Principles and Techniques Of Modern Radar Systems
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Lecture - 40
Detection in Radar Receiver (Contd.)

Key Concepts: Determination of velocity resolution for rectangular pulse, range and velocity resolution for Gaussian pulse, derivation of the ambiguity function for a Gaussian pulse

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar Systems. So, we are discussing the resolution concepts. So, we have seen the ambiguity function for the pulse, rectangular pulse, we have in the last class, we have seen the range ambiguity function, we have seen the frequency correlation function we have plotted them.

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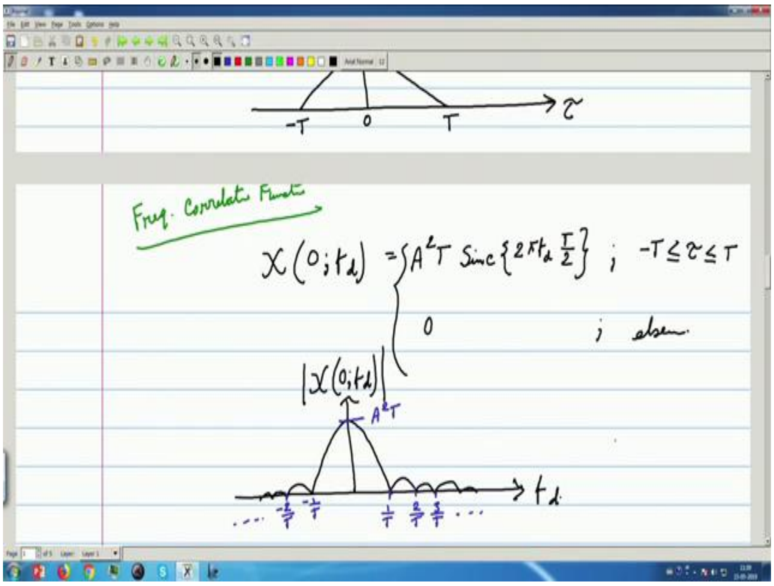

Now, I again refer you to that; that one thing you should note that actually if you go back and see the equation 38, then this expression of x f x; that means, the its a chi sometimes x f this f d. So, equation 38 you see what is equation what equation we say 38 x f f d its expression there we have written that minus infinity to infinity star f, then x f minus f d d f. And also that we have proved that was in time domain we can write that as x t square e to the power j 2 pi f d t d t. This was our equation 38.

$$X_f(t_d) = \int_{-\infty}^{\infty} \tilde{x}^*(t)\,\tilde{x}(t-t_d)\,dt = \int_{-\infty}^{\infty} |\tilde{x}(t)|^2 e^{j2\pi t_d t}\,dt$$

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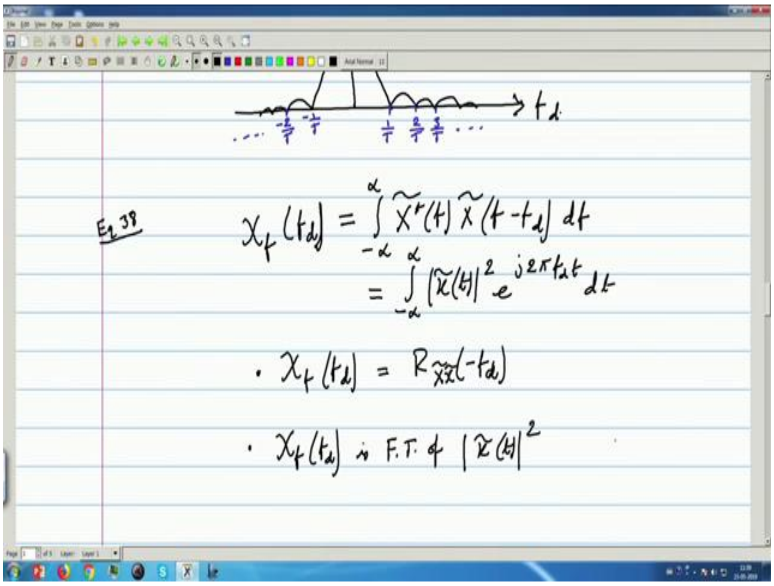

Now, you see can I say that, so the frequency correlation function is actually the auto correlation function of x, but according to our definition this should be written as minus f d because here the delay is like this and also. So, these are the two points that we should note because it will be later used so; that means, the frequency correlation function is the; sorry this is the auto correlation function of the Fourier transform of the transmitted signal.

And also if I look at this second part of equation 38 that frequency correlation function is can I say the Fourier transform of x t square. So, I can say that the frequency correlation function is Fourier transform of what is actually x t square or let me write that Fourier transform of the x t magnitude square.

$$x_f(t_d) = R_{\tilde{x}\tilde{x}}(-t_d)$$

$x_f(t_d)$ is F.T. of $|\tilde{x}(t)|^2$

That means something like the energy of the transmitted signal it is said, but you can see that one thing is that there will be a phase a thing because if I want to do it that there is a instead of e to the power negative sign it is plus thing, but it is something like that. So, there is a relation between the frequency correlation function and this I can say the x t square d t that will be energy. So, the energy in time domain of the transmitted signal and the frequency correlation function is something related this we will actually explore later.

Now, the question is that what we observe that for both the range ambiguity function and frequency correlation function we ideally would have been happy if we have got a delta function sort of thing, but in real life you can see rectangular pulse does not give us that it has a finite width and also in case of frequency correlation function, it has side lobes and it goes on extending.

Though with diminishing thing, but it has the ambiguity function has side lobes so; that means, there is a chance that it will disturb in the detection process because it will affect all subsequent targets. So, this is should have been avoided. So, we can if we can give a better design etcetera. Now, another thing I can say that if from this suppose actually real a thing always the null is not the width of the resolution because actually in resolution you want that from the peak of the.

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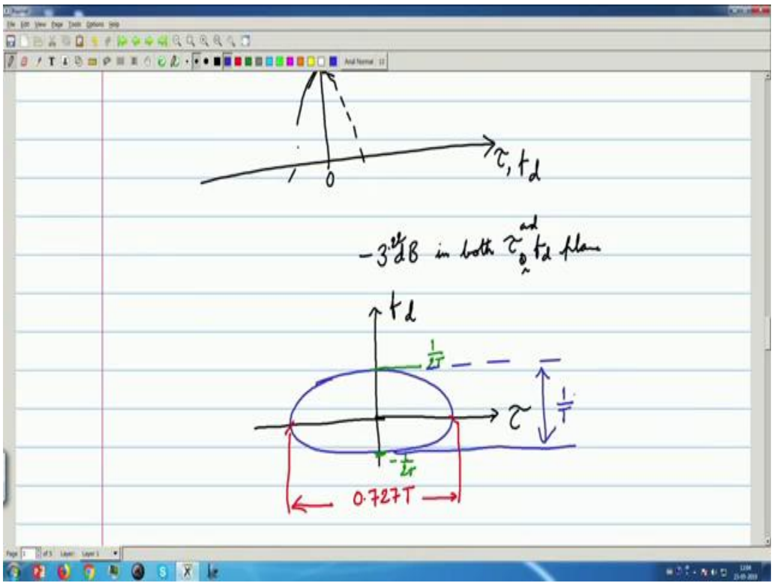

As I discussed in last class that the ambiguity function it should be peaking at 0 whatever may be, it may be tau or it may be in f d in any dimension and then it should come very close with whatever shape the roll off should be high. So, people generally do not say that always come to null here you can say that let us say 3 dB etcetera. So, one popular.

So, if we take a ambiguity function of the rectangular pulse, if we take a cut of let us say approximately at minus 3 dB in both tau and f d plane, if you take a cut if you plot it in matlab etcetera you can easily plot it. So, it will basically be a so; that means, if I take a three dB cut, then in and plot it in tau 3 dB plane then that thing will become an ellipse ok. So, in that ellipse let us say that these widths are important.

So, suppose we say that anything which is higher than this point we will consider there is a target, if the value lower than that then we dont consider the target. So, then there are in the range dimension there are these two. So, if you put it into the function we can easily find out that these points they are this separation of them that will be 0.727 T so; that means, the range ambiguity function has a width this for 3 dB. So, approximately 3 dB.

So, this actually to be precise people take it as minus 3.22 dB, then you get this 727 I think you are understanding. Similarly here in the frequency dimension actually this becomes 1 by 2 T and this becomes minus 1 by 2 T. So, this width is; that means, in the f d dimension the width becomes 1 by T. So, this is the resolution so; that means, delay resolution.

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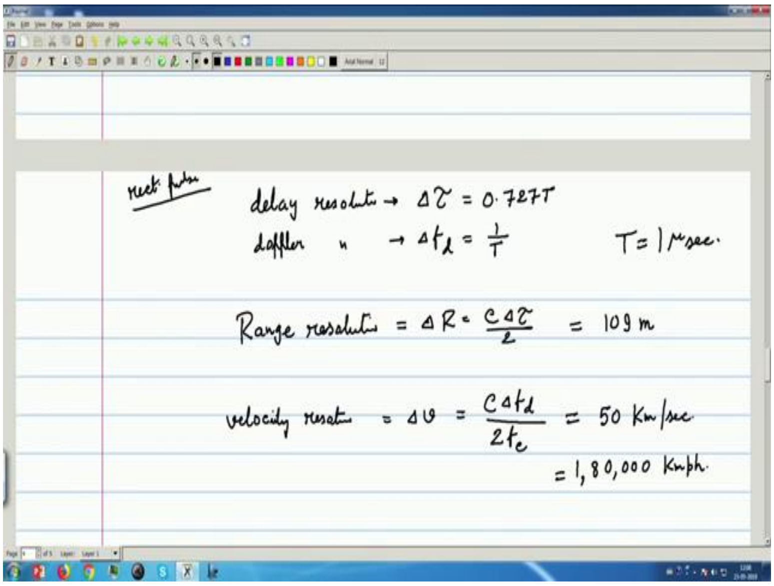

So, I can write now that delay resolution this is for rectangular pulse we are still discussing that rectangular pulse. So, delay resolution is; that means, delta tau that is 0.727 T and the Doppler resolution that is delta f d that will be 1 by T.

$$\text{delay resolution} \rightarrow \Delta\tau = 0.727T$$
$$\text{doppler resolution} \rightarrow \Delta f_d = \frac{1}{T}$$

So, we can easily then calculate what will be the range resolution. So, range resolution will be c delta tau by 2.

$$\text{Range resolution} = \Delta R = \frac{c\Delta\tau}{2}$$

So, if suppose I take a one microsecond pulse what is capital T? Capital T is the on time of the pulse rectangular pulse transmitted pulse. So, one microsecond is a typical value if we take that then delta R that will come as you can easily do it putting c as 3 into 10 to the power 8 meter per second you can find that it will come as 109 meters. Whereas, we know the Doppler resolution delta f d.

The physical meaning of Doppler means that if 2 targets their Doppler shift is separated by more than this value delta f d, then they will be resolved if they are less than that they would not be resolved as a that is the Doppler resolution. So, from there we can find what is the velocity resolution; velocity resolution; that means, delta v that we know that that is c delta f d by 2 f c this thing we have seen at the beginning that what is the relation between velocity and Doppler shift. So, in that expression we are just putting that the differentials.

$$\text{velocity resolution} = \Delta v = \frac{c \Delta f_d}{2 f_c}$$

So, that; so for this one microsecond pulse this thing if you do it will come as 50 kilometre per second to give you more idea 50 kilometres again is a huge speed actually if we convert it to kilometre per hour it will be kilometre per hour. So, you see that the typically our air craft etcetera their speed is 1000 kilometre per hour, 1100 kilometre per hour etcetera. So, in velocity resolution rectangular pulse is very poor.

Range resolution is quiet good because 109 meter; that means, 100 meters. So, if you can resolve ranges for a target for a thing because your range is several kilometres several 100s of kilometres in that range resolution of this for a one microsecond pulse is good, but this is very poor. So, this is ultimately you see unless and until we have done all those mathematics we could not have understood, that this is the picture in velocity resolution. So, something that should needs to be done etcetera.

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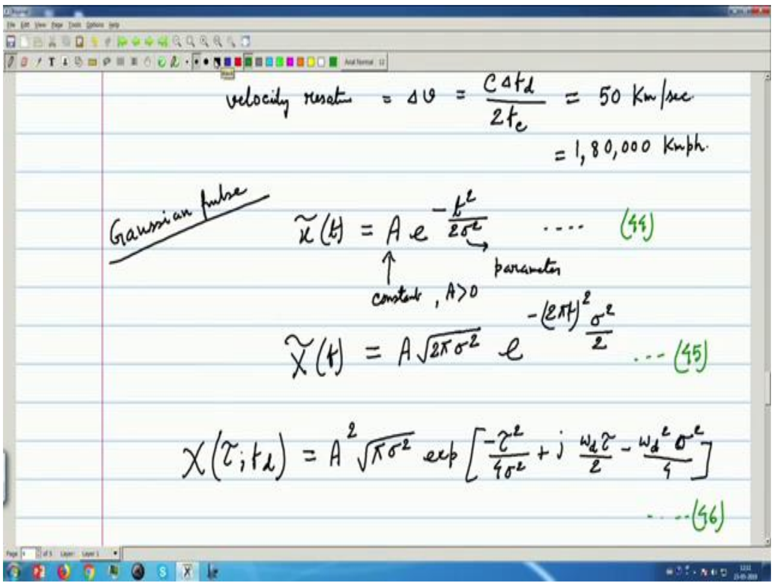

Now, before going further again we do this exercise very hurriedly for another pulse that is a Gaussian pulse; that means, instead of rectangular if I take a smoother pulse and what will be its ambiguity function let us see. So, Gaussian pulse; so what is a Gaussian pulse?

I define it mathematically you know mathematically Gaussian function is more easy to track. So, this is my Gaussian pulse let me write it as again equation 44.

$$\tilde{x}(t) = A e^{-\frac{t^2}{2\sigma^2}}$$

And so where this is a constant and also A is greater than 0 positive constant and this you know what it is, but let me call it is a parameter because this standard deviation of the Gaussian pulse, so its a parameter. So, now, Gaussian function we can always find Fourier spectrum.

So, first is what is the Fourier transform of this that if you do. I am not going into that that is pretty easy, if you compare this with the standard normal distribution you can get it minus 2 pi f whole square sigma square by 2.

$$\tilde{X}(t) = A\sqrt{2\pi\sigma^2}\, e^{-(2\pi f)^2\frac{\sigma^2}{2}}$$

So, this again put me in a equation because later I will use this results and now this ambiguity function I am writing; ambiguity function because you will have to do whatever I did for rectangular pulse similar type of thing here, but here unfortunately that, but pulse is there.

So, you can understand that again break it into two parts and you can do, but this is not a mathematics class. So, you can do it yourself or in tutorials we will see whether you solve this. So, this will be our equation 46.

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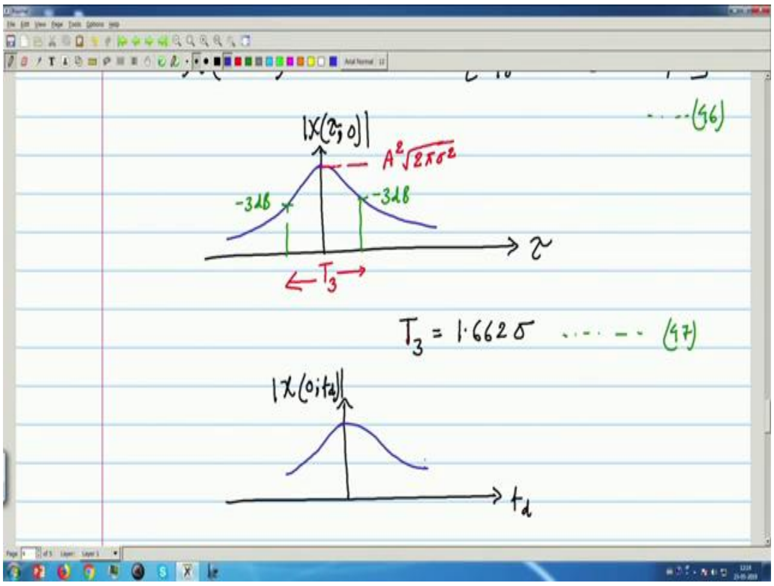

Now, we can easily write this a thing, but instead of writing we will draw what is the range ambiguity function that means, if I plot the tau 0, then it will be versus tau. So, that will be again a Gaussian type of function and this magnitude of the peak that is important that will be A square root over 2 pi sigma square as can be seen here and for a this if I take yes if I take a 3 dB cut; that means, from here if I take a point 3 dB down. So, I can say that this is minus 3 dB this is also.

Since it is the symmetric curve both side minus 3 dB and this separation let me call it T 3 and this also this T 3 in terms of the parameter of the pulse parameter of the pulse where A and tau. So, it will be if you do it from here it will come out to be 1.662 into sigma. So, this is again another equation. So, I should give it a name 47.

$$T_3 = 1.662\sigma \quad \ldots\ldots (47)$$

Similarly if I do the correlation function frequency correlation function.

So, that also is a Gaussian, that is a beauty that pulse the ambiguity functions both in range and in frequency they are of same shapes smooth shape. So, actually that is what is the idea for this pulse Gaussian pulse.

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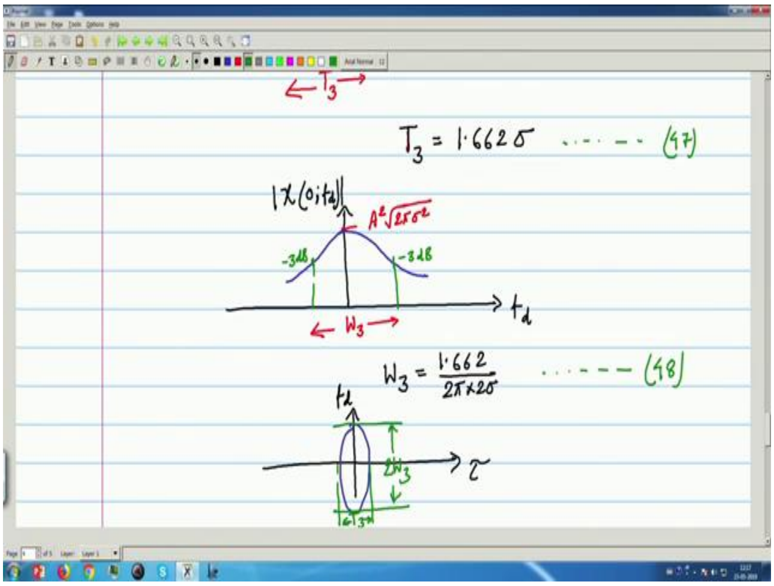

And here we can find what is the peak value again the peak is same you see here also the peak is a square root over 2 pi sigma square and this 3 dB points. So, let me call that 3 dB point W 3 in frequency. So, that W 3 if you calculate it will come as 1.662 by 2 pi into 2 sigma. So, this is our equation number 48.

$$W_3 = \frac{1.662}{2\pi \times 2\sigma}$$

So, again if we take the cut in the 2 dimensional picture of this so; that means, if I have a tau versus f d thing and if I take 3 dB cut there, then again I will have an ellipse sort of thing and whose this.

So, this point and this point I am calling it T 3 and this point and that point that will be 2 W 3 ok, so which we have already seen. So, if we want to compare with rectangular pulse. So, we choose the delay resolution same in both the cases and let us see whether velocity resolution for Gaussian pulse has improved or not. So, for that what we do; that means, we choose the same delay resolution for rectangular pulse; that means, here we choose that the T 3 that is same as; T 3 is same as that time I think we have chosen a delay resolution came to be this for one microsecond pulse.

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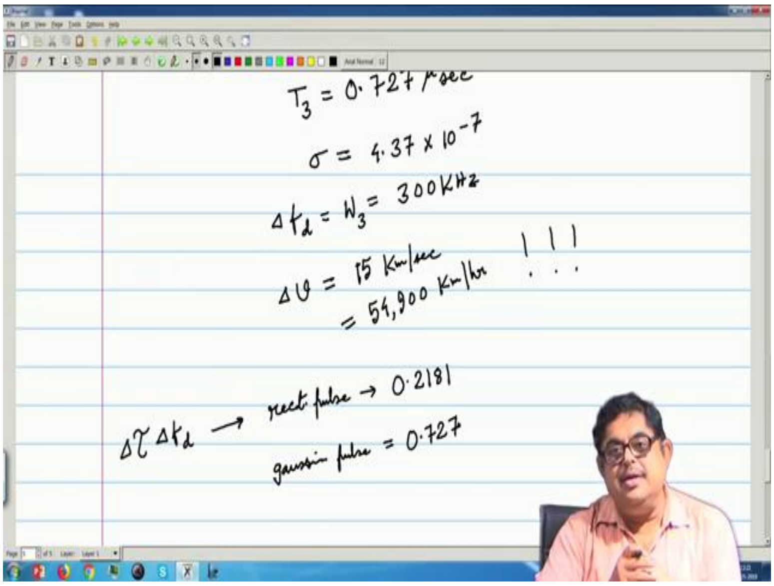

So, if we choose that then our sigma gets fixed, sigma is the parameter of this pulse and that turns out to be 4.37 into 10 to the power minus 7. So, if you now do the calculations that delta f d that will be that is nothing, but your W 3 and that expression we have given if you put that it will turn out to be 300 kilo Hertz.

And then if we get for this Doppler resolution we can get the velocity resolution. So, delta v; so you can put and that becomes 15 kilometre per second. So, a huge improvement 15 kilometre per second means what is 15 kilometre. So, from 15 it is 3 fold improvement more than 3 fold and but still that is 54,900 kilometre per hour. So, that is the resolution for this.

Now, let us also find out this product so; that means, you see that a various pulse shapes that changes the resolutions, but we need to understand that this two sides together we need to do. So, we should have a metric which takes care of both these resolutions. So, is it the product of that it is the product of the two resolutions. So, let us calculate what is the product of these two resolutions; that means, delta tau and delta f d.

So, this product for rectangular pulse you have the data if you do that. So, this will turn out to be 0.2181 and for Gaussian pulse you can from here you can find out this will be 0727. So, does it show anything? Till now, no. Because we have not theoretically made any performance matrix where to get an optimum pulse shape. That means, since both of these resolutions are desirable to me to minimise, but we are saying that we are haphazardly going for various pulse shapes I can calculate, but whether this is better than that I do not know somewhat I tried to do it with the product of the two resolutions you see from 2 it is coming 0.7. So, may be something is there because we know that generally it is called time band width product is same.

So, is it, but here we are saying that product is not same though it is not time band width it is delay and resolution, but they play the same role more or less. So, we are saying that this is not there. So, actually that made the theoreticians to ask for what will be the performance matrix for these because qualitatively what we understand that we should have a ambiguity function which is narrow in both the dimensions and peaky at its zero point or that means, when delay is 0 and doppler shift is 0.

That means, from my reference it will be having a high peak and it will have a very fast roll off and very small width for a particular falling down actually; that means, the roll off is high. So, this how to measure this criteria quantatively? So, that will lead us to a concept of effective this width concept. Basically we can compare then various pulses that what is their one thing is this joint thing and what is their width.

So, basically how much the signal is having the roll off. So, that its basically the area under that curve is very small over that width. So, most of the area it is covering there. So, that type of concept we will see in the next class actually that will lead us to the concept that what is the best choice. Because now without building that theoretical framework for a performance matrix we cannot compare whether these two pulses are good or bad ok. So, that we will see in the next class.

Thank you.