it into sub case a 1 will confuse. So, sub sorry sub case a, that 0 is less than t, less than tau.

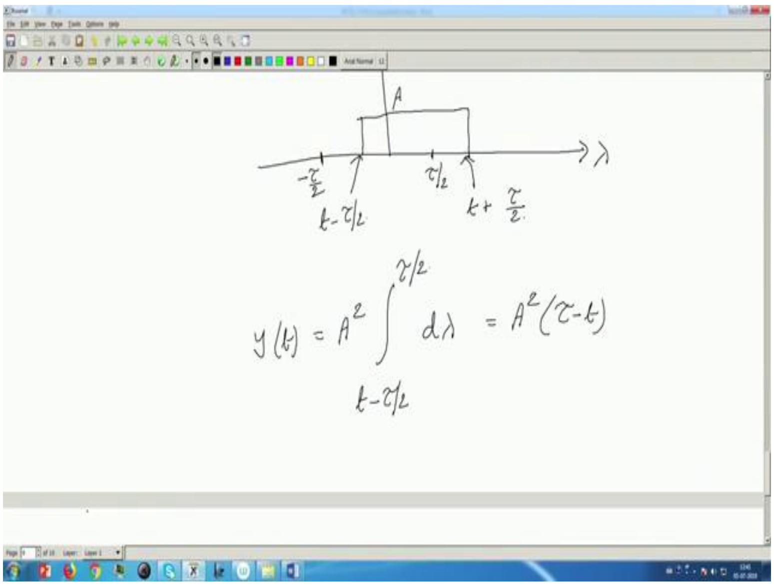
So, put what is s lambda h t minus lambda, this is lambda. So, this thing is A and somewhere here is tau.

(Refer Slide Time: 21:23)



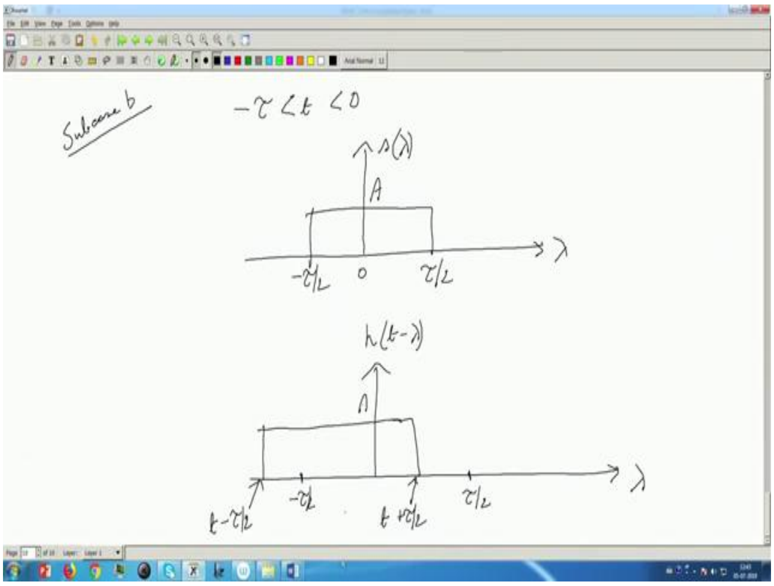
A, so, what is this point is really we can see t plus tau by 2 and what is this point? This point is t minus tau by 2, somewhere here will be minus tau by 2.

(Refer Slide Time: 22:45)



So, we can see that from here 2 tau by 2 there will be multiplication. So, we can say y t in this case will be; obviously, 2 s so, A square, but then this lower point t minus tau by 2 to plus tau by 2 d lambda. So, that will give us A square t minus like sorry A square tau minus t do it, then let us come to case b, sub case b.

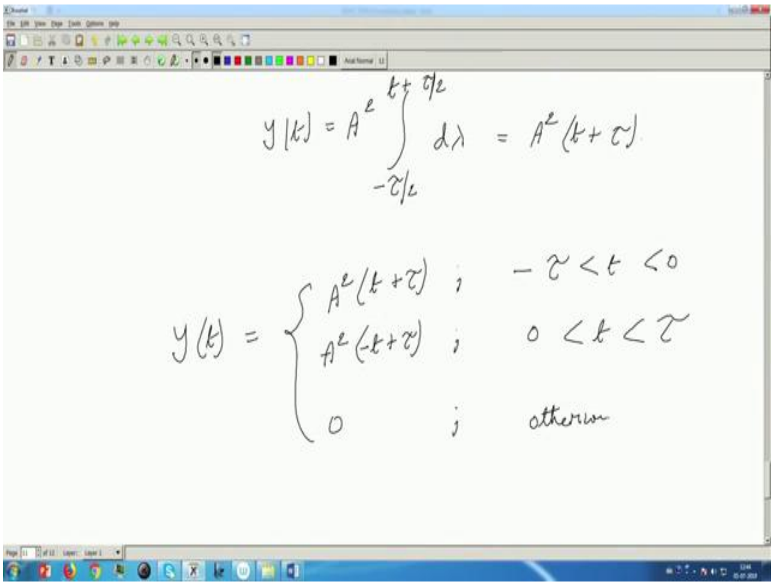
(Refer Slide Time: 23:25)



Sub case b that is minus tau. So, again as before s lambda, this is lambda, A minus tau by 2 plus 2 by 2 0. So, here it will be something like this, let us see this here will be tau by 2 which over this is h t minus lambda A, then this point; obviously, will be t minus tau by 2 somewhere in middle there will be minus tau by 2 and this point is t plus tau by 2.

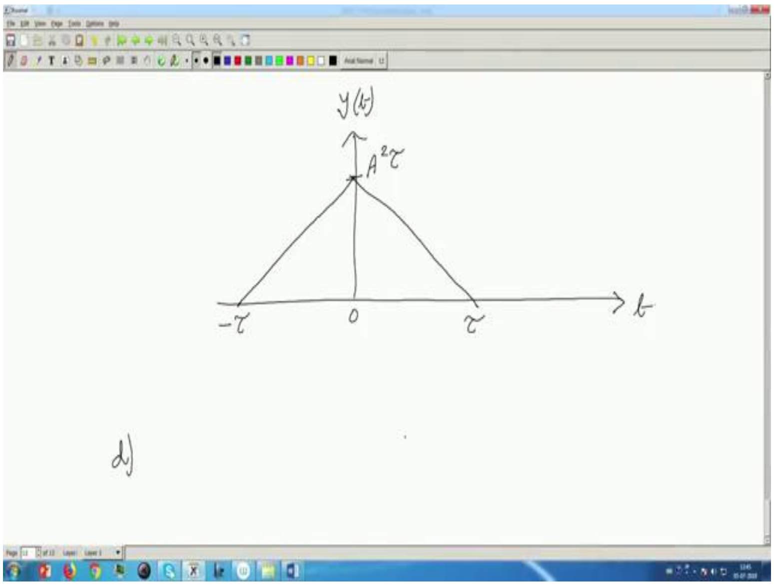
So; obviously, from minus tau by 2 to t plus tau by 2 we will have the multiplication.

(Refer Slide Time: 25:11)



So, y t will be A square minus tau by 2 to t plus tau by 2 d lambda that will be A square t plus tau that is all. So, now, I can write putting all those together y t is A square t plus tau minus tau less than t less than 0, A square minus t plus tau is 0 less than t less than tau and 0 otherwise or elsewhere. So, I can now sketch this y t versus t.

(Refer Slide Time: 26:19)



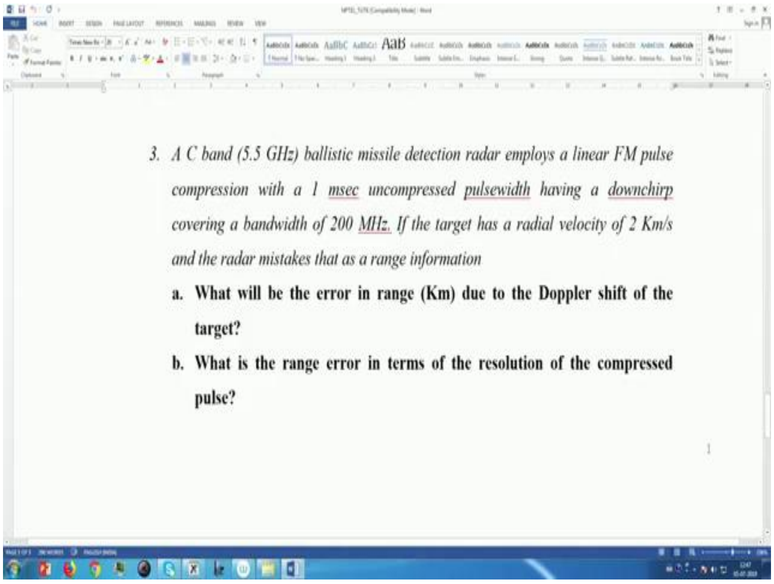
So, we can see t plus tau A square is a constant that is nothing t plus tau; that means, it is a linear function. So, A square tau here it is 0. So, this will come down this will be linear, this is tau and this is minus tau this is the shape. Now, next part is that negative frequency actually you know that negative frequency concept started from Fourier, because we are taking transfer function etcetera those are Fourier transform.

So, for mathematical soundness because Fourier tried to break a real signal into it is Eigen components. And, that is why the complex things came and that also introduced this negative frequency concept. So, this negative frequency concept does not have the physical meaning, but for mathematical reality it is there.

Now, it is well known that if we chop up the negative spectrum then will the signal will no more be real, because half of the power is in the positive spectrum, half of the power is in the negative spectrum, that makes the whole signal thing real. If, we cut off the negative thing that we cannot do, but you know that we can do that complex envelop concept is there, which actually shifts the whole spectrum to the positive side.

So, if you are uncomfortable with the negative side, you can work with the complex envelop which you do for bandpass signals, but otherwise the physical meaning of negative frequency is not there spectrum is a mapping. So, in that map mathematics dictates that it does not have any physical meaning ok. I think this problem we have seen.

(Refer Slide Time: 28:51)

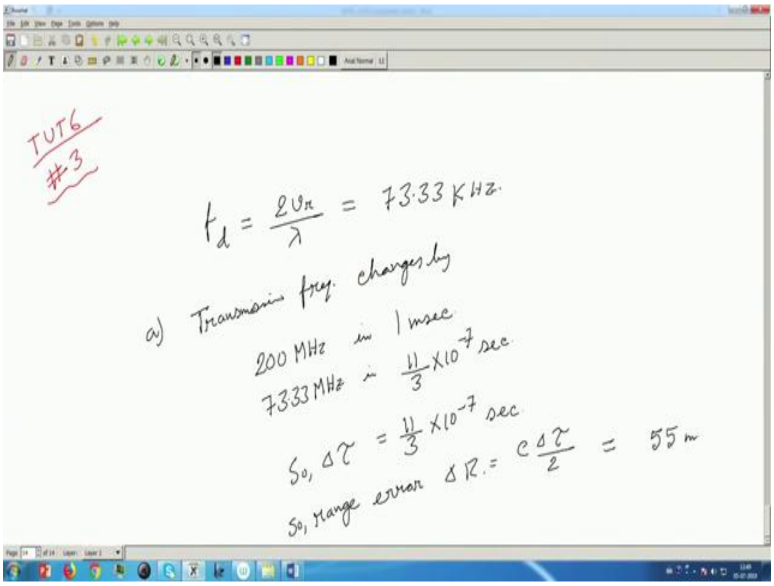


Let us see the last problem. This is a problem A C band ballistic missile detection radar employs a linear FM pulse compression with a 1 millisecond uncompressed pulse with having a down chirp covering a bandwidth of 200 Megahertz. If the target has a radial velocity of 2 Kilometre per second and the radar mistakes that as a range information.

Because, target has a velocity, but radar is mistaking that as a range information what will be the error in range due to the Doppler shift of the target? What is the range error in terms of the resolution of the compressed pulse? Good problem.

So, first actual let us see the solution. So, this is Tutorial 6 which you have problem 3.

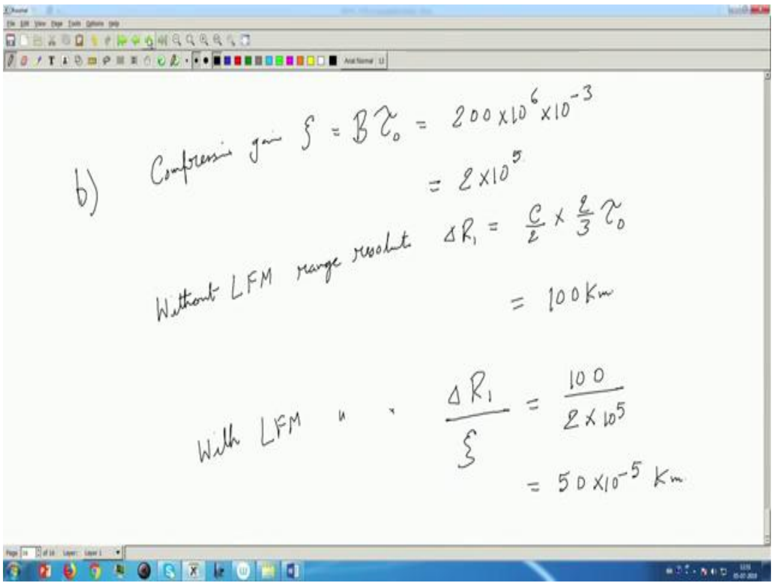
(Refer Slide Time: 30:01)



So, first find out the Doppler. So, radial velocity said the frequency said. So, you can calculate this will be 73.33 Kilohertz. Now, first is you see the due to LFM the transmission frequency changes by 200 Megahertz in 1 millisecond.

So, how much time it will take to change it to 73 in 11 by 3 into 10 to the power 7 second. So, we can say that delta tau that is 11 by 3 into 10 to the power minus 7 second. So, we can say that this will be given the range error correspondingly. So, range error delta R will be c delta tau by 2. So, that will be 55 Meter. Now range error is so, actually radar is mistakenly thinking that as the though the frequency is changing that radar is thinking this due to Doppler it is having so, that is why the range error is erroneous.

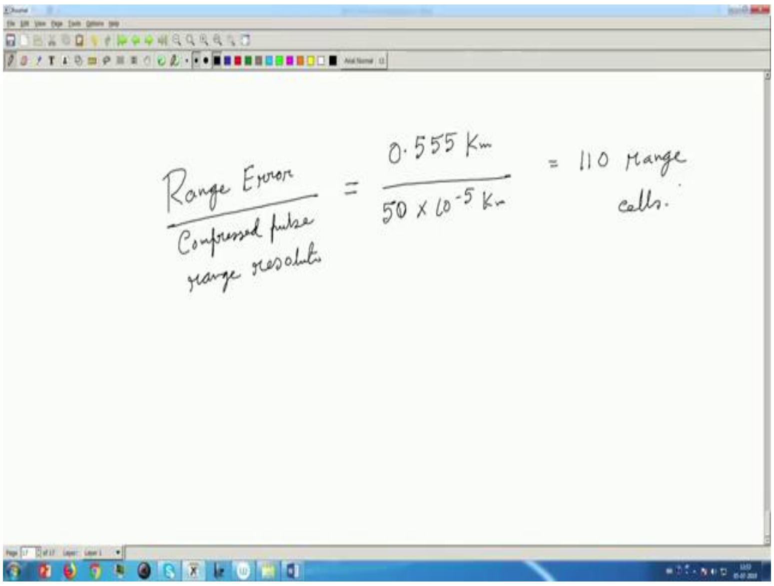
(Refer Slide Time: 32:07)



Now, that is I think I will have to increase the page and, part b the compression gain of the LFM compression gain I that we know is given by B tau naught and here B is 200 in Megahertz and tau naught is 1 millisecond. So, it is 2 into 10 to the power 5. So, now, we can say without LFM range resolution was how much. Let us call it delta R 1 that was c by 2 into two-third here we are taking the exact value of resolution.

So, if you put these it will come as 100 kilometer and with LFM range resolution will be let us call it delta R 1 by the compression gain. So, that will be 100 by 2 into 10 to the power 5. So, it is 50 in to 10 to the power minus 5 kilometer the drastic improvement.

(Refer Slide Time: 33:43)



So, now we can calculate, what is the range error divided by compressed pulse range resolution that will be range error was 55 meter. So, that is 0.055 kilo meter this also is 50 into 10 to the power minus 5 kilo meter. So, that will be 110 range cells. So, it will this error will lead to 110 range cells the range will change. So, this is something that the if you use LFM it should be aware that you are having a frequency change, otherwise it thinks that that is due to the Doppler. And, that makes the range calculation difficult, this is the concept of range migration type of thing ok.

So, good problems so, with that the detection in radar receiver we have seen. In finally, when we will see the statistical theory, because there are the environment there are statistical fluctuations that time again we will revisit this radar detector that will be in future tutorial there we will again come back here in the detection theory.

Thank you.