Principles And Techniques Of Modern Radar Systems
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Lecture - 46
Detection in Radar Receiver (Contd.)

Key Concepts: Improvement of the range resolution by pulse compression, Doppler resolution improvement factor for coherent pulses, properties of the ambiguity function

Welcome to this NPTEL lecture on Techniques and Principles of Modern Radar Systems.

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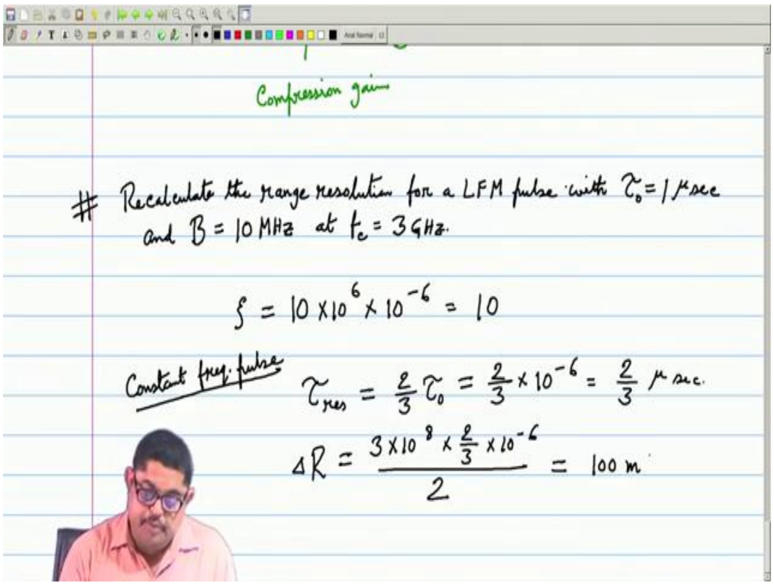

We were seeing the range ambiguity function of linear frequency modulated pulse. So, we have found out the expressions. Now, already for a single frequency pulse , we have earlier calculated the range resolution. Now, let us say that instead of a constant frequency pulse. Suppose, we have a LFM pulse with tau naught is equal to 1 microsecond where the signal frequency changes by 10 megahertz and the carrier frequency is 3 gigahertz. So, recalculate that.

So, you can easily find out that, what is this pulse compression gain? That will be B so, B is 10 megahertz 10 into 10 to the power 6 and tau naught is 10 to the power minus 6.

So, pulse compression gain is 10 and if you remember that earlier also we have taken this same thing. So, for a constant frequency pulse I can write constant frequency pulse.

Now, the range resolution or the delay resolution that came as two-third into tau naught. So, two-third into tau naught is 1 microsecond. So, that is two-third microsecond. So, that keeps you if you do the delta R that will be 3 into 10 to the power 8 into two-third into 10 to the power minus 6 by 2 these expressions we have earlier seen.

So, if you do that it will come to be almost 100 m.

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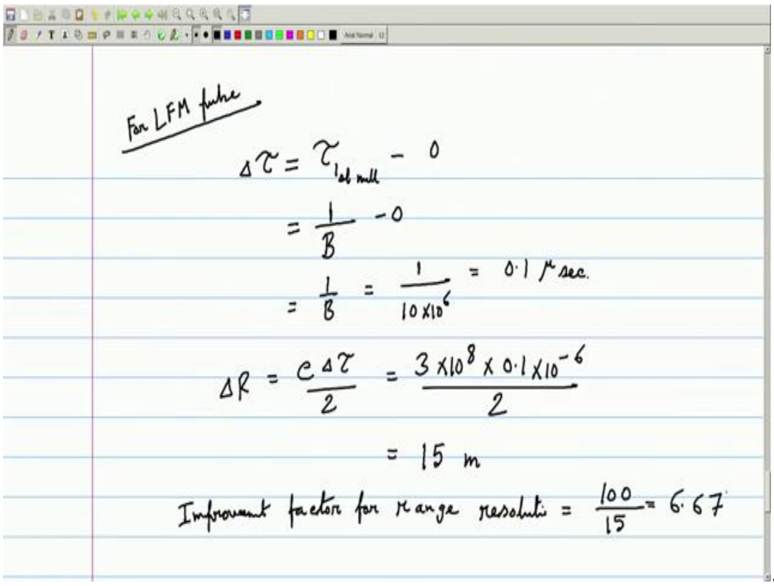

And, for LFM pulse. So, we can find what is the delta T. Now, that will be tau first null minus 0. Now, tau first null we have proved for a LFM it is 1 by B minus 0. So, basically it is 1 by B. So, 1 by B means 1 by 10 megahertz 1 by 10 into 10 to the power 6. So, that will be 0.1 microsecond.

So, what will be delta R, delta R is nothing, but C delta T by 2. So, that is 3 into 10 to the power 8 into delta T is 0.1 into 10 to the power minus 6 by 2. So, it becomes 15 meter. So, the improvement factor for range resolution, that is 100 by 15.

So, it is 6.67 times by giving LFM pulse we have improved ok. Now, this is the story for the a thing, there are other techniques there are other linear frequency, because we have given a particular 1 mu T square. Now, people can give T cube T forth etcetera also there are digital techniques which are called barcodes.

So, this LFM is a pulse compression by digital means those are all areas of research I am not going into that, but very simply I tried to give you an understanding that how it can improve resolution. Actually this method is called pulse compression. Actually, you see that by giving the phase in the phase there is an exponential variation with time actually you are making a large pulse behaving as a short pulse.

So, that it is ambiguity function is getting an it resolution is improving, but you are not losing the benefit of the large pulse, because large pulse can carry large amount of power. If, you make the pulse width very small actual reality that power will not be taken. So, the range etcetera will suffer. So, we are having their large pulse, but by giving the modulation in the frequency, we are making it behave in the as far as range resolution is concerned as a short pulse, that is why it is called pulse compression.

Now, what happens to the Doppler resolution, there we will see that this method is not very suitable.

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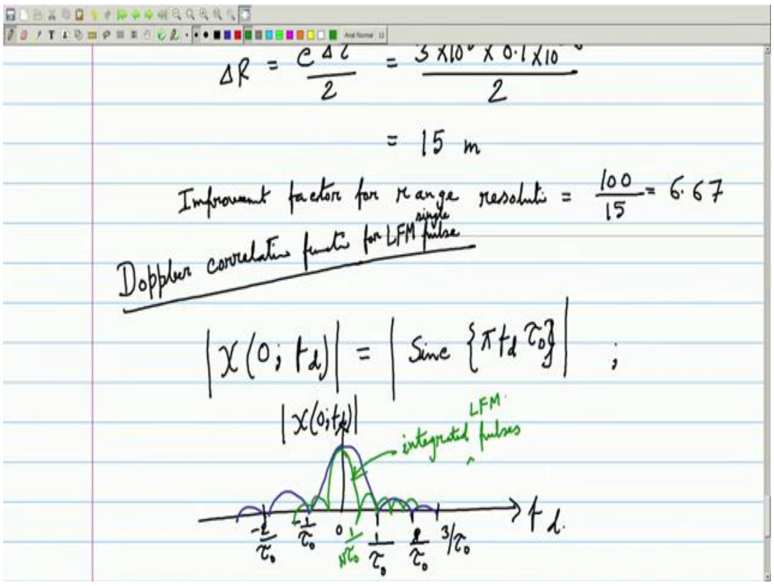

So, let us do that. So, what is the Doppler correlation function, because we have the expression for ambiguity function, in ambiguity function if we put delay is equal to 0 that will be Doppler correlation function. So, let us see for what is that? So, in that expression I can show you which expression I am not now I am not giving the equation number. So, I will just show you what expression. This expression.

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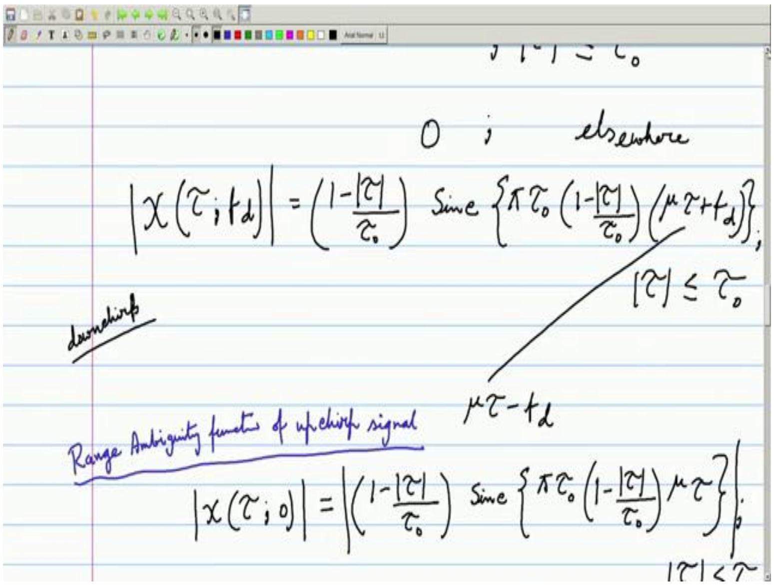

This is for up chart this is the ambiguity function here you put tau is equal to 0.

So, this first bracket term become 1, here also this become 1 tau is equal to 0 here. So, it is only f d. So, it is a sinc function that is why we will write that, we can write that for LFM Doppler correlation function for LFM pulse. Remember this is LFM single pulse I should say, because we have taken a single pulse here say LFM it will be sinc pi f d tau naught again for.

$$|\chi(0;f_d)| = |\text{Sinc}\{\pi f_d \tau_0\}|$$

So; that means, if I plot this that suppose I am plotting this thing versus f d, then the plot will be a sinc function. So, this point is 1 by tau naught this point will be 2 by tau naught 3 by tau naught, this is minus 1 by tau naught minus 2 by tau naught etcetera so.

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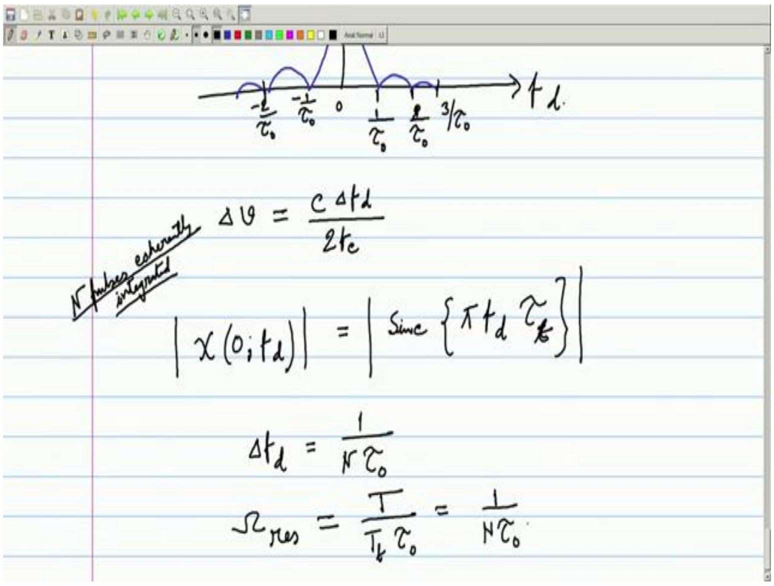

And, so, what is your delta v that will be as before c delta f d by 2 fc.

$$\Delta v = \frac{c \Delta f_d}{2 f_c}$$

So, here you can say that basically we are not gaining much actually pulse compression is not meant for gaining anything in Doppler.

But, remember one thing that actually the any radar pulse radar that does a integration of pulse. So, with coherent integration of n pulses this Doppler correlation. So, actually this if we have N pulses coherently integrated, then the expression for the Doppler correlation function that will be something like sinc pi f d. Now, this tau naught instead of that it will be something a because the pulse width that time is the total pulse width for N pulses.

So, I can say tau instead of tau naught I am calling it tau t total

$$|\chi(0; f_d)| = |\text{Sinc}\{\pi f_d \tau_t\}|$$

And we have so; that means, the this will have a null at a point the first null will be at 1 by tau t. And, tau t is N into tau naught so; that means, the delta f d for this case instead of tau naught it will be 1 by N tau naught. So, by that it will improve by a factor of N.

$$\Delta f_d = \frac{1}{N\tau_0}$$

I can say that whatever I am saying it can be validated, because in previous class I have also found out what is a resolution in the Doppler domain and that time if you recall, if you compare that with these parameters we have written like this and this is nothing, but 1 by N tau naught.

$$\Omega_{res} = \frac{T}{T_f\tau_0} = \frac{1}{N\tau_0}$$

So, if you have increase the pulse width; that means, N number of pulses so, N into tau naught. So, that will automatically improve it. So, suppose you are having 18 times you are integrating 18 pulses you are integrating in the Doppler resolution we will improve by a factor of 18.

So; that means, there I can say sorry that if Doppler correlation function will have this narrowing what is this point this point is 1 by N into tau naught. So, this is width integrated pulses integrated LFM pulses. So, that benefit you will always get. So, we can do that calculation also the example we are seeing. So, there we can see that.

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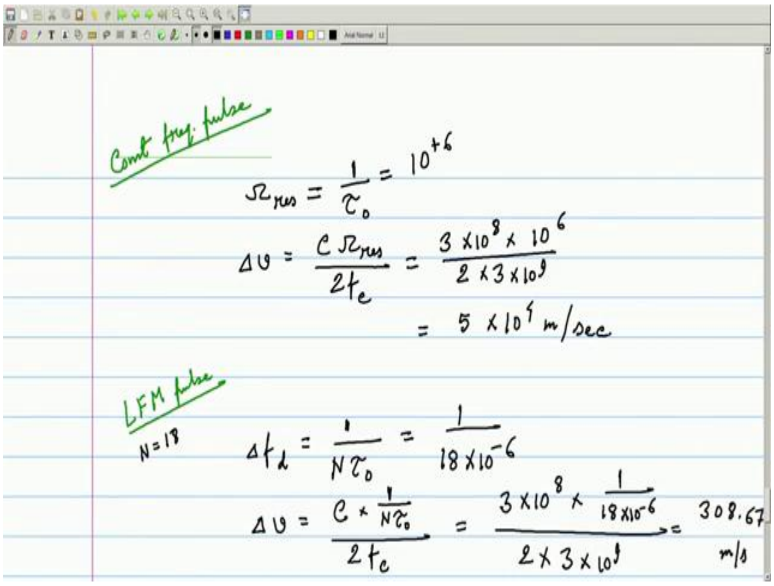

Suppose for a constant frequency pulse what will be our velocity resolution let us calculate that, we know that constant frequency pulse our velocity resolution is 1 by tau naught and delta v is c the resolution by 2 f c. So, that is 3 into 10 to the power 8 and this thing is 1 by tau naught. So, tau naught is in that example we have taken 1 micro second; that means, 10 to the power minus 6.

So, 10 to the power not 10 to the power plus 6 it will be 10 to the power plus 6 by 2 into f c is 3 gigahertz 10 to the power 9. So, that will be 5 into 10 to the power 4 meter per second.

Now, let us make LFM pulse. So, for LFM now we can say that, delta f d will be 1 by N into tau naught, let me take N is equal to 18; that means, 18 pulses I am integrating. So, it will be 1 by 18 into 10 to the power minus 6. And, so, my delta v will be c into 1 by N tau naught by 2 f c. So, that will be 3 into 10 to the power 8 into 1 by 18 into 10 to the power minus 6 by 2 into 3 into 10 to the power 9, basically that will give you a 308.67 meter per second.

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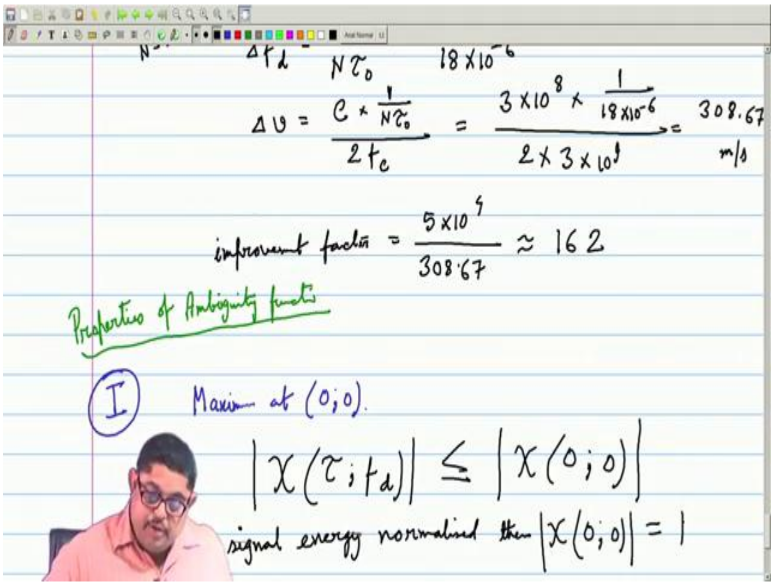

So, you see that with 18 pulse the improvement main factor is 5 into 10 to the power 4 divided by this 308.67. So, almost a 162 times improvement you are getting in the LFM pulse also provided you integrate some pulses. So, it is beneficial, but we will see that there is another technique that we will discuss next by where this integration is done, but by more processing you can have any order of improvement in the this frequency domain.

And actually this is a cross range case, because the generally the range is called down range. And, the other one thing so, in the cross range domain there will be Doppler processing, but this frequency 1 by integration generally it is sufficient for getting a good resolution.

So, before going there I will show you another thing that actually these ambiguity functions. They have some property and by using those properties sometimes we can for LFM signals, we can very easily find the property. If, we have constant frequency signals ambiguity function, we can very easily derive the ambiguity function for LFM signals. Though, I have given you from the first principles I have derived it always it is not so necessary. So, I will just show you 4 properties without proof, but you should know that because ambiguity function you will be using. So, you should know those important properties of ambiguity function.

So, that I will come the first property so, I will say that properties of ambiguity function. The property one, it says that the maximum of any ambiguity function is always at (0,0).

So, what it says this is the statement mathematically actually it can be proved we are not proving it, but this is an important thing that ambiguity function, it is always less than equal to ambiguity function at (0,0); that means, for 0 delay and 0 Doppler, what is the value that is the peak always it has the peak at the central position.

And, so, ambiguity function can nowhere be higher than that at the origin, and if signal energy is normalized if signal energy normalized, then the ambiguity function peak value is 1, it cannot be more than 1.

I Maximum at $(0;0)$.

$$|\chi(\tau;f_d)| \leq |\chi(0;0)|$$

If signal energy normalised then $|\chi(0;0)| = 1$

So, this is an important property always you should check, because sometimes if you get some value. And, if it does not satisfy this property we will understand that there is something erroneous.

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$$|\chi(\tau;f_d)| \leq |\chi(0;0)|$$

If signal energy normalised then $|\chi(0;0)| = 1$

II Constant Volume

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\chi(\tau;f_d)|^2 \, d\tau \, df_d = 1$$

for normalised signal energy

I & II $\Rightarrow$ if we try to squeeze the ambiguity functn to a narrow peak at the origin, the peak cannot exceed 1 and the volume squeezed out of the

The second property it says that, it is a constant volume property. So, what it says is this that first let me write the mathematics. If, we integrate the ambiguity function square. So, what it says that total volume under the normal normalized; that means, here also actually this property is for normalized signal energy. What it says, that total volume under the ambiguity surface squared equals unity irrespective of the signal waveform.

II Constant Volume

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\chi(\tau; t_d)|^2 \, d\tau \, dt_d = 1$$

for normalized signal energy

So, whatever wave form you use always this ambiguity function squared and then the total volume under that, this square curve it is a 2 dimensional curve. So, under that the total volume that is always one. So, this is also an important property.

Now, we can say that or let me use that I property I and property II, they together save a very important thing I am writing it, that if we squeeze, if we rather say try to squeeze the ambiguity function to a narrow peak at the origin, the peak cannot exceed 1 and the volume squeezed out of the peak must reappear somewhere else.

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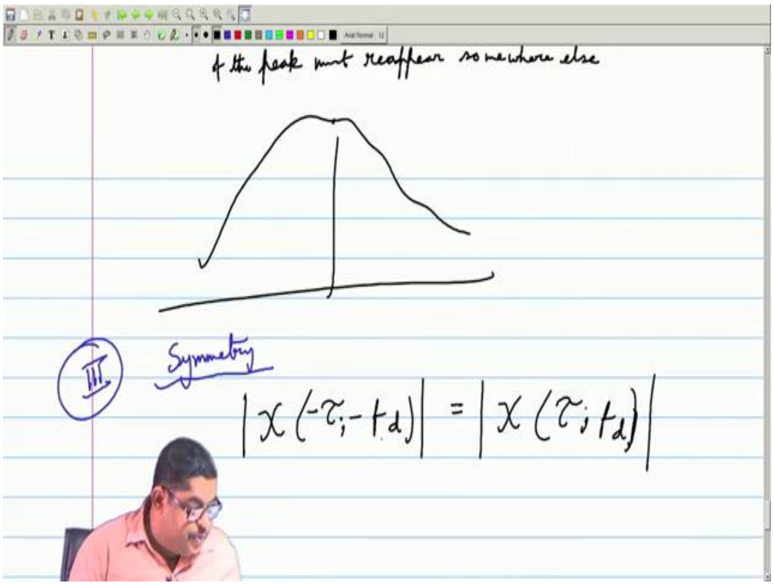
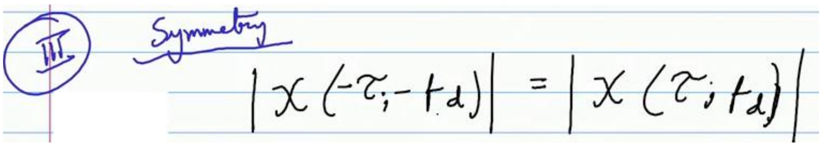


The meaning is suppose I have this. So, peak is here I want to make it more peaky here.

So, this volume will decrease if I want to peaky here more this will the peak will decrease. So, volume skews the out of that peak must reappear somewhere else. So, this is an important thing that if you want to make it more than whatever is coming, then it will reappear somewhere and that will disturb your resolution etcetera.

Now, there are two more properties next two properties are for both for normalized or unnormalized signal energy, but they are very important the third property is the symmetry property. So, symmetry property says that. So, what it says that it is sufficient to analyze only two adjacent quadrants of the ambiguity function. Generally first and second quadrants; that means, positive Doppler's if you find the other from symmetry you can always show.



So, as we have done in that our derivation of that ambiguity function of chirp signal etcetera. So, or always we have done for all possible cases, but actually using symmetry knowing one, you can tell the other that is one thing and fourth property is very important very relevant for us that is called LFM property or linear FM property.

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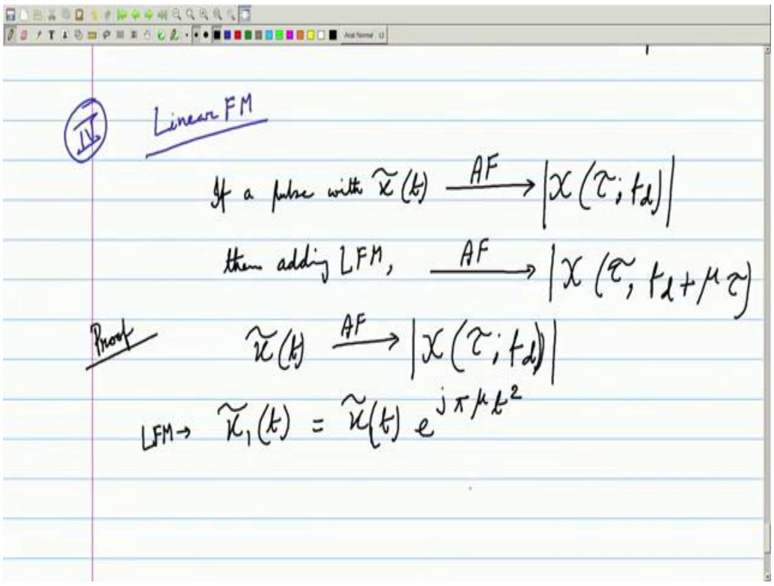
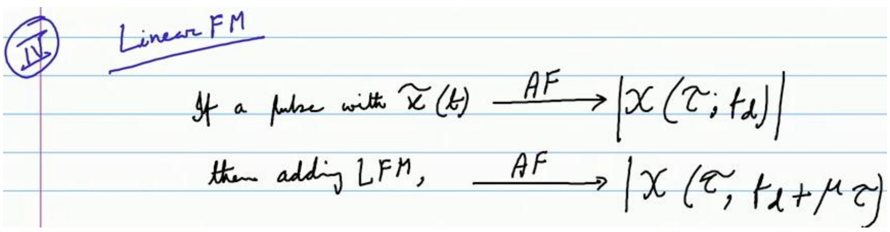


So, what it says that if a if a pulse with complex envelop x t has an ambiguity function given by x tau f d, then adding LFM, then adding LFM, the ambiguity function becomes tau f d plus mu into tau, you see that where mu is the LFM parameter.



So; that means, to prove this property we can easily prove this property mathematically. So, what we are doing we are saying that, the first part we are keeping x t is it is ambiguity function is and let us say the LFM signal let us call. So, LFM signal let us call instead of x t x 1 t. So, what is x 1 t, x l t it is x tilde t e to the power j pi mu t square.

Proof

$$\tilde{x}(t) \xrightarrow{AF} |\chi(\tau; f_d)|$$

$$\text{LFM} \rightarrow \tilde{x}_1(t) = \tilde{x}(t)\, e^{j\pi\mu t^2}$$

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$$\text{LFM} \rightarrow \tilde{x}_1(t) = \tilde{x}(t)\, e^{j\pi\mu t^2}$$

$$|\tilde{\chi}_1(\tau; f_d)| = \int_{-\infty}^{\infty} \tilde{x}_1(t)\, \tilde{x}_1^*(t-\tau)\, e^{j2\pi f_d t}\, dt$$

$$= \int_{-\infty}^{\infty} \tilde{x}(t)\, e^{j\pi\mu t^2}\, \tilde{x}^*(t-\tau)\, e^{-j\pi\mu(t-\tau)^2}\, e^{j2\pi f_d t}\, dt$$

$$= e^{-j\pi\mu\tau^2} \int_{-\infty}^{\infty} \tilde{x}(t)\, \tilde{x}^*(t-\tau)\, e^{j2\pi(f_d+\mu\tau)t}\, dt$$

$$= e^{-j\pi\mu\tau^2}\, \chi(\tau; f_d+\mu\tau)$$

So, what will be x 1 tau f d? The ambiguity function for the LFM signal that will be minus infinity to infinity x 1 t x 1 t minus tau e to the power j 2 pi fd t dt ok.

Now, this is minus infinity to infinity. Now, x 1 t we can put the value xt e to the power j pi mu t square, then t minus tau e to the power. Since, it is complex and a minus j pi mu t minus tau square e to the power j 2 pi f d t dt.

$$|\tilde{\chi}_1(\tau; f_d)| = \int_{-\infty}^{\infty} \tilde{x}_1(t)\, \tilde{x}_1^*(t-\tau)\, e^{j2\pi f_d t}\, dt$$

$$= \int_{-\infty}^{\infty} \tilde{x}(t)\, e^{j\pi\mu t^2}\, \tilde{x}^*(t-\tau)\, e^{-j\pi\mu(t-\tau)^2}\, e^{j2\pi f_d t}\, dt$$

So, you can take e to the power minus j pi mu tau square this term tau square you can take it out.

Because, that is not a function of t and this will be x t, then x star t minus tau e to the power j 2 pi fd plus mu tau t dt is equal to e to the power minus j pi tau square, this thing you can easily recognize as the ambiguity function for tau and fd plus mu tau.

$$= e^{-j\pi\mu\tau^2} \int_{-\infty}^{\infty} \tilde{x}(t)\, \tilde{x}^*(t-\tau)\, e^{j2\pi(f_d+\mu\tau)t}\, dt$$

$$= e^{-j\pi\mu\tau^2}\, \chi(\tau; f_d+\mu\tau)$$

Take absolute values of both sides.

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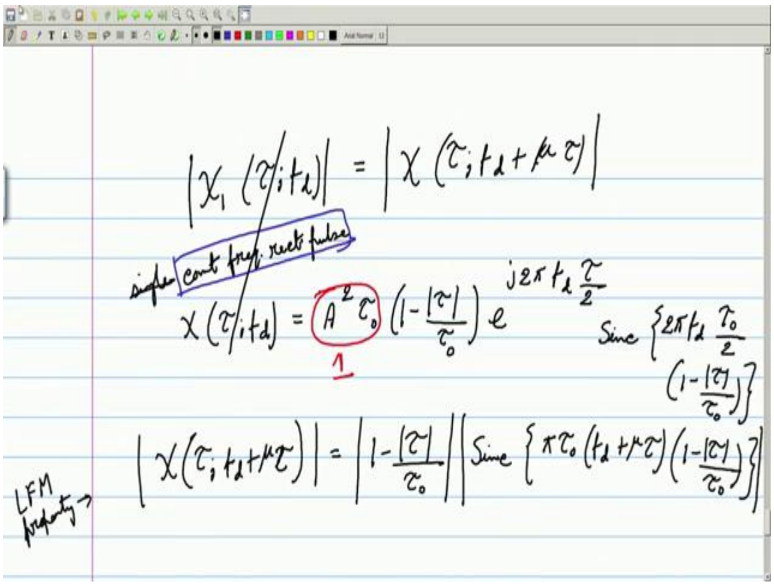

This equation you take absolute values of both sides that will give you.

$$|\chi_1(\tau;f_d)| = |\chi(\tau;f_d+\mu\tau)|$$

So, this property is very helpful and you can see that, actually earlier we have derived the ambiguity function of a rectangular constant pulse, you can see the equation I do not remember the equation.

So, from that directly without doing all those first principal things, you can find out that I think what I remember that we have found for a single, not single constant frequency, constant frequency rectangular pulse. I think the ambiguity function was something like this a square tau naught 1 minus tau by t naught e to the power j 2 pi fd tau by 2, sinc 2 pi fd tau naught by 2 1 minus tau by tau naught.

$$\chi(\tau; f_d) = A^2 \tau_0 \left(1 - \frac{|\tau|}{\tau_0}\right) e^{j 2\pi f_d \frac{\tau}{2}} \operatorname{Sinc}\left\{2\pi f_d \frac{\tau_0}{2} \left(1 - \frac{|\tau|}{\tau_0}\right)\right\}$$

Now, I think we have derived something like this for a constant frequency rectangular pulse. However, it was not an normalized thing, if I want to normalize a square tau naught should go to 1, because a square tau naught is the energy. So, it should go to 1 and actually you can now make that, these should go this will become 1. And, so, without this so, now, here if you use the LFM property, what you get, what is tau fd plus mu tau, that will be 1 minus tau by tau naught sinc pi tau naught fd plus mu tau again 1 minus tau by tau naught.

$$\left|\chi(\tau; f_d + \mu\tau)\right| = \left|1 - \frac{|\tau|}{\tau_0}\right| \left|\operatorname{Sinc}\left\{\pi \tau_0 (f_d + \mu\tau)\left(1 - \frac{|\tau|}{\tau_0}\right)\right\}\right|$$

So, this is LFM property. From LFM property I am writing and you can cross check that already from the first principle we derived. So, this so, this is the same expression that we could derive. Here, we have derived from first principles there we could have done it by using the property of LFM that if you give a constant frequency modulation; that means, say phase of e to the power j pi mu t square then by using that property in one line you can derive.

So, by that actually people derive the ambiguity functions of various new type of signals and find out their effects as far as the….

Also please remember that in the case of LFM you require a modulator, you require a frequency source frequency synthesizer, which can synthesize it within a very short time like. For example, the example we have seen that in one microsecond you need to sweep the frequency by a 10 megahertz thing.

Now, in today's electronics this is not a problem. So, that is why you get a sizable compression gain and that is now possible. And, this is a technique basically by which, you see that we can improve the down range resolution for cross range how to resolve or how to improve the cross range resolution, that technique is called synthetic aperture actually you need to increase the aperture; that means, you need to have an very large antenna, but large antenna is not feasible in moving systems.

So, we will see a synthetic way of having a large aperture by that the cross range resolution; that means, perpendicular to the range there are 2 perpendiculars can be drawn to a range direction. So, there are 2 cross ranges to a down range. So, those 2 cross range directions you can have synthetic aperture processing by which you can improve that we will see.

So, by this today we end the detection radar detection, there we have seen that how the what is the best filter for detection and by what it detects, it is output is not only having the best detection possible best SNR it gives, but also it preserves the resolution characteristic and if we properly design our signals we can improve our range resolution to whatever benefit we have and with integration we can have the frequency resolution Doppler resolution. Now, we will see that for cross range how to do that is our next topic that will be synthetic aperture processing.

Thank you.