Principles And Techniques Of Modern Radar Systems

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Lecture – 41
Detection in Radar Receiver (Contd.)

Key Concepts: Introduction to the performance metric of delay resolution, i.e. delay resolution time constant, mathematical model for delay resolution time constant in both the time and frequency domains, delay resolution time constant for a single rectangular pulse

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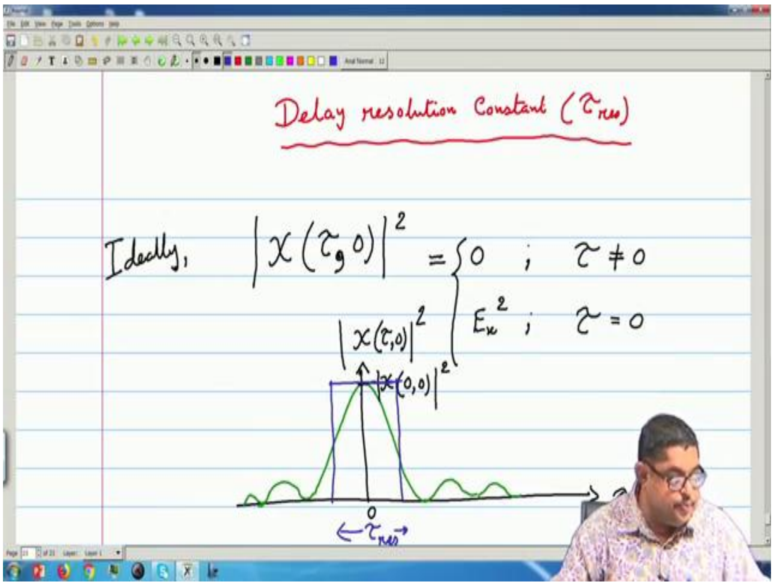

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar Systems. Up to the previous class, we have seen in detail the ambiguity functions and in the last class we have said that we want a performance metrics for both delay resolution and Doppler resolution, so that we can compare which wave form is better, and then we can appropriately designed the radar pulse shape. So, that we will do today. First we will see the concept of delay resolution constant.

Now, ideally what I am writing now. Let us in ambiguity function we are putting the Doppler portion off, so it has only delay. So, this is the range ambiguity function. So, ideally we know that magnitude square of this range ambiguity function should be peaky, so at tau is equal to 0 it should be having the peak and that peak should be equal to the whole energy of the signal. And at other places it should be 0; that means, it should be an ideal impulse type of thing, the.

And we know that range ambiguity function is nothing, but auto correlation of the sending pulse transmitted pulse. So that means, what we are saying is we will have to transmit a impulse type or very short pulse type which is covering the entire energy, but this is impractical because it will limit the transmission capability because every transmitter has a power handling capability. So, I cannot send any amount of power there. I cannot make it very peaky, in a very short time I cannot transmit the whole energy.

So, this is impractical, that is why we will see that any practical range ambiguity function will have a; see, this is magnitude square it will be always peaky and always positive, but something like this. So, in these I am putting tau and this is my x tau 0 square. So, it will have a spread and then also it will have side lobes. So, this is any practical correlation function can be.

Now, to and we also know that these value is nothing, but x 0 0 square because tau is equal to 0 here. Now, if I want to say that how fast the role off is or where within which range the energy is confined we can use a concept similar to something we have done if you remember in communication classes, the noise, white noise if it is passed through a filter it becomes filtered noise and it takes the shape of the filter transfer function.

Now, effective bandwidth for that we define with a equivalent rectangular function the equivalence is in terms of energy. Similar concept we will take here. We will draw a rectangle, the rectangle height is same as the energy of the pulse or xi 0 0 square, but its width is such that the energy contained in the rectangular function and energy contained in the range ambiguity function they are the same. So, this rectangle let it has a width, this width is called the delay resolution constant tau resolution.

So, we will have to now. So, what we have done conceptually? The energy in the range ambiguity function graph is make equal to the energy of a rectangular function whose width is tau resolution and height is xi 0 0 square; that means, range ambiguity functions value at 0,0, ok.

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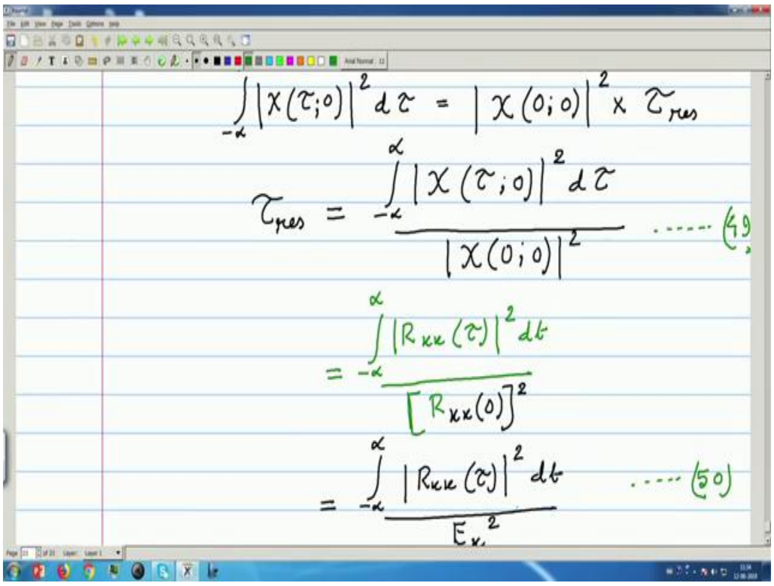

So, with this now mathematically we can easily relate these two. So, I can write that for the range ambiguity function that; this will be semi colons.

$$\int_{-\infty}^{\infty} |\chi(\tau;0)|^2 d\tau = |\chi(0;0)|^2 \times \tau_{res}$$

So, this easily tells us that tau is the delay resolution constant, so it is nothing, but this simple expression.

$$\tau_{res} = \frac{\int_{-\infty}^{\infty} |\chi(\tau;0)|^2 d\tau}{|\chi(0;0)|^2}$$

So, these thing I can give if I continue the equation number, this is our equation 49. It is an important thing. So, these now instead of range ambiguity function, I can also write these equation in terms of the auto-correlation function because ultimately from a given time wove from that autocorrelation function will matter.

So, we know that this is nothing but, should be a small x, and also going one step further I can also say that this I can also write as the these R x say 0 auto correlation with 0 a that is the energy of the signal this is a well-known thing. So, these thing if I know the expression of the transmitted signal in time domain, I can easily calculate these equation 50, so from that I can get the delay resolution.

$$= \frac{\int_{-\infty}^{\infty} |R_{xx}(\tau)|^2 \, dt}{[R_{xx}(0)]^2}$$

$$= \frac{\int_{-\infty}^{\infty} |R_{xx}(\tau)|^2 \, dt}{E_x^2}$$

Now, sometimes actually instead of the signal calculating these autocorrelation in time domain sometimes frequency domain calculation is easier. So, we will just; this is sufficient if you know the time domain expression you can find, but always that is not available. So, sometimes frequency domain calculation is easier. So, what we will do now? We will do some jugglery here, mathematical jugglery, but those are simple and you can we will have an expression that if you do not know the time domain expressions, but you know the frequency domain expressions of the signal or the spectrum then you can find that.

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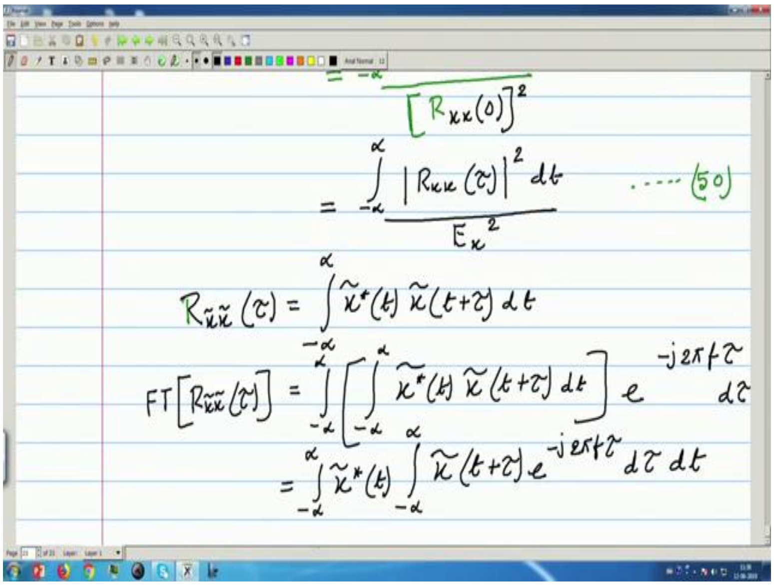

So, for that what we will do? That what we know is auto-correlation we know the definition that we are using from the start is something like this.

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} \tilde{x}^*(t)\,\tilde{x}(t+\tau)\,dt$$

Now, if I take the Fourier transform of these. So, both sides if I take the Fourier transform, actually I should have written like this. So, then it will be this. This is d tau, let me write it, ok.

$$FT\left[R_{\tilde{x}\tilde{x}}(\tau)\right] = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} \tilde{x}^*(t)\,\tilde{x}(t+\tau)\,dt\right] e^{-j2\pi f\tau}\,d\tau$$

So, I can, you see this is an the running variable is running variable of integration is tau. So, this x t, x tilde t can be taken out and this boils down to.

$$= \int_{-\infty}^{\infty} \tilde{x}^*(t) \int_{-\infty}^{\infty} \tilde{x}(t+\tau)\, e^{-j2\pi f\tau}\, d\tau\, dt$$

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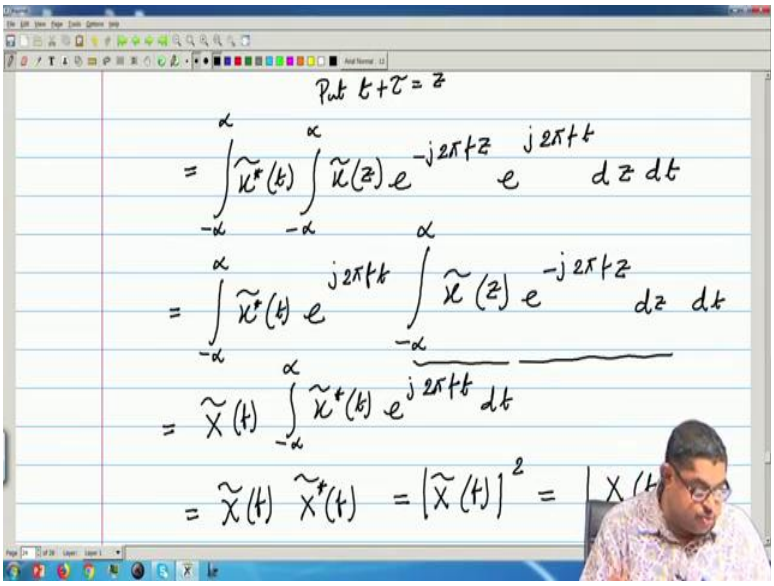

So, in that now we can put a change of variable t plus tau is equal to z, in the second integral, so that will give us. So, you see this 2 pi f d this is can be taken out. So, it becomes e to the powered j 2 pi f t, and this whole thing I can. First let me write and say that this is. So, this portion I think all of you will agree that this is nothing but the Fourier transform of x's, so this is x f.

Put $t+\tau = z$

$$= \int_{-\infty}^{\infty} \tilde{x}^*(t) \int_{-\infty}^{\infty} \tilde{x}(z)\, e^{-j2\pi f z}\, e^{j2\pi f t}\, dz\, dt$$

$$= \int_{-\infty}^{\infty} \tilde{x}^*(t)\, e^{j2\pi f t} \int_{-\infty}^{\infty} \tilde{x}(z)\, e^{-j2\pi f z}\, dz\, dt$$

So, I can write it as, take out as X f and this thing what is left behind is this minus infinity to infinity, and this is again the Fourier transform of this conjugate and we know that this is nothing, but magnitude square. And this is complex envelopment magnitude square, we know that this is also can be written as the original functions magnitude square.

$$= \tilde{X}(f) \int_{-\infty}^{\infty} \tilde{x}^*(t)\, e^{j2\pi f t}\, dt$$

$$= \tilde{X}(f)\, \tilde{X}^*(f) = \left|\tilde{X}(f)\right|^2 = \left|X(f)\right|^2$$

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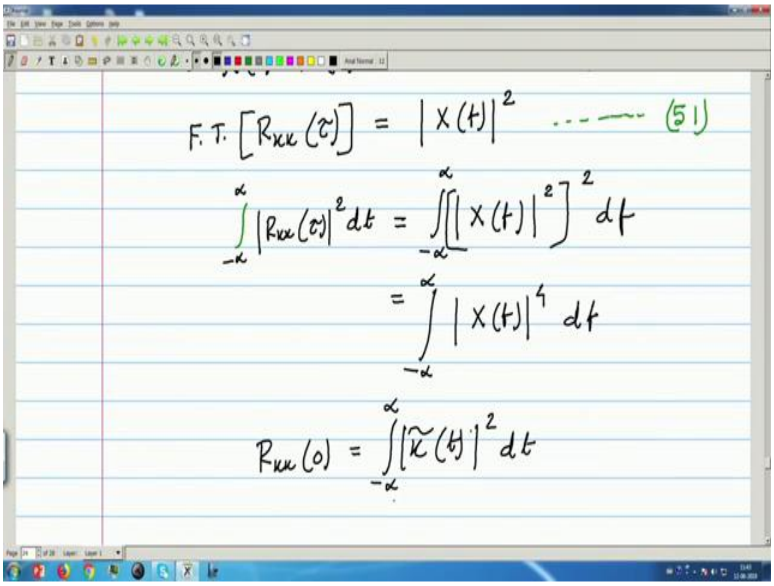

So, this is an important relation. That means, I can say that the Fourier transform of auto-correlation function of the signal, transmitted signal is equal to the magnitude square of the spectrum of the signal. So, this should be given a name a number, this is equation 51.

$$F.T.\left[R_{xx}(\tau)\right] = \left|X(f)\right|^2$$

So, if we see our resolution thing that delay resolution concept, so there is a oh sorry term minus infinity to infinity R xx tau square d tau. So, this now can be written as that minus infinity to infinity, ok. And this is nothing, but minus infinity to infinity X f whole to the power 4 df.

$$\int_{-\infty}^{\infty} |R_{xx}(\tau)|^2 d\tau = \int_{-\infty}^{\infty} \left[|X(f)|^2 \right]^2 df$$
$$= \int_{-\infty}^{\infty} |X(f)|^4 df$$

So, I can say that; also what is the numerator? We have seen that R xx 0 that is we know that auto correlation for 0 delay that is given by dt.

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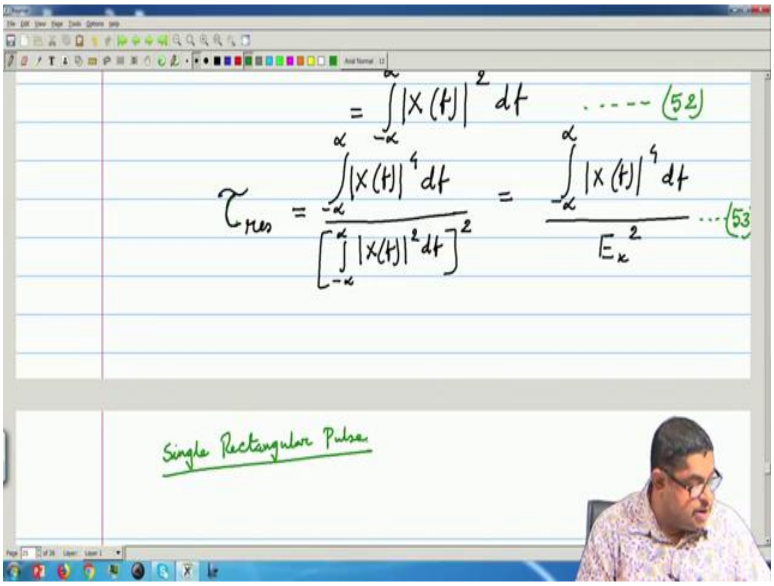

So, and by Parseval's theorem, we know that this can be written as Parseval's theorem and also knowledge of complex envelop this can be said as X f square df. So, let me call this equation 52.

$$R_{xx}(0) = \int_{-\infty}^{\infty} |\tilde{x}(t)|^2 dt$$
$$= \int_{-\infty}^{\infty} |X(f)|^2 df$$

So, 51 and 52 are the Fourier domain expressions for the numerator and denominator of the delay resolution constant. So, now, I will say sorry, the delay resolution constant is in frequency domain its expression is.

$$\tau_{res} = \frac{\int_{-\infty}^{\infty} |x(t)|^4 \, dt}{\left[\int_{-\infty}^{\infty} |x(t)|^2 \, dt\right]^2} = \frac{\int_{-\infty}^{\infty} |x(t)|^4 \, dt}{E_x^2}$$

Now, this sometimes this is energy of the signal is known. So, this second part also sometimes comes handy. So, this is our equation 53. So, using this we can easily find out. So, now, we have got a performance metric and we have seen that how narrow or how good a range ambiguity function in rolling down, you can see that you will have to go to because range ambiguity function is x something squared. So, we are going to a higher order and except whole to the power 4 integrating that throughout and comparing that with square of the energy and that is giving us the resolution thing.

So, smaller this thing we will have better this. So, now, we will put it into the rectangular pulse and we will find out what is this resolution and we will see a surprising thing. So, single rectangular pulse. We have seen this before, now we will find its delay resolution constant.

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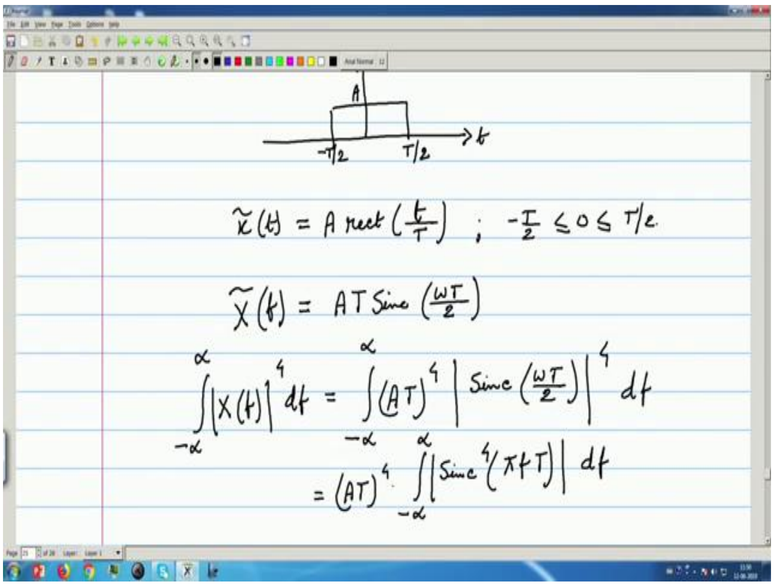

So, we know that the pulse is something like this is the x t, this is t, this is A, this is minus T by 2, this is plus T by 2. So, we know that its, analytic expression is A, and previously we have found its spectrum also because we will be requiring actually you can calculate from this directly we can calculate the A thing, also in the spectrum domain it may become easier. So, we know its spectrum is given by A T sinc function omega T by 2.

$$\tilde{x}(t) = A\,\text{rect}\left(\frac{t}{T}\right) \quad ; \quad -\frac{T}{2} \le 0 \le T/2$$

$$\tilde{X}(f) = AT\,\text{Sinc}\left(\frac{\omega T}{2}\right)$$

So, you see that instead of finding the auto-correlation function of this signal, that also you can do. Here I will show you that if I take the frequency domain expression that X f to the power 4 df, so that will be minus infinity to infinity, A T your constant, A T to the power 4 and I have sinc omega T by 2 whole to the power 4 df. Now, omega, so this thing I can take out A T whole to the power 4 and sinc of omega is 2 pi f. So, this will be pi f T whole to the power 4, that means sinc 4, magnitude of sinc function is always positive.

$$\int_{-\infty}^{\infty} |X(f)|^4 df = \int_{-\infty}^{\infty} (AT)^4 \left|\text{Sinc}\left(\frac{\omega T}{2}\right)\right|^4 df$$

$$= (AT)^4 \int_{-\infty}^{\infty} \left|\text{Sinc}^4(\pi f T)\right| df$$

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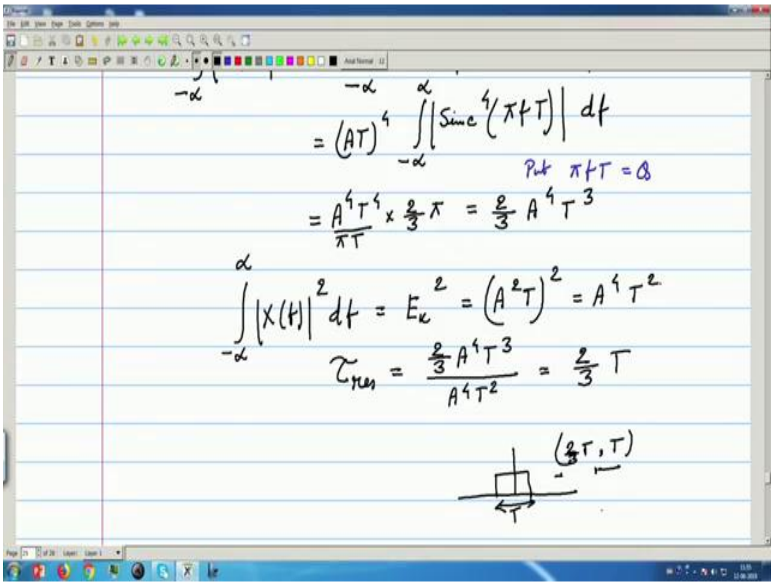

So, we can easily do this integration that it is A 4 T 4 and then I can pi f T you can put as, you can put pi f T, I put pi f T is equal to theta. You can do this and then the answer will be that part will give you, this is the simple integration I am not going to two-third pi. So, that will give you two-third A 4 T 4 by that thing, A 4 T 4 by that pi and T will come here, two-third pi; this full theta because that d theta that pi f will come here. So, this will become two-third, A 4th T cube, pi will go, ok. So, X f 4.

$$= (AT)^4 \int_{-\alpha}^{\alpha} \left| \mathrm{Sinc}^4(\pi f T) \right| df$$

Put $\pi f T = \theta$

$$= \frac{A^4 T^4}{\pi T} \times \frac{2}{3}\pi = \frac{2}{3} A^4 T^3$$

And what is the energy of the signal? We know that denominator is the energy which is nothing but E x square and we know that in a rectangular function the energy is A square T, so it is square. So, that is A 4th T square. So, what is your delay resolution constant? So, you can see the delay resolution expression the numerator is this thing. So, numerator will be two-third, A 4th T cube divided by A 4th T square, so it is two-third T.

$$\int_{-\alpha}^{\alpha} |X(f)|^2 df = E_x^2 = (A^2 T)^2 = A^4 T^2$$

$$\tau_{res} = \frac{\frac{2}{3} A^4 T^3}{A^4 T^2} = \frac{2}{3} T$$

So, you see that it says that the delay resolution is not the time period. Actually, if you remember in our early classes when we have said the fundamentals of pulse radar or various concepts of pulse radar, there I myself in this class said that the time period of the pulse the resolution is something like that because that time the if during that time any echo comes the radar cannot discriminate. But now that is okay for most of the time, but when we are very specifically mathematically saying then a rectangular pulse that resolution; that means, the what is the resolution, the two targets whose delay are two-third by 2 the radar won’t able to say that there are two targets.

But if their delay is between two-third by 2 to T then the radar will be able to tell that they are two different things. So, it is not simple T; that means, this is an important thing;

that means, if two targets, suppose I have this is the transmitted pulse this pulse on time is T, but here it is the two targets delay is something the there, due to their range the corresponding delays between two-third T and T in this range then also the radar will be able to say.

So, this is the thing that we got from here, a very important point. And now we will later again come to this concept then we will have to see the other part; that means, this is about the delay or range part. Now, what is the Doppler resolution constant? And we will have to see that what is the common pulses, like rectangular pulses or periodic rectangular pulses pulse train they are those from metrics and from that we will see that how you can design, where both this is resolutions you can choose by properly designing the signals shape. That we will see in the next class.

Thank you.