Principles and Techniques of Modern Radar Systems
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Lecture – 43
Detection in Radar Receiver (Contd.)

Key Concepts: Determination of the delay resolution time constant and the Doppler resolution constant of a coherent train of carrier pulses, SNR gain of a matched filter from the concept of Doppler resolution constant and delay resolution time constant, introduction to the LFM signal (chirp), upchirp and downchirp.

Welcome to this NPTEL lecture on Techniques and Principles of Modern Radar Systems. We have seen in the previous class the concept of delay resolution constant and Doppler resolution constant we have evaluated that for a single rectangular pulse today we will evaluate for it, for a more realistic case of coherent train of carrier modulated pulses.

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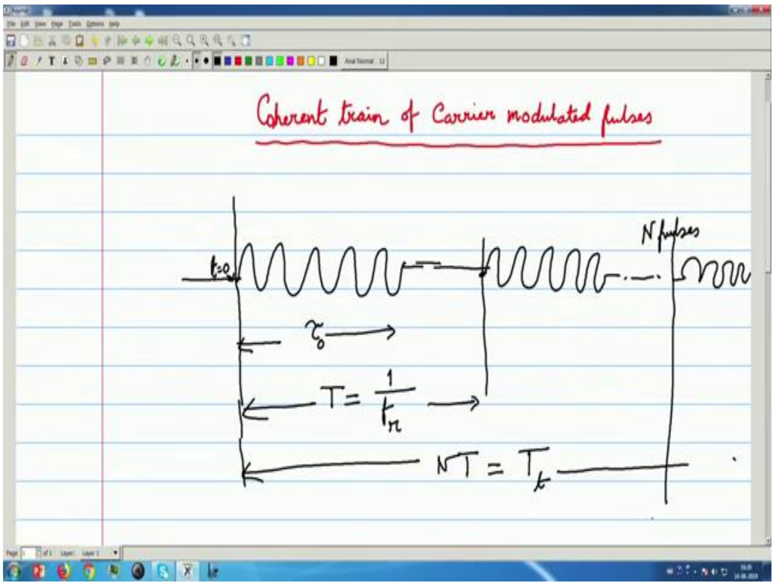

So, now this is a coherent train of you see that high frequency signals, the envelope is a rectangular envelope and there is an on time there is an off time there is a high frequency signal. So, we can take that this is the time instant t is equal to 0. And this on time that we can say that this is our tau naught.

Let us say, because here there will be many confusing things actually because the tau generally we call the delay, but this also when separately we tell these that is why I am taking tau naught to be the on time of the envelope and the time period that I can say like this that this time is the time period let me call that capital T and that is nothing, but this is PRI or the PRF usual things for a pulse radar and I am considering capital N number of pulses.

So, generally we integrate N number of pulses in pulse radar. So, one time period is T that is why capital N into T this thing I am giving it a name T with a subscript of t so; that means, from here up to these or let us say that this is the Nth number of pulse. So, I can say up to these so; that means, that there are n number of pulses. So, we can it is. So, there are let me write it that there are N pulses. So, that is why this is N T. Now let us go to the expression.

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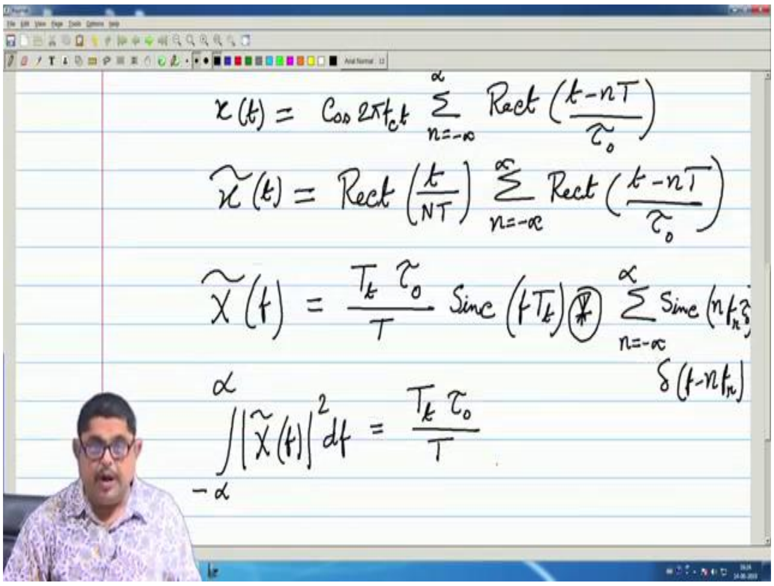

So, what is the transmitted pulse that I can write as cos 2 pi fct then n is equal to minus infinity to infinity rectangular t minus n T by tau naught.

$$x(t) = \cos 2\pi f_c t \sum_{n=-\infty}^{\infty} Rect\left(\frac{t-nT}{\tau_0}\right)$$

And so, if this is the case, we can say the complex envelope xt can be written as. This will be e to the power if you make then it will come out that the carrrier part will go t by capital NT this is as it is.

$$\tilde{x}(t) = Rect\left(\frac{t}{NT}\right) \sum_{n=-\infty}^{\infty} Rect\left(\frac{t-nT}{\tau_0}\right)$$

So, we know this is the we can take the Fourier transform the X tilde f that is the rectangular function.

So, we know separately if we take this Tt by tau naught by T we know that rectangular function means that will be a sinc function if you do it will come out like this, then these two. So, by the property of the Fourier transform this will be a convolution relation sinc n f r tau naught into for every rectangular thing there will be a delta function coming.

$$\tilde{X}(f) = \frac{T_t \tau_0}{T} Sinc(fT_t) \circledast \sum_{n=-\infty}^{\infty} Sinc(n f_r \tau_0) \delta(f - n f_r)$$

So, this is the spectrum you see that a sinc function convolved with a delta function, which is again whose envelope is given by a sinc function.

So, total energy for this signal we can easily calculate actually you see this is the beauty that we can find this that what is the because instead of finding the thing here we can find the total energy, that will be minus infinity to infinity. This if we do it will come simply as this.

$$\int_{-\infty}^{\infty} \left|\tilde{X}(f)\right|^2 df = \frac{T_t \tau_0}{T}$$

Also to find the delay resolution constant we need to find the fourth radar term.

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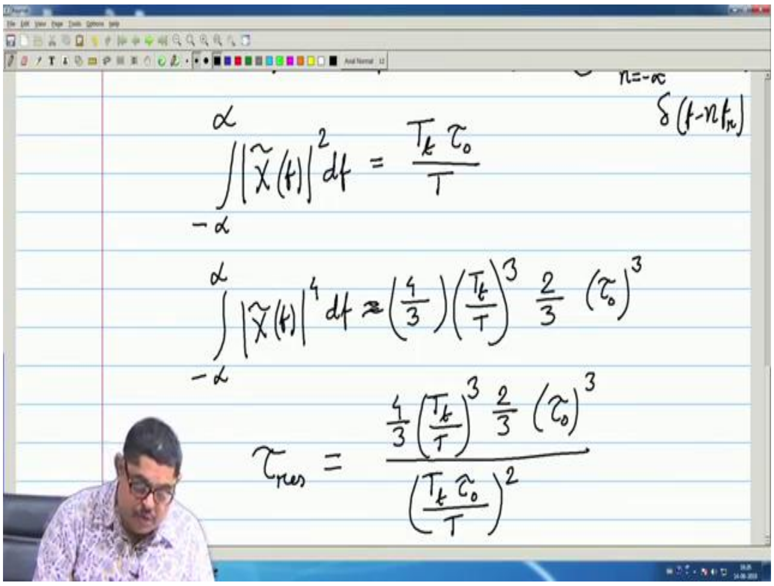
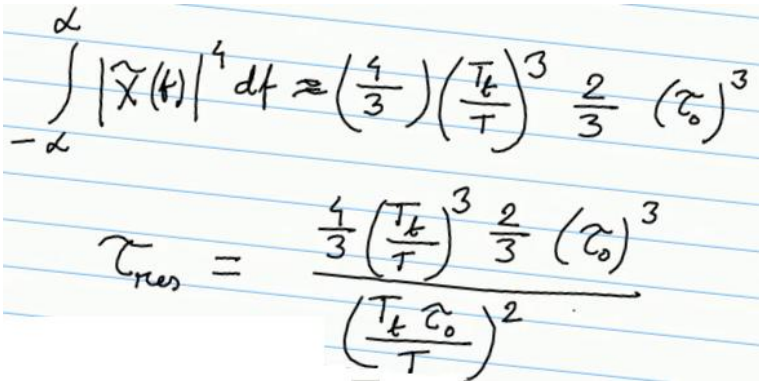


So, if we do that, it will be approximately it will come as 4 by 3 T t by T whole cube two third. So, we can find the delay resolution constant to be the these divided by this square. So, that will be 4 by 3 T t by t whole cube two third tau naught whole cube divided by T t tau naught by t whole square.



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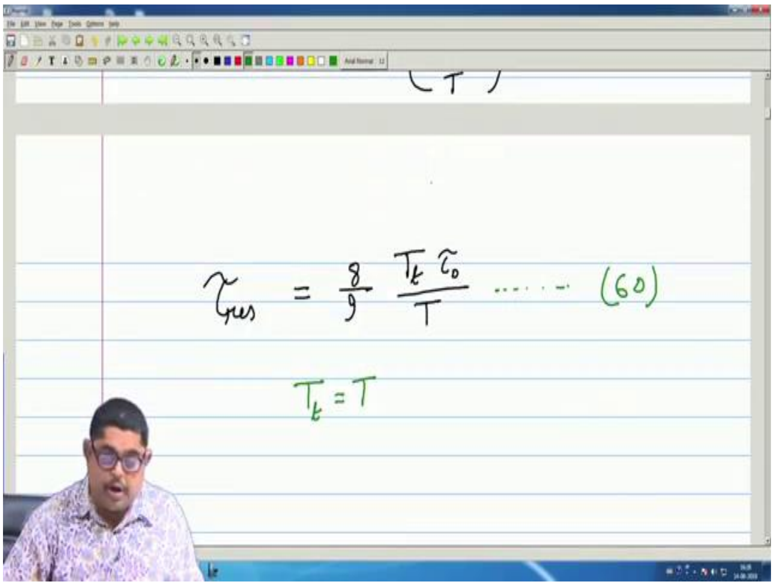

So, that becomes the resolution constants become 8 by 9 T t tau naught by T.

$$\tau_{res} = \frac{8}{9} \frac{T_t \tau_0}{T}$$

This is an important relation. So, I can give it a number 60 and so, it shows that delay resolution constant of the finite pulse train that increases as the length of the train; length of the train means T subscript t; that means, the total duration if that increases; that means, if I integrate more number of pulses the resolution get poorer now when T t becomes T when T t is equal to T basically there is a single a case.

So, that time you see this becomes 8 by 9, but if you see here that time this 4 by 3 also will not come. So, it is becoming two third correctly this four third is coming because of the whole n number of those things the delta functions, the rectangular functions convolution that part will not come..

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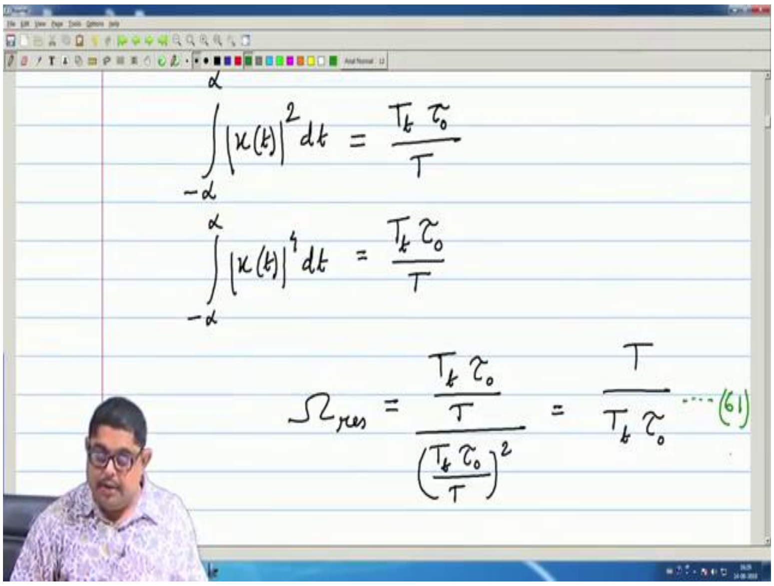

Now similarly we can find the Doppler resolution for that we need to find these terms x t square dt. So, this was simply the signal energy. So, from Fourier domain by Parseval's relation we can get it because already this has been calculated earlier.

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{T_b \tau_0}{T}$$

And one more thing we want that this thing will be.

$$\int_{-\infty}^{\infty} |x(t)|^4 dt = \frac{T_b \tau_0}{T}$$

So, the Doppler resolution in this case becomes. So, this again I can keep a number. So, this will be 61.

$$\Omega_{res} = \frac{\frac{T_b \tau_0}{T}}{\left(\frac{T_b \tau_0}{T}\right)^2} = \frac{T}{T_b \tau_0} \quad \cdots (61)$$

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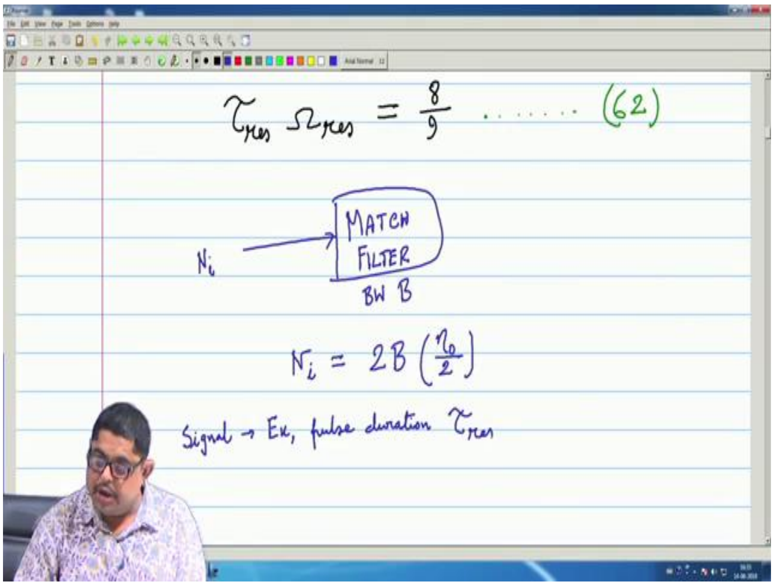

And so now, I can take this product that what is the delay resolution constant and the Doppler resolution constant product that will be if you do, it will be 8 by 9.

$$\tau_{res}\,\Omega_{res} = \frac{8}{9}$$

So, this is simply 62. So, you see that in case of single rectangular pulse it is two thirds, in case of this chain of pulses it is 8 by 9 so; that means, since this is a the product is fixed number. So, if I want to improve the this resolution; that means, if I want to decrease this value, this value will increase; that means, what I achieve in the range domain the same thing in the same proportion I will have to forgo in the velocity domain.

So, this is the problem for this signal so; that means, no flexibility in the design of the signal. So, also we will see another aspect of this product. So, let us again go to the match filter and consider that the case. Suppose I have a match filter and let the bandwidth of this match filter is B. So, the input noise power N i. So, I can say that input noise power N i will be 2B if I assume the white noise double sided white noise. So, it will be eta naught by 2.

$$N_i = 2B\left(\frac{\eta_0}{2}\right)$$

And if the signal is having energy E x and suppose the signal is having energy E x and the pulse duration of that tau resolution.

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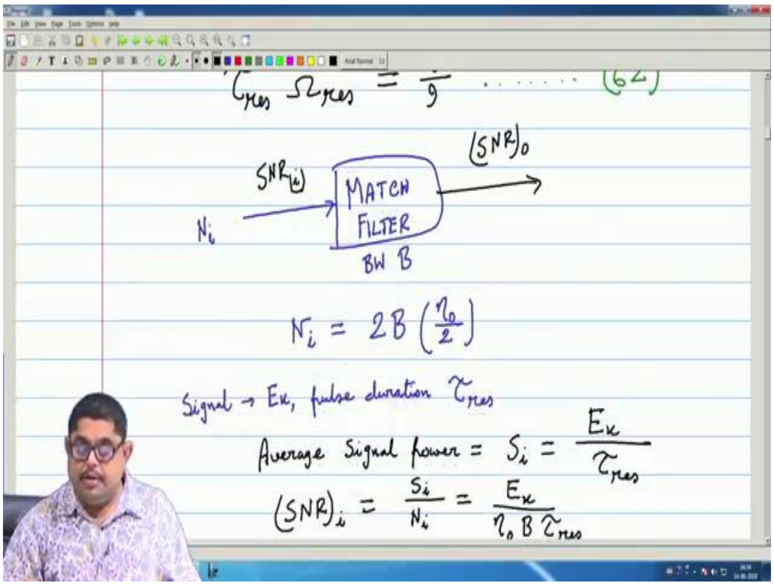

Then I can say the average signal power is S i and that S i will be E x by tau resolution.

$$\text{Average Signal power} = S_i = \frac{E_x}{\tau_{res}}$$

So, what is the SNR at the input; that means, I can say that SNR i. So, SNR i will be Si by Ni. So, that will be E x by eta naught B tau resolution.

$$(SNR)_i = \frac{S_i}{N_i} = \frac{E_x}{\eta_0 B \tau_{res}}$$

Also let me call that what is coming out that SNR output.

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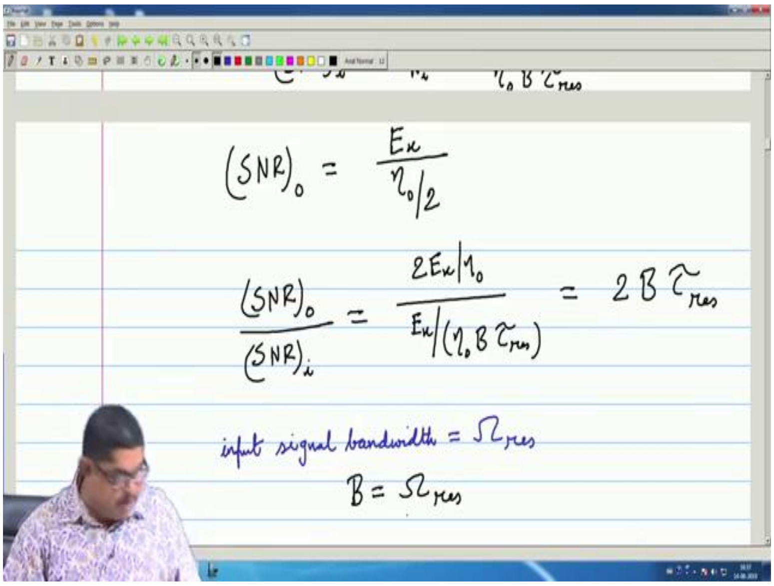

Now, if for a match filter we know that if the input signal has an energy E x and if it is sampled at the proper delay time T 0 then SNR out that becomes we know E x by eta naught by 2 this we have already seen.

$$(SNR)_0 = \frac{E_x}{\eta_0/2}$$

So, now, what I will do? I will make what is the SNR gain sort of thing? SNR out by SNR in of the match filter provided I sample it at a proper time t is equal to t naught.

So, this ratio the SNR out we know 2 E x by eta naught and SNR in this expression E x by eta naught B resolution. So, this turns out to be 2 B tau resolution.

$$\frac{(SNR)_0}{(SNR)_i} = \frac{2E_x/\eta_0}{E_x/(\eta_0 B \tau_{res})} = 2B\tau_{res}$$

Now the input signal you see that we know the if we take it the signal Doppler's frequency constant if it is omega resolution. So, we can say that signal the input signal bandwidth, we can take it as omega resolution then the match filter bandwidth should be to avoid distortion it should be greater than that or at least equal to that. So, this dictates that B we can take as gamma resolution means the minimum bandwidth will definitely make that because otherwise more noise will come.

$$\text{input signal bandwidth} = \Omega_{res}$$
$$B = \Omega_{res}$$

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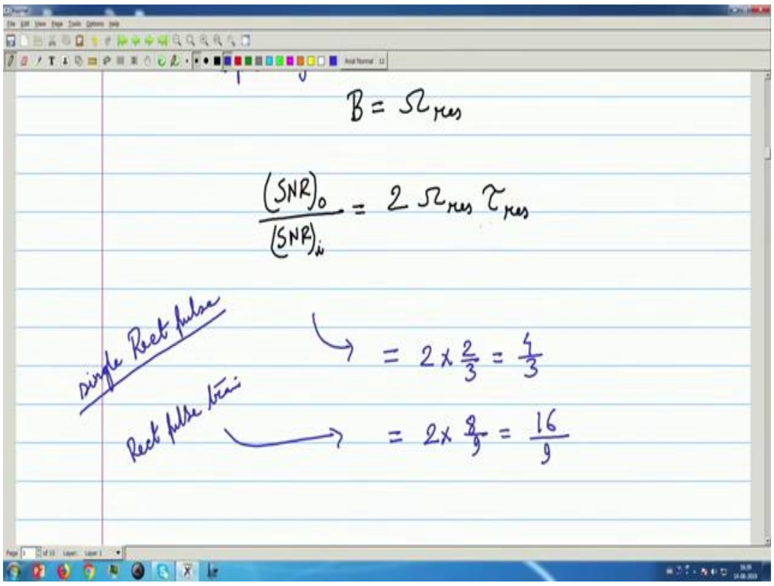

So, if we take this then this ratio becomes SNR o by SNR i this becomes 2.

$$\frac{(SNR)_o}{(SNR)_i} = 2\,\Omega_{res}\,\tau_{res}$$

So, what is this? This thing can be called as the match filter SNR gain. That means the maximum gain you can get in terms of SNR from these. So, you see that for a this is an important thing; obviously, here we will try to maximize it and we have seen that for a rectangular pulse single rectangular pulse write single rectangular pulse then this gain will be before that okay. So, 2 into two third. So, that will be 4 by 3. So, you see that this product gamma resolution into tau resolution that has an implication that gives the SNR gain.

So, how much SNR gain I can get maximum from the if I use a single rectangular pulse? 4 by 3. For pulse train rectangular pulse train pulse train this gain will be 2 into 8 by 9 that is 16 by 9. So, this is 1.33 this is almost 1.8 something, but these are fixed.

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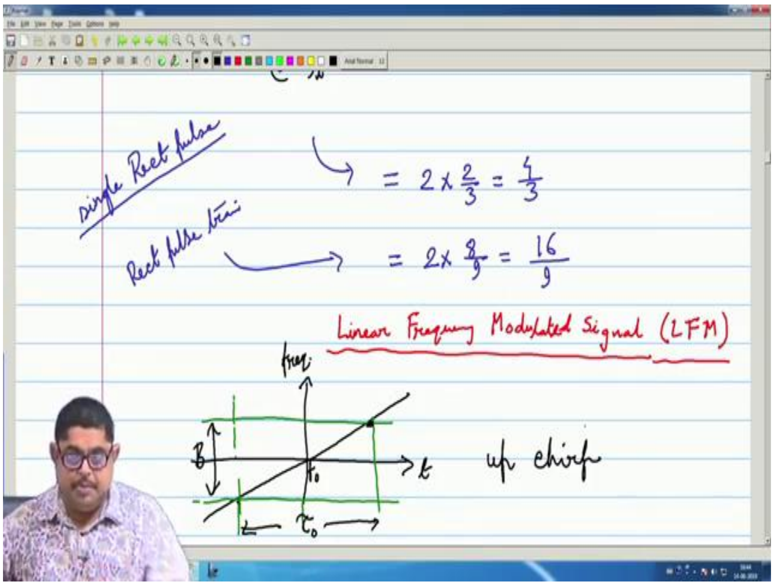

So, the point is this product omega resolution into tau resolution this is an important parameter. It gives good detection this product if we can make more, also we have seen that if we can minimize these then the resolution improves and also we should be able to do that independently. Independently we should be able to make this omega resolution smaller and that should not affect the tau resolution.

Tau resolution we should be able to handle independently these, but that is not possible in these scenarios, but if we use these rectangular pulses or pulse trains etcetera. So, can we do something that will now see actually that is an important thing that concept the concept will go later, but we see a particular class of signal called linear frequency modulated signal LFM signal. Its a simple signal.

Now suppose the signal at is 0 time its frequency is f naught, then with a constant slope the frequency is increased let us increase to this extent; that means, f naught plus B by 2 and in minus time; that means, from here it is decreased in the same slope up to my f naught minus B by 2 or I can say that from this time from f naught minus B by 2 to f naught plus B by 2 it changes; that means, the bandwidth of the signal is B in a time tau naught.

So; that means, if the frequency is swept linearly across the pulse width and in if it goes in the up; that means, frequency goes up as the time advances that is called up LFM or sometimes, it called up chirp, chirp is the bad sound very musical sound. So, if you see if this signal is given to a loudspeaker, it will give you a chirp birds chirping.

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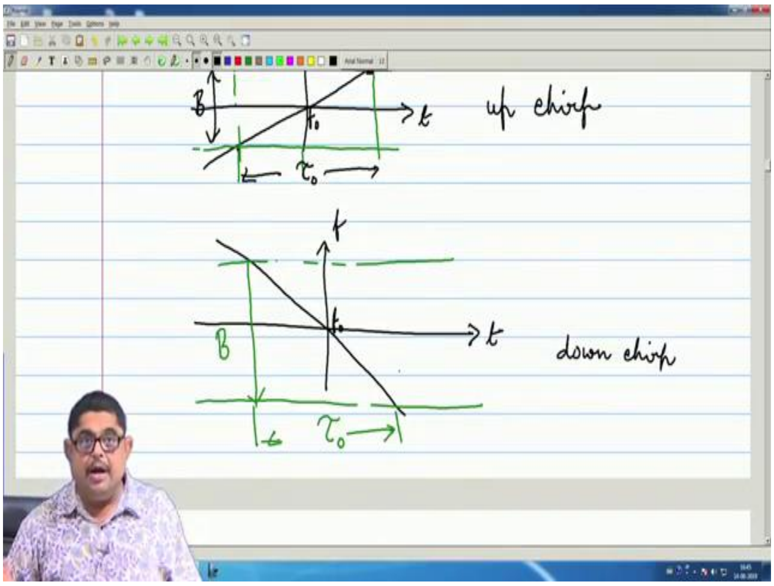

So, this is the up chirping similarly you can have a down chirp signal; that means, this is your frequency, this is your time and again. So, this is your B and this is your tau naught. So, this is your down chirp again this will be your f naught I think yes all the parameters. So, up chirp sharp and down chirp. So, if we you see this is something like our fm signal, but.

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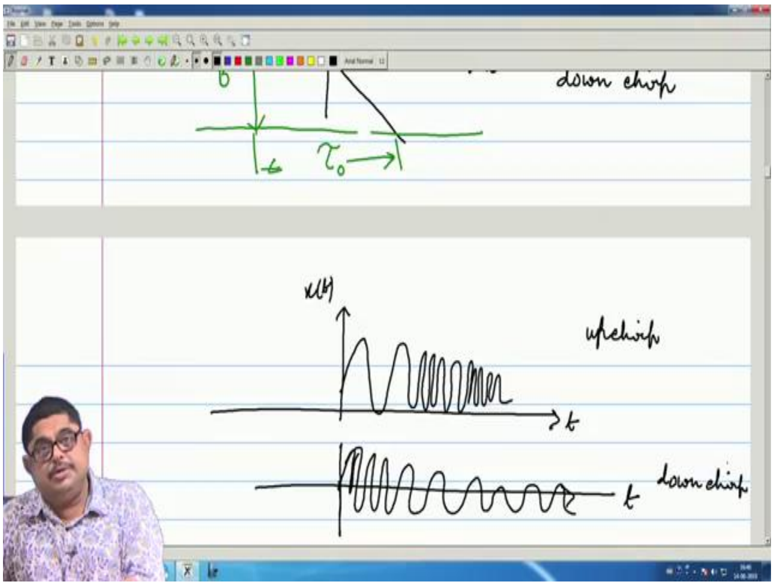

Not exactly that. So, if I see this signal in time domain, if it is x t then with t the up chirp signal will be something like this; that means, frequency is increasing with time and so, I will say this is my up chirp signal and if I have a down chirp I think. So, this is a down chirp signal. So, continuously in a particular fashion you can change it, but this is we will see in the next class the behavior of this signal as a transmitted signal.

So, this is not exactly the fm signal that we see in communication why we will explain that, and we will see that what is its effect on the detection or resolution of the radar concepts. So, that we will see in the next class.

Thank you.